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Fault-tolerant dissipative synchronization for chaotic systems based on fuzzy mixed delayed feedback☆

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ABSTRACT

This paper deals with the problem of fault-tolerant dissipativity-based synchronization for chaotic systems. A new synchronization strategy, namely a mixed fuzzy delayed feedback dissipativity-based synchronization strategy, is proposed for chaotic systems with external disturbance. The aim is to design a mixed fuzzy delayed feedback controller such that the synchronization error system is dissipative. Using Lyapunov functional approach, a sufficient condition for the existence of admissible controller is presented. Finally, a practical example is provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

Since the seminal paper of Pecora and Carroll in 1990 [1], much attention has been focused on the synchronization of chaotic dynamical systems. It has been proved that chaos synchronization (CS) has potential applications in many fields, such as secure communication, biomedical, laser physics, chemical reactor [2–4]. So it is not surprising that the many efforts have been devoted to studying the CS of many types of chaotic systems and many important advances have been presented during the past years [5–8]. The purpose of chaos synchronization is to make two systems oscillated in a synchronized manner under drive-response configuration [8]. When the chaotic dynamical systems with external interferences were taken into account, some well-known optimal control schemes, such as H_∞ control, passive control and positive real control, have been presented for the solvability of the CS problem. It is worth emphasizing that each of these control schemes is a special case of the dissipativity-based control scheme [9–19]. Therefore, the importance of the dissipativity-based CS problem has been undoubted. Nevertheless, little effort has been devoted to such an issue, which is one of our main motivations.

It is known that recent discussion on the safety and reliability of dynamical systems has drawn increasingly attention, and consequently the fault-tolerant control (FTC) problem has been an active research topic during the past two decades [20–24]. Roughly speaking, the design of fault-tolerant controllers aims to increase availability capable of maintaining performance in despite the occurrence of faults [25]. Considerable attention has been devoted to solving the FTC problems [20–24,26]. When the faults happen in chaotic systems, the fault-tolerant controllers were designed to guarantee that the CS was achieved in [27,28], regardless of whether the faults happened or not. On the other hand, the use of delayed feedback control in feedback loop has shown advantages for chaotic systems [5,29]. Therefore, a natural question is how to investigate the fault-tolerant dissipativity-based synchronization (FTDBS) problem for chaotic systems by using the delayed feedback method? To the best of our knowledge, such a problem is still seldom researched.

Motivated by the above discussions, in this paper we deal with the FTDBS problem for a class of nonlinear chaotic systems via delayed feedback. A fuzzy-model-based approach is employed to represent the considered chaotic systems. The purpose is to synthesize a mixed delayed fuzzy controller such that the resulting synchronization error system is dissipative. Finally, the well-known Lorenz system is employed to demonstrate the effectiveness of the proposed control method. The main contributions of this paper are two-fold: (1) the addressed FTDBS problem is new for nonlinear chaotic systems; (2) different from [5,29], the fuzzy mixed delayed feedback control scheme is used in the study of CS.

The rest of this paper is organized as follows. In Section 2, we describe the fault-tolerant dissipative synchronization for chaotic

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systems based on the Takagi–Sugeno (T–S) Fuzzy model and formulate the problem. The controller is proposed for chaotic systems with external disturbance and its design is developed in Section 3. The simulation results are provided in Section 4. Finally, conclusions are presented in Section 5.

Notation: The following notations will be used throughout the paper: I is the identity matrix with appropriate dimension. The notation A^T stands for the transpose of the matrix A. The notation "$*$" is used as an ellipsis for terms that are induced by symmetry. The $R^{n\times n}$ represents the set of all $n\times n$ real matrices. $M<0$ is used to denote a symmetric negative definite matrix. If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

In this paper, we consider the following nonlinear chaotic system:

$$\dot{x}(t)=g(x(t),t),$$

where $x(t)\in R^n$ is the state vector; the nonlinear function $g(x(t),t)$ is known and continuous. Inspired by [6, 30–32], the considered chaotic system can be represented as the following continue-time T–S fuzzy model (Σ) :

Fuzzy Rule i:

IF $\vartheta_1(t)$ is μ_{i1} and... $\vartheta_s(t)$ is μ_{is} THEN

$$\dot{x}(t)=A_i x(t)+\eta_i(t) \tag{1}$$

$$z(t)=Dx(t), \quad i=1,\dots,r, \tag{2}$$

where $x(t)\in R^n$, $z(t)\in R^n$ are the state vectors, $A_i\in R^{n\times n}$ and $D\in R^{n\times n}$ are known constant matrices with appropriate dimensions, $\eta_i(t)\in R^n$ represents a bias term generated by the fuzzy modeling procedure, $\vartheta_j (j=1,\dots,s)$ is the premise variable, $\mu_{ij} (j=1,\dots,s)$ is the fuzzy set that is formed by a membership function, r is the number of IF–THEN rules, and s is the number of premise variables.

In this work, we use a fuzzifier, fuzzy inference, and weighted average defuzzifier, the system (1) and (2) is inferred as follows:

$$\dot{x}(t)=\sum_{i=1}^{r} h_i(\vartheta(t))\left[A_i x(t)+\eta_i(t)\right] \tag{3}$$

$$z(t)=Dx(t) \tag{4}$$

where

$$\vartheta(t)=\left[\vartheta_1(t),\dots,\vartheta_s(t)\right], \quad h_i(\vartheta(t))=\frac{\varpi_i(\vartheta(t))}{\sum_{i=1}^{r}\varpi_j(\vartheta(t))},$$

$$\varpi_i(\vartheta(t))=\prod_{j=1}^{s}\mu_{ij}\left(\vartheta_j(t)\right),$$

with $\mu_{ij}\left(\vartheta_j(t)\right)$ being the grade of membership of $\vartheta_j(t)$ in μ_{ij}. Suppose $\varpi_i(\vartheta(t))\ge 0$, $i=1,2,\dots,r$, $\sum_{i=1}^{r}\varpi_i(\vartheta(t))>0$ for all t. Therefore, for $i=1,2,\dots,r$, and for all t, $h_i(\vartheta(t))$ can be deemed as the normalized weight of each IF–THEN rule and it satisfies

$$h_i(\vartheta(t))\ge 0, \qquad \sum_{i=1}^{r} h_i(\vartheta(t))=1.$$

In this paper, the system (3) and (4) is considered as a drive system. The synchronization problem of system (3) and (4) is here addressed under the drive-response configuration. According to the drive-response concept, the controlled fuzzy response system is described by the following rules:

Fuzzy Rule i:

IF $\vartheta_1(t)$ *is* μ_{i1} and $\dots\vartheta_s(t)$ *is* μ_{is} *THEN*

$$\dot{\hat{x}}(t)=A_i\hat{x}(t)+\eta_i(t)+u(t)+G_i d(t), \tag{5}$$

$$\hat{z}(t)=D\hat{x}(t)+Eu(t) \tag{6}$$

where $\hat{x}(t)\in R^n$ is the state vector of the response system, $\hat{z}(t)\in R^n$ is the state vector of the restrain system, $u(t)\in R^n$ is the control input, $d(t)\in R^k$ is the external disturbance, and $G_i\in R^{n\times k}$, $E\in R^{n\times n}$ are known constant matrices. The system can be further inferred as

$$\dot{\hat{x}}(t)=\sum_{i=1}^{r} h_i(\vartheta(t))\left[A_i\hat{x}(t)+\eta_i(t)+u(t)+G_i d(t)\right] \tag{7}$$

$$\hat{z}(t)=D\hat{x}(t)+Eu(t). \tag{8}$$

Define the synchronization error $e(t)=\hat{x}(t)-x(t)$, and $\tilde{z}(t)=\hat{z}(t)-z(t)$, one can easily obtain the resulting synchronization error system as follows:

$$\begin{aligned}\dot{e}(t)=&\sum_{i=1}^{r} h_i(\vartheta(t))\left[A_i\hat{x}(t)+\eta_i(t)+u(t)+G_i d(t)\right]\\ &-\sum_{i=1}^{r} h_i(\vartheta(t))\left[A_i x(t)+\eta_i(t)\right]\\ =&\sum_{i=1}^{r} h_i(\vartheta(t))\left[A_i e(t)+u(t)+G_i d(t)\right],\end{aligned} \tag{9}$$

$$\tilde{z}(t)=\hat{z}(t)-z(t)=De(t)+Eu(t). \tag{10}$$

In this paper, we consider the following mixed delayed controller can be constructed via the parallel distributed compensation, which is described by the following rules:

Fuzzy Rule j:

IF $\vartheta_1(t)$ *is* μ_{ij} and $\dots\vartheta_s(t)$ *is* μ_{ij} *THEN*

$$u(t)=K_j\left[e(t)-\xi_1 e(t-\tau)-\xi_2\int_{t-\tau}^{t} e(s)\,ds\right],$$

where $K_j\in R^{n\times n}$ is the gain matrix of the controller for the fuzzy rule j; ξ_1 and ξ_2 are regulatory factors which are used to regulate the strength of delayed feedback. In a similar way the fuzzy controller can be inferred as

$$u(t)=\sum_{j=1}^{r} h_j(\vartheta(t))K_j\left[e(t)-\xi_1 e(t-\tau)-\xi_2\int_{t-\tau}^{t} e(s)\,ds\right]. \tag{11}$$

It is noted that chaotic system is sensitive to initial conditions, where it is quite common that the actuator suffers from failures. This case will be considered in this paper, and the following actuator fault model derived from [33] is employed. To be more specific, for the control input $u(t)$, we use $u_f(t)$ to represent the signal sent from the actuator, and satisfies

$$u_f(t)=Fu(t) \tag{12}$$

where $F=\mathrm{diag}\left\{f^1,f^2,\dots,f^n\right\}$ is the actuator faults function matrix, and $0\le f_-^l\le f^l\le f_+^l\le 1, l=1,2,\dots,n$ with known real constants f_-^l and f_+^l.

By setting

$$f_0^l=\frac{f_-^l+f_+^l}{2}, \quad m^l=\frac{f^l-f_0^l}{f_0^l}, \quad h^l=\frac{f_+^l-f_-^l}{f_+^l+f_-^l},$$

then, one can readily obtain that

$$F=F_0(I+M), \quad |M|\le H\le I, \tag{13}$$

where

$$F_0=\mathrm{diag}\left\{f_0^1,f_0^2,\dots,f_0^n\right\},$$

$$|M|=\mathrm{diag}\{|m^1|,|m^2|,\dots,|m^n|\},$$

$$H=\mathrm{diag}\left\{h^1,h^2,\dots,h^n\right\}.$$