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Control synthesis of Roesser type discrete-time 2-D T–S fuzzy systems via a multi-instant fuzzy state-feedback control scheme

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ABSTRACT

This paper is concerned with further relaxations for control synthesis of the Roesser type discrete-time 2-D T–S fuzzy systems. A novel multi-instant fuzzy state-feedback control scheme and a new multi-instant Lyapunov function, which are homogeneous polynomially parameter-dependent on both the current-time normalized fuzzy weighting functions and the past-time normalized fuzzy weighting functions along two independent directions, are developed to stabilize the underlying 2-D T–S fuzzy system with less conservatism. Because more useful information about both current-time and past-time normalized fuzzy weighting functions is involved into control synthesis, the relaxation quality of control synthesis could be improved significantly. Finally, a numerical example is provided to illustrate the effectiveness of the proposed results.

1. Introduction

Over the past two decades, the two-dimensional (2-D) models [1,2] have attracted considerable attention due to their extensive applications of practical systems, such as those in image data processing and transmission, thermal process and signal filtering. Recently, the 2-D modeling theory is also frequently applied as an analysis tool to some control problems, e.g., iterative learning control [3], repetitive process control [4] and PI control of discrete linear repetitive processes [5], etc. In [6], the problem of stability and l_1-gain analysis for positive 2D T–S fuzzy state-delayed systems in the second FM model has also been investigated. Due to its application in modeling hybrid systems, H_∞ filtering for 2-D Markovian jump systems has also been investigated in [7]. Moreover, the problem of stability analysis of 2-D discrete systems described by the FM second model with state saturation is investigated in [8]. However, it is worth noting that the aforementioned results are only for linear 2-D systems. As is well known, most of the actual 2-D systems belong to nonlinear systems and the above results fail to work in dealing with the problem of control synthesis of nonlinear 2-D systems [9].

On the other hand, the well-known Takagi–Sugeno (T–S) fuzzy model [10] has attracted much attention from scientists of control science, essentially because it can effectively approximate a wide class of general nonlinear systems. By exploiting both the single Lyapunov function and the so-called parallel distributed compensation (PDC) control law [11], quadratic stabilization conditions for T–S fuzzy control systems have been addressed in the past two decades and many extensive results have been given in terms of LMIs, e.g., [12,13]. However, those existing quadratic stabilization conditions are very conservative and thus numerous slack variable methods have been proposed for further releasing its conservatism [14–25]. As far as the problem of control synthesis of discrete-time T–S fuzzy systems is concerned, the usage of both the non-quadratic Lyapunov function and the non-PDC control law has been fully investigated and several kinds of relaxed non-quadratic stabilization conditions have been proposed in the existing literature. It is worth noting that all the above fuzzy control laws are only dependent on the current-time normalized fuzzy weighting functions, for example, the PDC control law [11] is linearly dependent on the current-time normalized fuzzy weighting functions and the non-PDC case [14] is quadratically dependent on the current-time normalized fuzzy weighting functions. Indeed, the usual fuzzy Lyapuonv functions are also linearly dependent on the current-time normalized fuzzy weighting functions. One will naturally raise a question: whether the conservatism could be further reduced if we adopt different design structures of either control laws or fuzzy Lyapuonv functions? Therefore, researches in this area should be very important and useful for researchers and

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designers in this field, which motivates us to carry out positive answers to this question.

Recently, based on the T–S fuzzy modeling method, the problem of control synthesis of nonlinear Roesser type 2-D systems has been investigated by applying some new fuzzy relaxed techniques in [9], i.e., the authors in [9] have extended both the non-quadratic control scheme and the so-called right-hand-side slack variables approach [13] into stabilization of Roesser type 2-D T–S fuzzy systems with less conservatism. More recently, with the purpose of further reducing the conservatism, some improved homogeneous polynomial techniques have been developed in dealing with the problem of relaxed stabilization of Roesser type discrete-time 2-D T–S fuzzy systems [26]. In the above relaxed result [26], both the control scheme and the non-quadratic Lyapunov function are homogeneous polynomially parameter-dependent on current-time normalized fuzzy weighting functions and the usage of them plays an important role in reducing the conservatism. Now, it seems that the challenge is how to significantly improve the quality of the relaxations. For instance, in terms of efficiency, i.e. the ability to obtain less conservative results via designing an efficient approach, or the easiness to extend a given stabilization condition to deal with other similar problems. As stated in [26], there is still some conservatism to be lifted if we change 'something' in the process of control synthesis, such as the form of control law or Lyapunov function, or further exploiting the algebraic property of normalized fuzzy weighting functions, which motivates us to carry out the present work.

In this paper, the problem of relaxed stabilization for Roesser type discrete-time 2-D T–S fuzzy systems will be investigated. To do this, a new multi-instant fuzzy control scheme and a new class of fuzzy Lyapunov function are proposed, which are homogenous polynomially parameter-dependent on both the current-time normalized fuzzy weighting functions and the past-time normalized fuzzy weighting functions. Compared with the existing results in [9,26], more additional decision variables are introduced and consequently more flexibility is generated for further releasing the underlying conservatism. Furthermore, for any given value of the degree of the underlying homogenous polynomials matrix, convergent stabilization conditions are developed by exploiting the algebraic property of multi-instant normalized fuzzy weighting functions, i.e., attaining asymptotically necessary and sufficient stabilization conditions in the convergent sense. Indeed, it is worth noting that the underlying 2-D system's information is propagated along two independent directions and this fact makes the problem of stabilization more complicated than the usual T–S system. In the proof of our main result, several pairs of multi-instant normalized fuzzy weighting functions (along horizonal and vertical directions, respectively) are produced. As a result, the Polya's Theorem should be applied along two independent directions and across different instants, and thus those related coefficients of the homogeneous polynomially parameter-dependent matrices should be elaborately arranged.

The rest of this paper is organized as follows: in Section 1, problem formulation and preliminaries are given in details. In Section 2, less conservative stabilization conditions are developed via a novel multi-instant fuzzy state-feedback control scheme. In Section 3, an numerical example is given to demonstrate the effectiveness of the result proposed in this paper. Finally, some conclusions are given in Section 4.

For simplicity, the notations used are fair standard. For example, $X>0$ (or $X\geq 0$) means the matrix X is symmetric and positive definite(or symmetric and positive semidefinite). For a square matrix E, He(E) is defined as $E+E^T$. X^{T} denotes the transpose of X. The symbol I represents the identity matrix with appropriate dimension. A star $*$ in a symmetric matrix denotes the transposed element in the symmetric position. For a matrix P, min(P) (respectively, max(P)) means the smallest (respectively, largest) eigenvalue of P. $\mathbb{Z}^+$ denotes the set of negative integers $\{0,1,2,\ldots\}$ and $M!$ represents factorial, i.e. $M!=M(M-1)(M-2)\cdots(2)(1)$ for $M\in\mathbb{Z}^+$ with $0!=1$.

2. Problem formulation and preliminaries

2.1. Roesser type discrete-time 2-D T–S fuzzy model [9]

Consider a class of Roesser type discrete-time nonlinear 2-D system described as follows [9]:

$$\boldsymbol{x}^+(s,l)=\mathcal{Z}(\boldsymbol{x}(s,l))+\mathcal{S}(\boldsymbol{x}(s,l))\boldsymbol{u}(s,l) \tag{1}$$

$$\boldsymbol{x}^h(0,l)=f(l),\boldsymbol{x}^v(s,0)=g(s) \tag{2}$$

with

$$\boldsymbol{x}(s,l)=\begin{bmatrix}\boldsymbol{x}^h(s,l)\\ \boldsymbol{x}^v(s,l)\end{bmatrix},\quad \boldsymbol{x}^+(s,l)=\begin{bmatrix}\boldsymbol{x}^h(s+1,l)\\ \boldsymbol{x}^v(s,l+1)\end{bmatrix},$$

where $\boldsymbol{x}^h(\cdot)$ is the horizonal state in $\mathbf{R}^{n_1}$, $\boldsymbol{x}^v(\cdot)$ is the vertical state in $\mathbf{R}^{n_2}$, $\boldsymbol{u}(\cdot)$ is the control input in $\mathbf{R}^m$. $\mathcal{Z}(\cdot)$ and $\mathcal{S}(\cdot)$ are general nonlinear functions satisfying $\mathcal{Z},\mathcal{S}\in C^1$. s, l are two integers in $\mathbb{Z}^+$. $f(l)$ and $g(s)$ are corresponding boundary conditions along two independent directions.

By extending the usual 1-D T–S fuzzy modeling approach into the 2-D model, a Roesser type discrete-time 2-D T–S fuzzy model described by the following rules could be proposed to represent the Roesser type discrete-time nonlinear 2-D system (1) [9]:

If $z_1(s,l)$ is M_{i1}, and…, and $z_L(s,l)$ is M_{iL}, then,

$$\begin{aligned}&\boldsymbol{x}^+(s,l)=A_i\boldsymbol{x}(s,l)+B_i\boldsymbol{u}(s,l), i=1,\ldots,r\\ &\boldsymbol{x}^h(0,l)=f(l),\boldsymbol{x}^v(s,0)=g(s)\end{aligned} \tag{3}$$

with

$$A_i=\begin{bmatrix}A_i^{11} & A_i^{12}\\ A_i^{21} & A_i^{22}\end{bmatrix},\quad B_i=\begin{bmatrix}B_i^1\\ B_i^2\end{bmatrix},$$

where $z_p(s,l)$, for $p=1,\ldots,L$ are the premise variables, M_{ip} is the fuzzy set, r is the number of IF-THEN rules. $A_i^{11}\in\mathbf{R}^{n_1\times n_1}, A_i^{12}\in\mathbf{R}^{n_1\times n_2}, A_i^{21}\in\mathbf{R}^{n_2\times n_1}, A_i^{22}\in\mathbf{R}^{n_2\times n_2}, B_i^1\in\mathbf{R}^{n_1\times m}, B_i^2\in\mathbf{R}^{n_2\times m}$.

By using product of inference, singleton fuzzifier, and center-average defuzzifer, the overall Roesser type discrete-time 2-D T–S fuzzy systems can be expressed as follows:

$$\begin{aligned}&\boldsymbol{x}^+(s,l)=\sum_{i=1}^r h_i(z(s,l))\{A_i\boldsymbol{x}(s,l)+B_i\boldsymbol{u}(s,l)\},\\ &\boldsymbol{x}^h(0,l)=f(l),\boldsymbol{x}^v(s,0)=g(s),\end{aligned} \tag{4}$$

where

$$h_i(z(s,l))=\frac{\mu_i(z(s,l))}{\sum_{i=1}^r\mu_i(z(s,l))},$$

$$\mu_j(z(s,l))=\Pi_{m=1}^L M_{mj}(z(s,l)).$$

Denote $X_r=\sup\{\|\boldsymbol{x}(s,l)\|:r=s+l\}$, and next we give the definition of asymptotical stability of the above system (4).

Definition 1 (*Xie and Zhang [9]*). The Roesser type discrete-time 2-D T–S fuzzy systems (4) is asymptotically stable if $\lim_{r\to\infty}X_r=0$ with the initial and boundary conditions (2).

In this paper, for a matrix X_i, the following notations will be adopted for simplicity:

$$h_i=h_i(z(s,l)),\quad h=(h_1(z(s,l)),\ \ldots,\ h_r(z(s,l))),$$

$$X_z=\sum_{i=1}^r h_iX_i,\quad h(z(s-1,l))=(h_1(z(s-1,l)),\ \ldots,\ h_r(z(s-1,l))),$$

$$X_z^{-1}=\left(\sum_{i=1}^r h_iX_i\right)^{-1},\quad h(z(s,l-1))=(h_1(z(s,l-1)),\ \ldots,\ h_r(z(s,l-1))).$$