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New global exponential stability conditions for inertial Cohen–Grossberg neural networks with time delays ☆

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ABSTRACT

In this paper, global exponential stability of inertial Cohen–Grossberg neural networks with time delays is investigated. By using Homeomorphism theorem and inequality technique, a LMI-based global exponential stability condition and inequality form global exponential stability condition are obtained for the above neural networks. In our result, the assumptions for the differentiability and monotonicity on the behaved functions in Ke and Miao (2013) [23] are removed. Thus our results are less conservative than those obtained in Ke and Miao (2013) [23]. Hence, we obtain new global exponential stability for this neural network.

1. Introduction

In recent decades, much attention has been devoted to the studies of artificial neural networks because of the fact that neural networks can be applied to signal processing, image processing, pattern recognition, control and optimization problems. In particular, the Cohen–Grossberg neural network proposed in 1983 [1] has been a focal research subject. In the past years, the global stability problem for a class of Cohen–Grossberg neural networks

$$\frac{dx_i(t)}{dt} = -\alpha_i(x_i(t))\left[h_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + I_i\right] \tag{1.1}$$

has received much research attention, and many interesting and good results have been obtained, see [2–11,25,26–29].

On the other hand, some researchers investigated inertial neural networks and obtained some results. For example, Liu et al. [12,13] found chaotic behavior of the inertial two-neuron system with time through numerical simulation, and gave that the system will lose its stability when the time delay is increased and will rise a quasi-periodic motion and chaos under the interaction of the periodic excitation. Wheeler and Schieve [14] studied an inertial continuous-time, Hopfield effective-neuron system which is shown to exhibit chaos, this system is of the following form:

$$x_1'' = -a_{11}x_1' - a_{12}x_1 + a_{13}\tanh(x_1) + a_{14}\tanh(x_2),$$

$$x_2'' = -b_{11}x_2' - b_{12}x_2 + b_{13}\tanh(x_1) + b_{14}\tanh(x_2),$$

Babcocka and Westervelt [15] investigated the electronic neural networks with added inertial and found that when the neuron couplings are of an inertial nature, the dynamics can be complex, in contrast to the simpler behavior displayed when they are of the standard resistor–capacitor variety. Juhong and Jing [16] considered an inertial four-neuron delayed bidirectional associative memory model. Weak resonant double Hopf bifurcations are completely analyzed in the parameter space of the coupling weight and the coupling delay by the perturbation-incremental scheme. In [17], authors studied a kinematical description of traveling waves in the oscillations in the networks which is extended to networks with inertia. Liu et al. [18,19] investigated the Hopf bifurcation and dynamics of an inertial two-neuron system or in a single inertial neuron mode. Zhao et al. [20] studied the stability and the bifurcation of a class of inertial neural networks. The authors Ke and Miao [21,22] investigated stability of equilibrium point and periodic solutions in inertial BAM neural networks with time delays and unbounded delays under the assumptions that the activation functions satisfy global Lipschitz condition and boundedness condition, respectively.

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In [23], the authors studied the following inertial Cohen–Grossberg neural networks with time delays:

$$\frac{d^2x_i(t)}{dt^2} = -\beta_i\frac{dx_i(t)}{dt} - \alpha_i(x_i(t))\left[h_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) - \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_{ij})) + I_i\right], \tag{1.2}$$

for $i=1,2,\ldots,n$, where the second derivative is called an inertial term of system (1.2), $\beta_i>0$ are constants, $x_i(t)$ denotes the states variable of the ith neuron at the time $t, \alpha_i(\cdot)$ denotes an amplification function; $h_i(\cdot)$ is the behaved function, a_{ij} and b_{ij} are the connection weights of the neural networks; f_j denotes the activation function of jth neuron at time $t;\tau_{ij}$ is the time delay of jth neuron at time t and satisfies $0\le\tau_{ij}\le\tau; I_i$ denotes the external inputs on the ith neuron at time t.

The initial values of system (1.2) are

$$x_i(s)=\phi_i(s),\quad \frac{dx_i(s)}{ds}=\psi_i(s),\quad -\tau\le s\le 0, \tag{1.3}$$

where $\phi_i(s)$ and $\psi_i(s)$ are the bounded and continuous functions, respectively.

From the viewpoints of mathematics and physics, the system (1.2) is a class of nonlinear second-order dynamical system where $\alpha_i>0$ is a damping coefficient, then the system (1.2) can be considered as a model overdamped. However, in some practical problems, we need to consider the existence and stability of the system when it has damping. For example, pendulum equation with dissipation term

$$\frac{d^2x(t)}{dt^2} = -\alpha\frac{dx(t)}{dt} - \beta x - \gamma\ \sin\ t,$$

and forced Duffing equation

$$\frac{d^2x(t)}{dt^2} = -\alpha\frac{dx(t)}{dt} - x(\beta x+\gamma x^2) + \delta\ \cos\ vt,$$

which have applied background.

In [23], the global exponential stability of inertial Cohen–Grossberg neural networks with time delays is investigated. By using Homeomorphism theory and inequality technique, some sufficient conditions in inequality form which can ensure the global exponential stability of the system are obtained under the assumptions that the behaved functions satisfy differentiability condition and monotonicity condition, and the activation functions satisfy global Lipschitz condition.

So in this paper, our purpose is to establish a LMI-based condition and an inequality form condition on global exponential stability for system (1.2) under the assumptions that the behaved functions do not satisfy the differentiability condition and monotonicity condition, while only satisfies global Lipschitz condition and the activation functions do not satisfy boundedness condition, while only satisfy global Lipschitz condition. Thus our global exponential result will be less conservative than those obtained in [23]. Since the assumption on the behaved functions in our paper is different from that in [23], then more effective technique will be introduced to solve the stability problem of system (1.2).

The paper is organized as follows. In the next section, we introduce some preliminaries. In Section 3, the LMI-based sufficient condition is derived for the global exponential stability of inertial Cohen–Grossberg neural networks with time delays by constructing a suitable Lyapunov function and using some inequality techniques.

2. Preliminaries

For arbitrary matrix A, A^T stands for the transpose of A, A^{-1} denotes the inverse of A. If A is a symmetric matrix, $A>0$ ($A\ge 0$) means that A is positive definite (positive semidefinite). Similarly, $A<0$ ($A\le 0$) means that A is negative definite (negative semidefinite), $\lambda_m(A), \lambda_M(A)$ denotes the minimum and maximum eigenvalue of a square matrix A respectively. For any $A=(a_{ij})_{m\times m}\in R^{m\times m}$, we define $\|A\|=\sqrt{\lambda_M(A^TA)}$. Let R^m be an m-dimensional Euclidean space, which is endowed with a norm $\|\cdot\|$ and inner product $(\cdot,\cdot)$, respectively. Given column vector $x=(x_1,x_2,\ldots,x_m)^T\in R^m$, the norm is the Euclidean vector norm, i.e., $\|x\|=(\sum_{i=1}^m x_i^2)^{1/2}$. $|\cdot|$ denotes the Euclidean norm in R. $|x|=(|x_1|,|x_2|,\ldots,|x_m|)$. We cite the following notations:

$$r_1=\max_{1\le i\le n}\sum_{j=1}^{n} a_{ij}^2,\quad r_2=\max_{1\le i\le n}\sum_{j=1}^{n} b_{ij}^2,$$
$$r_3=\max_{1\le i\le n}\{p_{1i}^2, p_{2i}^2(1-\beta_i)^2+\overline{\alpha_i}^2\}.$$

Throughout this paper, we make the following assumptions:

(H_1) For each $i=1,2,\ldots,n$, functions $\alpha_i(x)$ are continuous and bounded, satisfy $0<\underline{\alpha_i}\le\alpha_i(x)\le\overline{\alpha_i}$, for all $x\in R$.

(H_2) For each $i=1,2,\ldots,n$, there exist positive constants h_i such that for $\forall x,y\in R$,

$$|h_i(x)-h_i(y)|\le h_i|x-y|.$$

(H_3) The activation functions f_j satisfy the Lipschitz condition, i.e., there exists constant $l_j>0$ such that for $j=1,2,\ldots,n$,

$$|f_j(x)-f_j(y)|\le l_j|x-y|,\quad x,y\in R.$$

Let variable transformation: $y_i(t)=dx_i(t)/dt+x_i(t), i=1,2,\ldots,n$, then (1.2) and (1.3) can be rewritten as

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -x_i(t)+y_i(t), \\ \dfrac{dy_i(t)}{dt} = -(1-\beta_i)x_i(t)-(\beta_i-1)y_i(t)-\alpha_i(x_i(t)) \\ \quad\left[h_i(x_i(t))-\displaystyle\sum_{j=1}^{n} a_{ij}f_j(x_j(t))-\sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_{ij}))+I_i\right] \end{cases} \tag{2.1}$$

and

$$\begin{cases} x_i(s)=\phi_i(s), & \dfrac{dx_i(s)}{dt}=\psi_i(s), \\ y_i(s)=\phi_i(s)+\psi_i(s), \end{cases} \tag{2.2}$$

for $-\tau\le s\le 0, i=1,2,\ldots,n$.

Let $x^*=(x_1^*,x_2^*,\ldots,x_n^*)^T, y^*=(y_1^*,y_2^*,\ldots,y_n^*)^T$.

Definition 1. The point $((x^*)^T,(y^*)^T)^T$ is called an equilibrium point of system (1.2) if it satisfies the following equations for $i=1,2,\ldots,n$:

$$\begin{cases} -x_i^*+y_i^*=0 \\ h_i(x_i^*)-\displaystyle\sum_{j=1}^{n} a_{ij}f_j(x_j^*)-\sum_{j=1}^{n} b_{ij}f_j(x_j^*)+I_i=0. \end{cases} \tag{2.3}$$

Lemma 1 (*Forti and Tesi [24]*). *Let $H:R^n\to R^n$ be continuous. Assume that the H satisfies the following conditions:*

1. *$H(u)$ is injective on R^n,*
2. *$\|H(u)\|\to\infty$ as $\|u\|\to\infty$.*

Then H is a homeomorphism.