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Global exponential stability via inequality technique for inertial BAM neural networks with time delays ☆

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ABSTRACT

In this paper, the existence and global exponential stability of equilibrium point for inertial BAM neural networks with time delays are discussed. Firstly, by using homeomorphism theory and inequality technique, the LMI-based sufficient condition on the existence and uniqueness of equilibrium point for above inertial BAM neural networks is obtained. Secondly, a LMI-based condition which can ensure the global exponential stability of equilibrium point for the system is obtained by using LMI method and inequality technique. In our results, the boundedness assumption on the activation functions in Ke and Miao (2013) [19,20] is removed. Hence, our result on global exponential stability of equilibrium point for above system is less conservative.

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1. Introduction

The bidirectional associative memory (BAM) neural networks first introduced by Kosko [1–3] are composed of neurons arranged in two layers, the U-layer and V-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer. Through iterations of forward and backward information flows between the two layers, it serves as a two-way associative search for stored bipolar vector pairs and generalizes the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched hetero-associative circuits. Hence, BAM neural networks possess good application prospects in signal processing, image processing, pattern recognition, and associative memories. So far, the stability and periodic solutions of BAM neural network have been widely investigated and many good results have been obtained, for example, see [4–10] and their references in herein.

On the other hand, the inertia can be considered a useful tool that is added to help in the generation of chaos in neural system. In [11], by adding the inertia to one- and two-neuron systems, the chaos of the system was explored. Tani et al. [12,13] added inertia and a nonlinear oscillating resistance to neural equations as a way of chaotically searching for memories in neural networks. In [14], the bifurcation and chaos were considered in a single-inertial neuron model with both time delay and inertial term. In [15–17], the Hopf bifurcation and dynamics of an inertial two-neuron system and a single inertial neuron model were investigated. In [18], the authors studied the dynamical characteristics of a single inertial neuron model with time delay. In [19], the authors considered the stability of equilibrium point for a class of inertial Cohen–Grossberg neural networks and obtained some sufficient conditions in inequality form on the global exponential stability of equilibrium point for the inertial networks under the assumptions that the activation functions satisfied global Lipschitz conditions. Recently, in [20], the authors considered the existence and global exponential stability of periodic solutions for a class of inertial BAM neural networks and obtained some sufficient conditions in inequality form for the existence and global exponential stability of periodic solutions of the inertial BAM neural networks by using some analysis techniques under the assumptions that the activation functions satisfied boundedness conditions and global Lipschitz conditions.

However, the LMI-based global stability results for inertial BAM neural networks under the assumptions that the activation functions only satisfy global Lipschitz conditions have been not found. In thispaper, we are concerned with the following inertial BAM

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neural networks:

$$\begin{cases} \dfrac{d^2u_i(t)}{dt^2} = -\alpha_i\dfrac{du_i(t)}{dt} - a_iu_i(t) + \sum_{j=1}^{m} c_{ij}f_j(v_j(t)) \\ \qquad + \sum_{j=1}^{m} d_{ij}f_j(v_j(t-\tau_{ji})) + I_i, \\ \dfrac{d^2v_j(t)}{dt^2} = -\beta_j\dfrac{dv_j(t)}{dt} - b_jv_j(t) + \sum_{i=1}^{n} p_{ji}g_i(u_i(t)) \\ \qquad + \sum_{j=1}^{m} q_{ji}g_i(u_i(t-\sigma_{ij})) + J_j, \end{cases} \tag{1.1}$$

for $i,j = 1,2,\ldots,n$, where the second derivative is called an inertial term of system (1.1), $\alpha_i > 0$ and $\beta_j > 0$ are constants; $u_i(t)$ and $v_j(t)$ are the state of the ith neurons from the neural field F_U and the jth neurons from the neural field F_V at time t, respectively; $a_i > 0$, $b_j > 0$ denote the rate with which the ith neurons and the jth neurons will reset its potential to the resetting state in isolation when disconnected from the networks and external inputs, respectively; $c_{ij}, d_{ij}, p_{ji}, q_{ji}$ are constants and denote the connection strengths; f_i and g_i denote the activation functions of the jth neurons and the ith neurons at time t, respectively; τ_{ji} and σ_{ij} correspond to the transmission delays and satisfy $0 \le \tau_{ji} \le \tau$ and $0 \le \sigma_{ij} \le \sigma$; I_i and J_j denote the external inputs on the ith neurons from the neural field F_U and the jth neurons from the neural field F_V, respectively.

The initial values of system (1.1) are

$$\begin{cases} u_i(s) = \phi_{ui}(s), & \dfrac{du_i(s)}{dt} = \psi_{ui}(s), \quad -\tau \le s \le 0, \\ v_j(s) = \phi_{vj}(s), & \dfrac{dv_j(s)}{dt} = \psi_{vj}(s), \quad -\sigma \le s \le 0, \end{cases} \tag{1.2}$$

where $\phi_{ui}(s), \psi_{ui}(s), \phi_{vj}(s), \psi_{vj}(s)$ are bounded and continuous functions.

In this paper, our purpose is to obtain LMI-based sufficient conditions for the existence and global exponential stability of equilibrium point of system (1.1) with initial conditions (1.2) by removing the assumption for boundedness on the activation functions in [20].

This paper is organized as follows. Some preliminaries and lemmas are given in Section 2. In Section 3, the sufficient condition is derived for the existence of equilibrium points for inertial BAM neural networks (1.1). In Section 4, the sufficient condition is derived for the global exponential stability of equilibrium points for system (1.1). In Section 5, an illustrative example is given to show the effectiveness of the proposed theory. In Section 6, a conclusion is given to show the improvement on the proof of [19,20].

2. Preliminaries

For the sake of convenience, we introduce some notations and lemmas ass follows.

For arbitrary matrix A, A^T stands for the transpose of A, A^{-1} denotes the inverse of A. If A is a symmetric matrix, $A > 0$ ($A \ge 0$) means that A is positive definite (positive semidefinite). Similarly, $A < 0$ ($A \le 0$) means that A is negative definite (negative semidefinite), $\lambda_m(A), \lambda_M(A)$ denotes the minimum and maximum eigenvalue of a square matrix A respectively. For any $A = (a_{ij})_{m\times m} \in R^{m\times m}$, we define $\|A\| = \sqrt{\lambda_M(A^TA)}$. Let R^m be an m-dimensional Euclidean space, which is endowed with a norm $\|.\|$ and inner product $(.,.)$ respectively. Given column vector $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, the norm is the Euclidean vector norm, i.e., $\|x\| = (\sum_{i=1}^n x_i^2)^{1/2}$. $|\cdot|$ denotes the Euclidean norm in R. $|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$.

We cite the following notations:

$$r_k^* = \max_{1\le i\le n}\sum_{j=1}^n r_{kij}^2, \quad k = 1,2,5,6,9,10; \qquad r_l^* = \max_{1\le j\le n}\sum_{i=1}^n r_{lji}^2, \quad l = 3,4,7,8,11,12.$$

Throughout this paper, we make the following assumptions.

(H_1) The activation functions f_j, g_i $(i,j = 1,2,\ldots,n)$ satisfy Lipschitz condition, i.e., there exist constants $l_j > 0, k_i > 0$ such that for $\forall x, y \in R$,

$$|f_j(x) - f_j(y)| \le l_j|x-y|,$$

$$|g_i(x) - g_i(y)| \le k_i|x-y|.$$

Introducing variable transformation:

$$y_i(t) = \frac{du_i(t)}{dt} + u_i(t), \quad i = 1,2,\ldots,n,$$

$$z_j(t) = \frac{dv_j(t)}{dt} + v_j(t), \quad j = 1,2,\ldots,n,$$

then (1.1) and (1.2) can be rewritten as

$$\frac{du_i(t)}{dt} = -u_i(t) + y_i(t),$$

$$\frac{dy_i(t)}{dt} = -(a_i - \alpha_i + 1)u_i(t) - (\alpha_i - 1)y_i(t) + I_i + \sum_{j=1}^n c_{ij}f_j(v_j(t)) + \sum_{j=1}^n d_{ij}f_j(v_j(t-\tau_{ji})),$$

$$\frac{dv_j(t)}{dt} = -v_j(t) + z_j(t),$$

$$\frac{dz_j(t)}{dt} = -(b_j - \beta_j + 1)v_j(t) - (\beta_j - 1)z_j(t) + J_j + \sum_{i=1}^n p_{ji}g_i(u_i(t)) + \sum_{i=1}^n q_{ji}g_i(u_i(t-\sigma_{ij})) \tag{2.1}$$

and

$$\begin{aligned} u_i(s) &= \phi_{ui}(s), \quad \frac{du_i(s)}{ds} = \psi_{ui}(s), \quad -\tau \le s \le 0, \\ y_i(s) &= \phi_{ui}(s) + \psi_{ui}(s), \quad -\tau \le s \le 0, \\ v_j(s) &= \phi_{vj}(s), \quad \frac{dv_j(s)}{ds} = \psi_{vj}(s), \quad -\sigma \le s \le 0, \\ z_j(s) &= \phi_{vj}(s) + \psi_{vj}(s), \quad -\sigma \le s \le 0, \end{aligned} \tag{2.2}$$

for $i,j = 1,2,\ldots,n$.

Definition 1. The point $(u^*, y^*, v^*, z^*)^T$ with $u^* = (u_1, u_2, \ldots, u_n)^T$, $y^* = (y_1, y_2 \ldots, y_n)^T$, $v^* = (v_1, v_2, \ldots, v_n)^T$, $z^* = (z_1, z_2, \ldots, z_n)^T$ is called an equilibrium point of system (2.1), if

$$\begin{cases} y_i^* - u_i^* = 0, \\ -(a_i - \alpha_i + 1)u_i^* - (\alpha_i - 1)y_i^* + \sum_{j=1}^n c_{ij}f_j(v_j^*) \\ \quad + \sum_{j=1}^n d_{ij}f_j(v_j^*) + I_i = 0, \\ z_j^* - v_j^* = 0 \\ -(b_j - \beta_j + 1)v_j^* - (\beta_j - 1)z_j^* + \sum_{i=1}^n p_{ji}g_i(u_i^*) \\ \quad + \sum_{i=1}^n q_{ji}g_i(u_i^*) + J_j = 0. \end{cases}$$

Lemma 1 (*Fort and Tesi [21]*). *Let $H: R^{2n} \to R^{2n}$ be continuous. Assume that the H satisfies the following conditions:*

1. *$H(u)$ is injective on R^{2n},*
2. *$\|H(u)\| \to \infty$ as $\|u\| \to \infty$. Then H is a homeomorphism.*

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