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Stochastic finite-time state estimation for discrete time-delay neural networks with Markovian jumps

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ABSTRACT

This paper investigates the problem of stochastic finite-time state estimation for a class of uncertain discrete-time Markovian jump neural networks with time-varying delays. A state estimator is designed to estimate the network states through available output measurements such that the resulted error dynamics is stochastically finite-time stable. By stochastic Lyapunov–Krasovskii functional approach, sufficient conditions are derived for the error dynamics to be stochastic finite-time stable. The desired state estimator is designed via linear matrix inequality technique. Simulation examples are provided to illustrate the effectiveness of the obtained results.

1. Introduction

In the past few decades, neural networks have attracted considerable attention due to a variety of their applications, including signal processing, pattern recognition, target tracking, static image processing and associative memories [1–3]. Meanwhile, time delay, referred as a typical characteristic of the signal transmission between neurons, has been recognized as one of the major causes of instability and poor performance of network dynamics. Therefore, a great deal of works has been done for various types of neural networks with parameter uncertainties and time-delays, such as asymptotic stability, exponential stability, passivity analysis, and the existence of an equivalent point [4–9]. More results on the topic could be found in [10–12] and the references therein.

On the other hand, the information latching often happens in neural networks, which means that a neural network may have finite modes that may switch from one to another at different time. Therefore, the class of neural networks can be disposed by the theoretical framework of Markovian jump systems since the switching between different modes may be governed by a Markov chain. Markovian jump neural networks with time-delays have attracted a lot of research interests in mathematics and control communities, and many results have been reported in the literature, see, for example, [13,14] and the references therein. For instance, exponential stability criteria were derived for time-delay recurrent neural networks with Markovian jumping parameters in [15]. The passivity analysis was considered for discrete-time neural networks with Markovian jumps and mixed time-delays in [16]. Applying the input delay approach and linear matrix inequality (LMI) technique, the authors [17] studied the problem of sampled-data synchronization for Markovian jump neural networks with time-varying delay and variable samplings.

It should be pointed out that all the above-mentioned works assume that the considered neuron states are fully available. However, the assumption cannot always be satisfied in real applications. Therefore, it is important and necessary to estimate the neuron state through available output measurements to make full use of neural networks in practice. For instance, the authors [18] addressed state estimation for delayed neural networks employing the LMI technique, in which the neuron activation function of the measurement nonlinearity satisfies the standard Lipschitz condition. In [19], the state estimation problems were investigated for delayed neural networks with Markovian jump parameters under the same Lipschitz condition for the measurement nonlinearity. Very recently, the problem of state estimation was studied for continuous-time neural networks with Markovian jumps and distributed delays under the assumption that the neuron activation function and the

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nonlinearity satisfy the type of sector-bounded conditions in [20]. For more works of the literature related to state estimation of neural networks, readers can refer [21,22] and the references therein.

It is well known that classical control theory mainly tackles the asymptotic behavior of the system trajectories over an infinite interval. In practice, however, main attention may be related to the behavior of the dynamics over a fixed finite time interval, such as keeping the acceptable values in a prescribed bound in the presence of saturations [23]. In [24], finite-time stability or short time stability was first introduced in order to deal with the transient performance of control systems. Then, employing Lyapunov function approach and LMI technique, some important results related to the topic have been extended to finite-time stability, finite-time boundedness and finite-time stabilization for continuous- or discrete-time systems. For instance, the finite-time control problem was studied for discrete-time linear systems in [25]. In [26], the finite-time boundedness analysis was conducted for continuous-time neural networks with Markovian jumping parameters and time-delays. Recently, finite-time state estimation was investigated in [27] for continuous-time neural networks with time-varying delays by applying an augmented Lyapunov–Krasovskii functional. More results related to finite-time stability, finite-time boundedness and finite-time H_∞ control could be found in [28–35]. However, to date and to the best of our knowledge, the finite-time state estimation problem for discrete-time delayed neural networks with Markovian jumps has not been fully investigated in the literature yet and it motivates us to carry out the present study.

The main contribution of the paper is that we provide sufficient conditions for robust stochastic finite-time state estimation of uncertain discrete-time neural networks with Markovian jumps and time-varying delays. More specifically, under the assumption that the activation functions satisfying the sector-bounded conditions that are more general than the commonly used Lipschitz ones, a state estimator is designed such that the state of the error dynamics is bounded in mean-square sense in a specified time interval. Based on the stochastic Lyapunov–Krasovskii functional approach, sufficient criteria on stochastic finite-time state estimation are derived for the nominal and uncertain error dynamics. The conditions are reduced to LMIs-based feasibility problems. The rest of this paper is outlined as follows. Section 2 formulates the problem to be studied, and recalls some preliminaries. The main results are provided in Section 3. Numerical examples are given in Section 4 to illustrate the effectiveness of the obtained results, and the conclusions are drawn in Section 5.

Notations: In the sequel, $\mathbb{N}^+$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ represent the set of nonnegative integers, the sets of n component real vectors and $n\times m$ real matrices, respectively. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure. A^T stands for the transpose of the matrix or vector A, the symbol $*$ represents the transposed elements in the symmetric positions of a matrix, I represents the identity of appropriate dimension, I_n denotes the identity of n dimension, and diag$(\cdots)$ stands for a block-diagonal matrix. Matrices, without special stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the discrete-time time-delay neural network with Markovian jumps and uncertainties described as follows:

$$\begin{aligned} &x(k+1)=\mathcal{A}(r_k)x(k)+\mathcal{B}(r_k)f(x(k))+\mathcal{C}(r_k)g(x(k-h(k))),\\ &y(k)=H(r_k)x(k),\\ &x(j)=\varphi(j),\quad j\in\{-h_M,\ldots,-1,0\}, \end{aligned} \tag{1}$$

where $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T\in\mathbb{R}^n$ is the state vector of the neuron network, $y(k)\in\mathbb{R}^m$ is the measurement output, $h(k)$ represents the transmission delay satisfying $0<h_m\le h(k)\le h_M$, in which h_m and h_M are prescribed positive integers representing the lower and upper bounds of the delay, respectively. $f(x(k))$ and $g(x(k-h(k)))$ are the neuron activation functions. $\mathcal{A}(r_k),\mathcal{B}(r_k)$ and $\mathcal{C}(r_k)$ are coefficient matrices satisfying

$$[\mathcal{A}(r_k),\mathcal{B}(r_k),\mathcal{C}(r_k)]=[A(r_k),B(r_k),C(r_k)]+M(r_t)\Delta(r_k,k)[N_1(r_k),N_2(r_k),N_3(r_k)], \tag{2}$$

where $\Delta(r_k,k)$ is an unknown, time-varying matrix function and satisfies $\Delta^T(r_k,k)\Delta(r_k,k)\le I$ for all $k\in\mathbb{N}^+$. $A(r_k)=\mathrm{diag}(a_1(r_k), a_2(r_k),\ldots,a_n(r_k))$ is the known mode-dependent diagonal matrix. The mode-dependent matrices $B(r_k)$, $C(r_k)$ and $H(r_k)$ are the connection weight matrix, the delayed connection weight matrix and the gain matrix of the network measurement, respectively. $M(r_k),N_1(r_k),N_2(r_k)$ and $N_3(r_k)$ are known mode-dependent matrices. The matrices are functions of the stochastic jump process $\{r_k,k\ge 0\}$, which is a discrete-time, discrete-state Markov chain taking values in a finite set $\Lambda=\{1,2,\ldots,s\}$ with transition probabilities

$$\Pr\{r_{k+1}=j|r_k=i\}=\pi_{ij}, \tag{3}$$

where $\pi_{ij}\ge 0$ and $\sum_{j=1}^s\pi_{ij}=1$ for all $i\in\Lambda$.

For notational simplicity, in the sequel, for each possible $r_k=i, i\in\Lambda$, a matrix $G(r_k)$ will be denoted by G_i; for instance, $A(r_k)$ will be denoted by A_i, $B(r_k)$ by B_i, and so on. In addition, $\check{P}_i$ denotes $\sum_{j=1}^s\pi_{ij}P_j$.

In order to estimate the state of the neural network (1), we construct the following state estimator:

$$\begin{aligned} &\tilde{x}(k+1)=\mathcal{A}_i\tilde{x}(k)+\mathcal{B}_if(\tilde{x}(k))+\mathcal{C}_ig(\tilde{x}(k-h(k)))+K_i[y(k)-H_i\tilde{x}(k)],\\ &\tilde{x}(j)=\phi(j),\quad j\in\{-h_M,\ldots,-1,0\}, \end{aligned} \tag{4}$$

where $\tilde{x}(k)$ is the state estimation of the neuron network (1) and K_i is the state estimator gain matrix to be designed.

Define $e(k)=x(k)-\tilde{x}(k)$, $F(k)=f(x(k))-f(\tilde{x}(k))$ and $G(k-h(k))=g(x(k-h(k)))-g(\tilde{x}(k-h(k)))$. Then, the error dynamics of the neural network can be obtained from (1) and (4) as follows:

$$\begin{aligned} &e(k+1)=(\mathcal{A}_i-K_iH_i)e(k)+\mathcal{B}_iF(k)+\mathcal{C}_iG(k-h(k)),\\ &e(j)=\varphi(j)-\phi(j),\quad j\in\{-h_M,\ldots,-1,0\}. \end{aligned} \tag{5}$$

In this paper, we have the following assumption for the neuron activation functions.

Assumption 1 (*Chen and Zheng [20]*). The neuron state-based nonlinear functions $f(\cdot)$ and $g(\cdot)$ in (1) are continuous and satisfy $f(0)=0$ and $g(0)=0$ and the following sector-bounded conditions:

$$[f(x)-f(y)-U_1(x-y)]^T[f(x)-f(y)-U_2(x-y)]\le 0, \tag{6}$$

$$[g(x)-g(y)-V_1(x-y)]^T[g(x)-g(y)-V_2(x-y)]\le 0. \tag{7}$$

where U_1,U_2,V_1 and V_2 are real matrices of appropriate dimensions.

Remark 1. Note that, when $U_1=-U_2=U$ and $V_1=-V_2=V$, conditions (6) and (7) are respectively reduced to

$$\begin{aligned} &[f(x)-f(y)]^T[f(x)-f(y)]\le[x-y]^TU^TU[x-y],\\ &[g(x)-g(y)]^T[g(x)-g(y)]\le[x-y]^TV^TV[x-y]. \end{aligned}$$

That is to say that the standard Lipschitz conditions $|f(x)-f(y)|\le U|x-y|$ and $|g(x)-g(y)|\le V|x-y|$ hold when $U>0$ and $V>0$. Therefore, the neural network model (1) under Assumption 1 is more general than those described by [18,19].

Throughout the paper, we need the following definition and lemma.

Definition 1 (*Zhang et al. [29,31]*). The error dynamics (5) is said to be stochastically finite-time stable with respect to (δ,ϵ,R_i,N),