Neurocomputing 151 (2015) 288–295

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Letters

# Adaptive NN consensus tracking control of a class of nonlinear multi-agent systems

Jun Feng [a], Guo-Xing Wen [b,*]

[a] Department of Information Engineering, Binzhou University, Binzhou 256600, Shandong, PR China
[b] Department of mathematics, Binzhou University, Binzhou 256600, Shandong, PR China

ARTICLE INFO

*Article history:*
Received 18 May 2014
Received in revised form
5 August 2014
Accepted 15 September 2014
Communicated by H. Jiang
Available online 30 September 2014

*Keywords:*
Robust adaptive control
Leader-following consensus
Neural network
Nonlinear multi-agent systems

ABSTRACT

This paper proposes a novel robust adaptive consensus tracking control approach for a class of nonlinear multi-agent systems with modeling uncertainties and external disturbances. Radial Basis Function Neural Networks (RBFNNs) are used to approximate the unknown nonlinear function of agent's dynamic. Compared with existing NN consensus algorithms of nonlinear multi-agent systems, the proposed consensus control method only needs a small number of adjustable parameters, thus the online computation burden is greatly alleviated. In addition, by online updating the estimation of the NN approximation errors, the proposed consensus control approach can enhance the system robustness against modeling uncertainties. It is proven that all the signals of the multi-agent system are uniformly bounded and the consensus tracking errors converge to a small neighborhood of zero. A simulation is carried out to demonstrate the effectiveness of the proposed control method.



## 1. Introduction

In recent decades, cooperative control of multi-agent systems has received significant attention, because of its wide applications in diverse areas, such as multiple robots [1], unmanned air vehicles [2], autonomous underwater vehicles [3], spacecraft [4], and so on. A fundamental problem for coordinated control is consensus control, which means designing an appropriate network protocol so that all agents asymptotically reach an agreement. The consensus problem of multi-agent systems as a fresh research topic has become an important research direction of control theory due to its broad applications [5,6].

A particularly interesting topic of consensus control is the leader-following consensus problem of multi-agent systems. The leader is an independent agent and it is followed by all the other ones. In fact, the leader-following strategy is an energy saving mechanism in many biological systems [7], and it can also enhance the communication and orientation of the flock [8].

Recently, based on matrix theory, algebraic graph theory and control theory, many meaningful research results of the leader-following and leaderless consensus problems of linear multi-agent systems have been reported [9–15]. It is well known that nonlinearity, which is an inherent quality in most practical control systems, is more complex and challenging than linear nature. For nonlinear multi-agent systems, several valuable results have also emerged and received widespread attention [16–20]. In [16], Lu et al. proposed a signal impulsive control approach for a class of directed dynamical network with impulsive coupling. By several simple criteria, it is concluded that the single impulsive control can stabilize the class of impulsive dynamical networks. For a class of multi-agent systems with non-identical unknown nonlinear dynamics, a decentralized adaptive leader-following consensus algorithm is developed in [17] by using both relative position feedback and local consensus error feedback of neighboring agents. Su et al. [18] proposed a second-order consensus algorithm for a class of multiple nonlinear dynamical mobile agents with a virtual leader. A primary contribution of the research work is that nonlinear multi-agent with directed topology is developed. In [19,20], pinning-controlled consensus control method is studied for nonlinear multi-agent systems. Because a coupling term is contained in each agent's dynamic, the consensus objective can be realized by only controlling a small fraction of agents. A main advantage of the consensus strategy is that the control cost is greatly reduced. However, all these eminent consensus control approaches do not consider the improvement of multi-agent system robustness. Although adaptive robust control has been well-developed in tracking control, for examples [23–25], there are few research results to be reported about the adaptive robust consensus control.

Since it is proven that the neural networks (NNs) have the universal approximation property [21], the neural networks

* Corresponding author.
*E-mail address:* gxwen@live.cn (G.-X. Wen).

http://dx.doi.org/10.1016/j.neucom.2014.09.037
0925-2312/

become an attractive and powerful tool for stabilizing complex nonlinear dynamic systems. In addition, the fuzzy logic systems (FLS) are also the universal approximator [22]. Active research has been carried out in adaptive tracking control of nonlinear systems by using the fact that neural networks or fuzzy logic systems can approximate a wide range of nonlinear functions to any desired accuracy under certain conditions [23–28]. Recently, a neural network-based leader–follower consensus control for a class of nonlinear multi-agent system with modeling uncertainties has been proposed in [29], and received widespread attentions.

However, for the most of the existing adaptive NN consensus control of nonlinear multi-agent systems, such as [29–32], the quantity of the adjusted parameters depends on the number of input neurons of neural networks (or hidden layer nodes in the fuzzy systems). In general, the number of the input neurons or the hidden layer nodes must be sufficiently large in order to improve approximation accuracy. Therefore, if the schemes proposed in [29–32] are employed to stabilize the corresponding nonlinear multi-agent system, the online computation burden would be very heavy. From the viewpoint of the engineering application, the consensus control performance is affected, and the running cost is increased.

In this paper, a robust adaptive consensus tracking control for a class of nonlinear multi-agent systems with modeling uncertainties and external disturbances is researched. In the consensus control design, RBFNN is used to construct the adaptive approximators to counteract uncertainties which derive from the unknown nonlinear function of system dynamic. The main contributions of the research work are (1) Compared with existing consensus algorithms, the number of adaptive parameters is greatly decreased because only a scalar adjustable parameter, which is the estimation of the norm of the optimal neural weight vector, is adaptively updated for each agent; (2) By adaptive adjusting the estimations of unknown bounds of neural approximation errors, the adaptive consensus control method can enhance the system robustness against modeling uncertainties. Finally, it is proven that all the signals of the multi-agent system are uniformly bounded and the consensus tracking errors converge to a small neighborhood of zero.

The paper is organized as follows. In Section 2, a description of nonlinear multi-agent systems is given, and the relevant notation, assumption, lemma and preliminaries are stated. In Section 3, the robust adaptive consensus control is designed, and stability result is proven. In Section 4, a numerical simulation example is given to demonstrate the effectiveness of the proposed consensus control approach. In Section 5, the conclusion is made for this paper.

## 2. Problem statement and preliminaries

Consider a class of nonlinear multi-agent systems consisting of $n$ following agents and a leader. The dynamic of each following agent is described as follows:

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(t) + \Delta_i(t, x_i(t))$$
$$i = 1, 2, \dots, n \tag{1}$$

where $x_i(t) \in R$ is the agent's state, $f_i(x_i(t))$, $g_i(x(t)) : R \to R$ are unknown smooth nonlinear functions. $u_i(t) \in R$ is the control input. $\Delta_i(t, x_i(t)) : R \to R$ is the unknown external disturbance.

The leader of the nonlinear multi-agent system (1) is an independent agent and it is described by the following nonlinear dynamic:

$$\dot{x}_r(t) = f_r(t) \tag{2}$$

where $x_r(t) \in R$ is the leader's state, $f_r(t) \in R$ is an unknown smooth nonlinear function.

**Definition 1.** [14]: The leader-following consensus of the multi-agent system (1) with leader (2) is achieved if there is a local state feedback controller $u_i$ for each agent such that the close loop system satisfies $\lim_{t \to \infty} \|x_i(t) - x_r(t)\| = 0$, $i = 1, \dots, n$ for any initial condition, where $\|\bullet\|$ denotes the $2-$norm and the representation is also used in the whole paper.

**Remark 1.** It should be noticed that most existing literatures of nonlinear multi-agent systems only assume the control gain $g_i(\bullet)$ to equal to 1 or constant, such as [16–20,29–33]. In this paper, a more general condition that $g_i(\bullet)$ is unknown nonlinear function is considered.

**Remark 2.** It should be reminded that the well-known Young's inequality, $ab \le \frac{a^2}{2} + \frac{b^2}{2}$, is frequently used in the paper.

**Assumption 1.** For $1 \le i \le n$, the nonlinear function $g_i(x_i(t))$ is unknown, but its sign is known, and there exist constants $\underline{g}_i$ and $\overline{g}_i$ such that $\underline{g}_i \le g_i(x_i) \le \overline{g}_i$, $i = 1, \dots, n$. Without loss of generality, it is further assumed that $g_i(x_i) \ge \underline{g}_i > 0$, $i = 1, \dots, n$.

**Assumption 2.** For $1 \le i \le n$, there exist positive unknown continuous functions $\rho_i(x_i(t))$, $1 \le i \le n$ such that

$$|\Delta_i(t, x)| \le \rho_i(x_i(t)), \quad i = 1, \dots, n \tag{3}$$

**Assumption 3.** The leader's nonlinear vector function $f_r(t) \in R$ is bounded by a positive constant $\alpha > 0$, i.e.,$|f_r(t)| \le \alpha$, $\forall t \in R^+$.

In this paper, the control objective is to design the adaptive leader-following consensus controllers $u_i(t) \in R, i = 1, \dots, n$ for every agent such that the leader-following consensus behavior of the multi-agent system (1) can be obtained finally.

**Remark 3.** Assumption 1 is reasonable because $g_i(x_i(t))$ away from zero is the controllable condition of the multi-agent system (1). In Assumption 1, the constants $\underline{g}_i$, $\overline{g}_i$ are required just for analytical purposes, and their true values are unnecessary to be known for the consensus control design. Similarly, the constant $\alpha$ that is given in Assumption 3 is also unknown.

**Remark 4.** Some practical multi-agent systems can be described by nonlinear differential equations (1), such as formation control of unmanned air vehicle, sensor network, satellite clusters, robotic teams, complex networks, etc.

### 2.1. Graph theory

Let $G = (V, \varepsilon, A)$ be an undirected graph, where $V = \{v_1, v_2, \dots, v_n\}$ is the set of nodes, $\varepsilon \subseteq V \times V$ is the set of edges, and $A = [a_{ij}]$ is the weighted adjacency matrix with nonnegative elements. Node $v_i$ denotes the $i$th agent and the node indexes belong to a finite index set $I = \{1, 2, \dots, n\}$. An edge of $G$ is denoted by $e_{ij} = (v_j, v_i)$, where the node $v_j$ of the edge $e_{ij}$ is said to be the tail of the edge and node $v_i$ is the head of the edge $e_{ij}$. $e_{ij} = (v_j, v_i) \in \varepsilon$ if and only if there is the information exchange between agents $i$ and $j$, and the communication quality is described by the adjacency elements $a_{ij}$. It is stipulated that $a_{ii} = 0$ and $a_{ij} = a_{ji} \ge 0$ if and only if $e_{ij} \in \varepsilon$; $a_{ij} = 0$ if $e_{ij} \notin \varepsilon$. Let $b_i$ denote the communication weight between agent $i$ and leader, such that if there exists an information exchange between agent $i$ and leader, then $b_i > 0$, otherwise $b_i = 0$, in which there always exist at least one agent connected to the leader, i.e., $b_1 + b_2 + \dots + b_n > 0$. Node $v_i$ is a neighbor of node $v_j$ if the edge $e_{ji} = (v_i, v_j) \in \varepsilon$. A sequence of edges $(v_{i_1}, v_{i_2})$, $(v_{i_2}, v_{i_3}), \dots, (v_{i_{k-1}}, v_{i_k})$ is called a path from node $v_{i_1}$ to node $v_{i_k}$. If any two nodes $v_i, v_j \in V$, there exists a path between them, the undirected graph $G$ is called a connected graph.