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Global synchronization of nonlinear coupled complex dynamical networks with information exchanges at discrete-time

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ABSTRACT

This paper studies the problem of synchronization for a class of complex networks with discrete-time couplings. The intrinsic local dynamical behaviors of the nodes in the complex networks are varied continuously while the manners of information interaction between every two different nodes are discrete time-varying rather than proceeded continuously, that is, the communications of the nodes only active at some discrete instants. Similar to the sampled data control systems, we convert the discrete time coupling issue into an effective time-varying delayed coupling network. By constructing the Lyapunov function skillfully, sufficient conditions are derived to guarantee the realization of the synchronization pattern for all initial values based on the Lyapunov stability theorem and linear matrix inequalities. What is more, the maximum allowable sampling period for communication is obtained through a optimization problem. Numerical simulations are also exploited to demonstrate the effectiveness and validity of the main result.

1. Introduction

Sampled-data systems have attracted extensively and significantly attention over the past few decades for the widely applications in many information exchange and interaction situations, such as [1,2]. In system science, a sampled-data system is a control system in which continuous-time plant is controlled with a sampler and a hold digital device. Under periodic sampling, the sampled-data system is not only time-varying but also periodic, thus, it may be modeled by a simplified discrete-time system obtained by discretizing the plant. The sampler and the hold device are usually assumed to be synchronized in time and have a fixed sampling period. The analysis of sampled-data systems incorporating full-time information leads to challenging control problems with a rich mathematical structure.

Generally speaking, there are three approaches to tackle the synchronization and control problems of the sampled-data systems, which capture the special characteristics of this communication manner:

(1) The stability of the sampled-data system problem is converted into a finite-dimensional discrete-time issue based on *lifting*, on condition that the sampling intervals are already known [3,4];
(2) By a delayed control input, the discrete time sampled-data system is modeled as a continuous-time system [5,6];
(3) Using a time-varying periodic Lyapunov function, the sampled-data systems can be modeled impulsively [7,8].

Recently, the synchronization problems of the sampled-data systems are well investigated in the literature, such as [1,9–13]. Chen et al. studied the input–output stability of a continuous-time plant with a discrete-time controller in paper [1] by using the usual arrangement of periodic sampling and zero-order bold. The problems of exponential stability and stabilization for sampled data control systems with state feedback were discussed in [9], Zhu and Wang derived less conservative stability and stabilization results by an improved delay approach and a novel piecewise Lyapunov function as well as a convex combination technique. And in paper [11], Jia et al. discussed the synchronization of the networked multi-agent system with sampling communications. Different topologies on the maximum allowable sampling period for communications between agents were studied and the impacts of the topological structure on the maximum allowable sampling period on agent communications were reported. It was shown that this problem can be alleviated if a strictly causal stable continuous-time filter (e.g., anti-aliasing filter) was introduced prior to the sampler.

As well all know, the synchronization and control on complex dynamical networks have attracted too many scientists' attention from various fields such as sociology, biology, mathematics and physics

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[14–21]. Although the well-studied sampled-data systems are similar to the complex dynamical networks with discrete-time communication, the theoretical results on the synchronization and consensus of the sampled-data systems cannot be usually analyzed if the network with discrete-time information interchanges directly. The reason lies that the dynamics of the nodes in complex dynamical networks are always obtained in the form of nonlinear. For this purpose, Wang et al. in [22] did a good job. They considered the synchronization of continuous dynamical networks with discrete-time communications. Synchronization criterion was derived in terms of linear matrix inequalities by choosing a piecewise Lyapunov–Krasovskii functional to capture the characteristics of the discrete communication instants and using a convex combination technique with an upper bound for the communication intervals.

Inspired by the above works, we study the synchronization problem for a kind of nonlinear coupled complex dynamical networks with discrete time information exchanges in this paper. We emphasize that the dynamics of nodes are continuous and nonlinear, while the information interchange manners between each two nodes are discrete. Some global synchronization criteria are derived by skillfully choosing a concise piecewise Lyapunov–Krasovskii function, which makes the conditions in this paper relatively easier to implement than those in [22]. The Lyapunov Stability Theorem and the LMI are applied to ensure the realization of the asymptotical synchronization for all initial values. Moreover, the maximum allowable sampling period for communication is obtained through an optimization problem. At the same time, we present two corollaries to show the innovation and comprehensiveness on our main results compared to previous works. At last, numerical simulations are also exploited to demonstrate the effectiveness and validity of the main result.

The main improvements of this paper by considering [9,11,22] relatively are as the following aspects: (1) though we discuss a more complicated network model, there are 9 free-matrices needed to satisfy the LMI for the global synchronization rather than 10 in [22]; (2) the piecewise Lyapunov–Krasovskii function in this paper is selected simpler than those in previous works; (3) the dimensions of the criteria in the form of LMI are less than those in [22], so we reduce the complexity of the computing. Above all, one can know that it is easier to implement the global synchronization of the network with discrete communications asymptotically.

The rest of the paper is organized as follows. In Section 2, we present some preparatories and the complex continuous dynamical network with discrete communications. In Section 3, we analyze the synchronization of the network based on the Lyapunov Stability Theorem and linear matrix inequalities. Numerical simulations are given to verify our theoretical results in Section 4. Concluding remarks are drawn in Section 5.

Notations: The mark A^T denotes the transport of the matrix A. R^n denotes the n-dimensional Euclidean space. $R^{n\times n}$ are $n\times n$ real matrices. $diag\{\cdots\}$ stands for a diagonal matrix. The sign $\|\cdot\|$ stands for the Euclid norm of the matrix or the vector. A symmetric real matrix A is positive definite (semi-definite) if $x^TAx>0\ (\ge 0)$ for all nonzero x we denote this as $A>0 (A\ge 0)$. I stands for the identity matrix with proper dimension. We use $\lambda_{max}(\cdot)$ to denote the maximum eigenvalue of a real symmetric matrix. Some properties of the Kronecker product are used: $(A\otimes B)^T=A^T\otimes B^T$; $(A\otimes B)(C\otimes D)=(AC\otimes BD)$. In a symmetrical matrix $A=\begin{pmatrix} a_{11} & a_{12} \\ * & a_{22}\end{pmatrix}$, where $*=a_{12}^T=a_{21}$ stands for the symmetrical term. The dimension of these vectors and matrices will be cleared in the context.

2. Preliminaries and model description

In this section, some preliminaries, such as definitions and lemmas, will be firstly given, which are necessary throughout the paper. Then, we will present the complex dynamical network with continuous dynamics and discrete couplings, it will be converted into the complex network with time-varying delayed coupling by an effective assumption.

Definition 1 (*Zhang et al. [23]*). Synchronization manifold

$S=\{(x_1^T,\dots,x_N^T)^T\in R^{N\times n}: x_i=x_j,\ \ i,j=1,2,\dots,N\},$

where $x_i=(x_i^1,\dots,x_i^n)^T\in R^n$ for $i=1,2,\dots,N$.

Definition 2 (*He and Cao [24]*). Define $T(R,b)=\{$ matrices with entries such that the sum of the entries in each row is equal to $b\}$ for some $b\in R$.

Definition 3. *Lip* :(l) denotes the class of Lipschitz continuous functions with the Lipschitz constant l.

Lemma 1 (*Moon et al. [26]*). *(Schur Complement) The following linear matrix inequality (LMI)*

$$\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x)\end{pmatrix}>0$$

is equivalent to one of the following conditions

(1) $Q(x)>0, R(x)-S(x)^TQ(x)^{-1}S(x)>0;$
(2) $R(x)>0, Q(x)-S(x)R(x)^{-1}S(x)^T>0,$

where $Q(x)$ and $R(x)$ be symmetric matrices and $S(x)$ be a matrix with suitable dimensions.

Lemma 2 (*Wu and Chua [25]*). *Let G be an $N\times N$ matrix and $G\in T(R,b)$, define the matrix $H_0\in R^{(N-1)\times(N-1)}$ as $H_0=\tilde{M}G\Xi$, which satisfies $\tilde{M}G=H_0\tilde{M}$, where the matrices $\tilde{M}\in R^{(N-1)\times N}$ and $\Xi\in R^{N\times(N-1)}$ are defined as*

$$\tilde{M}=\begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & & & & & \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{pmatrix},\quad \Xi=\begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & 1 & \cdots & 1 & 1 \\ & & \cdot & \cdot & \cdot & \cdot \\ & & & & 1 & 1 \\ & & & & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

Lemma 3. *For any vectors $x,y\in R^n$, scalar $\epsilon>0$, and positive definite matrix $Q\in R^{n\times n}$, the following inequality holds:*

$2x^Ty\le \epsilon x^TQx+\epsilon^{-1}y^TQ^{-1}y.$

In general, the i-th node equation of the nonlinear coupled complex dynamical network with information communications at discrete-time can be described as

$$\dot{x}_i(t)=Ax_i(t)+f(x_i(t),t)+c\sum_{j=1}^{N}g_{ij}\Gamma H(x_j(t_k)),\quad t\in[t_k,t_{k+1})\ \ i=1,\dots,N, \tag{1}$$

where N is the number of nodes in the network. $t_k,\ k=0,1,2,\dots$, are the communication instants, and they satisfy the nondecreasing property as $0\le t_0<t_1<\cdots<t_k<\cdots$ and $\lim_{k\to+\infty}t_k=+\infty$. $x_i(t)=(x_i^1,\dots,x_i^n)^T\in R^n$ are the state variables of the i-th node, $f: R^n\times R^+\to R^n$ is a continuously nonlinear vector-valued function with initial value $f(0,t)=0\in R^n$ for any $t\ge t_0$. $A\in R^{n\times n}$ is a constant matrix. Importantly, $Ax_i(t)+f(x_i(t),t)$ describes the local dynamics of i-node in the network which is obviously continuous for any time. $\Gamma=diag\{r_1,r_2,\dots,r_n\}\in R^{n\times n}$ is inner-coupling matrix with $r_i\ge 0,\ i=1,2,\dots,n.$ $c\in R$ is the constant coupling strength. $G=(g_{ij})\in R^{N\times N}$ is the coupling matrix, and g_{ij} is defined as follows: if there is a connection from node i to node $j (j\ne i)$, then $g_{ij}>0$, otherwise, $g_{ij}=0$. We assume that the coupling matrix satisfies