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Finite-time control of linear systems under time-varying sampling ☆

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ABSTRACT

In this work, the finite-time control problem for sampled-data systems is investigated. The time-varying sampling is not required to be periodic, and the only assumption is that the distance between any two consecutive sampling instants does not exceed a given bound. First, the continuous-time systems with digital control are modeled as continuous-time systems with delayed control input. Then, a novel Lyapunov functional is used to solve the finite-time control problem. At last, sufficient conditions for the existence of state feedback finite-time controller are obtained by solving a set of linear matrix inequalities (LMIs). Different from many existing finite-time control designs of linear systems, this work extends the partial results to the sampled-data systems. The effectiveness of the proposed results is eventually illustrated by a numerical example.

1. Introduction

The concept of finite time stability (FTS) dates back to the sixties in the last century, when this idea was introduced in the literature [1]. A system is said to be finite time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a specified time interval [2,3]. It is important to recall that the concepts of Lyapunov asymptotic stability (LAS) and FTS are independent. Many results on control theory and applications are based on the concept of LAS. However, in many applications, LAS is not enough. This is the case, for instance, when the saturation is present, or when a linear model is used which results from a linearization of nonlinear one around an equilibrium. In these situations, the large values of the state are not allowable, and the FTS could be used. In [2,3], sufficient conditions for FTS and finite-time stabilization of continuous-time linear time-invariant systems are provided, by using state feedback and output feedback respectively. The results are also extended to the discrete-tine case [4]. For linear time-varying systems, some works dealing with FTS control problems have been recently published, see for example [5–7]. Recently, FTS has been further extended to stochastic systems [8], nonlinear systems [9] and time-delay systems [10].

With the rapid development of high-speed computers, modern control systems tend to be controlled by digital controllers, i.e., only the samples of the control input signals at discrete time instants will be employed. This is called sampled-data system, which has been a research subject for about three decades and numerous results have been reported in the literature, see. e.g. [11–14]. Recently, with the ever developing techniques for handing time-delay that frequently occur in various applications (see, e.g., [15,16], and the references therein), a popular approach dealing with sampled-data control problem is to transform the sampling period into a certain time-delay with finite bound [17]. This is also referred to as input delay approach. Then, the sampled-data systems are modeled as continuous-time systems with the delayed-control input and the techniques dealing with time-delay systems are used to study this one. In recent years, this method has been further studied. Using the small gain theorem [18,19] and impulsive system approach [20], the results in [17] are improved and less conservative results are obtained. Recently, [21] further refines those approaches and obtains tighter conditions. Following the input delay approach, the sampled-data stabilization [17], sampled-data H_∞ control [11] and synchronization control [12–14] have been extensively investigated.

However, although the sampled-data systems have been well studied in the recently years, the particular finite-time control of sampled-data systems was not proved before. The main goal of this work is to fill this gap, i.e., to extend the partial results of existing FTS of linear systems to the sampled-data systems. In this work, the sampling period is time-varying and does not exceed a given bound. First, the continuous-time systems with sampling control are modeled as continuous-time systems with delayed control input. Then, a novel Lyapunov functional is constructed to solve the finite-time control problem. By using a free-weighting matrix approach and solving a set of linear matrix inequalities (LMIs), a finite-time sampled-data controller is then obtained.

This work is organized as follows. In Section 2, the definition of finite-time stability is recalled. Then the problem formulation and some preliminaries are formally presented. In Section 3, the

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analysis and synthesis of finite-time control problem for linear systems with sampling control are provided. A set of sufficient conditions for the existence of state feedback controllers guaranteeing FTS of the closed-loop system is obtained. A numerical example is proposed to verify the effectiveness of our method in Section 4 and conclusions are drawn in Section 5.

Notation: The notation is standard. **N** denotes the set of nonnegative integer. $\mathbb{R}$ represents the set of real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, while $\mathbb{R}^{n\times m}$ refers to the set of all $n\times m$ real matrices. A^T represents the transpose of the matrix A, while A^{-1} denotes the inverse of A. For real symmetric matrices X and Y, the notation $X>Y$ (respectively $X\geq Y$) means that the matrix $X-Y$ is positive-definite (respectively, positive semidefinite). I is the identity matrix with appropriate dimensions. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote, respectively, the maximal and minimal eigenvalues of matrix P. For a symmetric matrix, $*$ denotes the entries implied by matrix symmetry.

2. Problem formulation and preliminaries

Consider the following continuous-time linear system:

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector and $u\in\mathbb{R}^m$ is the control input. A and B are constant matrices with appropriate dimensions. The control signal is assumed to be generated by a zero-order hold function with a sequence of hold times $0=t_0<t_1<\cdots<t_k<\cdots$

$$u(t)=u_d(t_k),\quad t\in[t_k,t_{k+1}) \tag{2}$$

where $u_d(\cdot)$ is a discrete-time control signal and $\lim_{k\to\infty}t_k=\infty$. Assume that the sampling instant $t_k, k\in\mathbf{N}$ satisfies

$$t_{k+1}-t_k=h_k\leq h,\quad \forall k\in\mathbf{N} \tag{3}$$

where $h>0$ is a scalar and represents the largest sampling interval. Consider a state-feedback control law of the form

$$u_d(t)=Kx(t_k),\quad t\in[t_k,t_{k+1}) \tag{4}$$

As [21], the digital control law is represented as a delayed control as follows:

$$u(t)=u_d(t_k)=u_d(t-\tau(t)),\quad \tau(t)=t-t_k,\ \ t\in[t_k,t_{k+1})$$

Here we have $0\leq\tau(t)<h$. Then the closed-loop system of (1) and (4) is given by

$$\dot{x}(t)=Ax(t)+BKx(t-\tau(t)),\quad \tau(t)=t-t_k,\ \ t\in[t_k,t_{k+1}) \tag{5}$$

This work concerns with the boundedness of the state of the closed-loop system (5) over a finite time interval for a set of given initial conditions, which is the concept of finite-time stability stated as follows.

Definition 1 (*Finite-time stability, FTS*). Given three positive scalars $\delta_1,\delta_2,T>0$ and a positive-definite matrix $R>0$, the linear system (1) with $u\equiv 0$ is said to be finite-time stability with respect to $(R,\ \delta_1,\ \delta_2,\ T)$ if

$$x^T(0)Rx(0)<\delta_1 \Rightarrow x^T(t)Rx(t)<\delta_2,\quad \forall t\in[0,T]$$

Remark 1. The concept of FTS concerns with the behavior of the system over a fixed finite time interval. The corresponding parameters R, δ_1, δ_2, T can be determined according to the actual situation. It is worth noting that the two concepts of FTS and asymptotical stability are independent. That is to say: a asymptotically stable system may not be FTS, while a system with FTS may not be asymptotic stability [2,3].

The aim of this work is to design a state-feedback control law (4) such that the closed-loop system (5) is FTS with respect to (R,δ_1,δ_2,T), which is also referred to as finite-time stabilization problem.

3. Main results

In this section, sufficient conditions will be established to solve the finite-time stabilization problem under time-varying sampling control. The following lemma presents a set of sufficient conditions for the FTS of the closed-loop system (5), which is fundamental to obtain the main results of this work.

Lemma 1. *Given four positive scalars $\alpha,\delta_1,\delta_2,T>0$ and a positive-definite matrix $R>0$, consider the following Lyapunov functional:*

$$V(x(t))=\sum_{i=1}^{3}V_i(x(t)),\quad t\in[t_k,t_{k+1}) \tag{6}$$

where

$$V_1(x(t))=x^T(t)Px(t)$$
$$V_2(x(t))=(t_{k+1}-t)\int_{t_k}^{t}e^{\alpha(t-s)}\dot{x}^T(s)U\dot{x}(s)\,ds$$
$$V_3(x(t))=(t_{k+1}-t)\begin{bmatrix}x(t)\\x(t_k)\end{bmatrix}^T\Pi\begin{bmatrix}x(t)\\x(t_k)\end{bmatrix}$$
$$\Pi=\begin{bmatrix}\frac{1}{2}(X+X^T) & -X+Y\\ * & -Y-Y^T+\frac{1}{2}(X+X^T)\end{bmatrix}$$

with the matrices $P>0, U>0, X, Y$ determined in the following. If the following relationships hold:

$$\dot{V}(x(t))-\alpha V(x(t))<0,\quad \forall t\in[0,T] \tag{7}$$

$$\delta_1 e^{\alpha T}\frac{\lambda_{\max}(R^{-1/2}PR^{-1/2})}{\lambda_{\min}(R^{-1/2}PR^{-1/2})}\leq\delta_2 \tag{8}$$

then the closed-loop system (5) is FTS with respect to (R,δ_1,δ_2,T).

Proof. Dividing both sides of (7) by $V(x(t))$, and integrating from 0 to $t, t\in[0,T]$, we have

$$\ln\frac{V(x(t))}{V(x(0))}<\alpha t,\quad \forall t\in[0,T] \tag{9}$$

From (9), it follows that

$$V(x(t))<e^{\alpha t}V(x(0)),\quad \forall t\in[0,T] \tag{10}$$

From (6), one has

$$\begin{aligned}V(x(t))&>x^T(t)Px(t)\\&=x^T(t)R^{1/2}R^{-1/2}PR^{-1/2}R^{1/2}x(t)\\&\geq\lambda_{\min}(R^{-1/2}PR^{-1/2})x^T(t)Rx(t)\end{aligned} \tag{11}$$

Moreover, one gets

$$\begin{aligned}e^{\alpha t}V(x(0))&=e^{\alpha t}x^T(0)R^{1/2}R^{-1/2}PR^{-1/2}R^{1/2}x(0)\\&\leq e^{\alpha t}\lambda_{\max}(R^{-1/2}PR^{-1/2})x^T(0)Rx(0)\\&\leq e^{\alpha T}\lambda_{\max}(R^{-1/2}PR^{-1/2})\delta_1\end{aligned} \tag{12}$$

From (10)–(12), one obtains

$$\begin{aligned}x^T(t)Rx(t)&\leq\frac{e^{\alpha t}V(x(0))}{\lambda_{\min}(R^{-1/2}PR^{-1/2})}\\&\leq\delta_1 e^{\alpha T}\frac{\lambda_{\max}(R^{-1/2}PR^{-1/2})}{\lambda_{\min}(R^{-1/2}PR^{-1/2})}\end{aligned} \tag{13}$$

From (8), we have $x^T(t)Rx(t)\leq\delta_2$ for $t\in[0,T]$. It follows from Definition 1 that the closed-loop system is FTS with respect to (R,δ_1,δ_2,T). □