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Exponential synchronization for a class of complex spatio-temporal networks with space-varying coefficients

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ABSTRACT

This paper addresses the problem of exponential synchronization for a class of complex spatio-temporal networks with space-varying coefficients, where the dynamics of nodes are described by coupled partial differential equations (PDEs). The goal of this research is to design distributed proportional-spatial derivative (P-sD) state feedback controllers to ensure exponential synchronization of the complex spatio-temporal network. Using Lyapunov's direct method, the problem of exponential synchronization of the complex spatio-temporal network is formulated as the feasibility problem of spatial differential linear matrix inequality (SDLMI) in space. The feasible solutions to this SDLMI in space can be approximately derived via the standard finite difference method and the linear matrix inequality (LMI) optimization technique. Finally, a numerical example is presented to demonstrate the effectiveness of the proposed design method.

1. Introduction

Complex dynamical networks can be used to describe many large-scale systems in many aspects such as nature and human societies, where nodes and edges of the network represent individuals in the system and the connection among them, respectively. Typical examples involve biological neural networks, ecosystems, electrical power grids, and the internet. Since their wide applications, there has been rapidly increasing interests in the studying of various complex dynamical networks in the past decades [1–3].

Ever since the discovery of the synchronization of two pendulum clocks by Christian Huygens [4,5], synchronization, a typical collective behavior and a basic motion in nature, has been studied for several centuries. Since then, synchronization has received a great attention for its potential applications in many fields like in secure communications, biological systems, chemical processes, and so on [6,7], and therefore a great number of studies have been devoted to the analysis of synchronization of various complex networks [8–10]. For the case of when one complex network cannot synchronize by itself, control design is desired to achieve the synchronization of the complex network. In this case, many effective design methods have been reported in [6–16]. However, the dynamical behavior of the networks in these results [6–16] is assumed to be described by ordinary differential equations (ODEs) or delay differential equations (DDEs).

In practice, the node dynamics in most complex networks like biological systems [17] is spatio-temporal in nature so that its behavior must depend on time as well as spatial position and therefore could be described by partial differential equations (PDEs). Many researchers have paid attention to the study of synchronization of complex PDE networks over the past decades, in which many analysis and design approaches have been proposed [18–26]. These design approaches can be classified into two types: "*reduce-then-design*" [18–20] and "*design-then-reduce*" [21–26]. The former one initially applies the spatial discretization techniques like finite difference method to complex PDE networks to obtain an approximate model consisting of complex ODE networks in time. Although it is easy to apply various concepts of finite-dimensional control theory and techniques for the control design of complex PDE networks, a potential drawback of the reduce-then-design approach is that the order of the resulting complex ODE networks may be very large for yielding the desired degree of approximation, resulting in complicated controller design and high dimensionality controllers different from the reduce-then-design approach, the design-then-make the best use of the merits of the original PDE model for the controller design and the resulting infinite-dimensional control solution is then lumped for implementation purpose. Hence, the research on the synchronization for complex PDE networks based on PDE model is very active [21–26]. However, the papers [21–26] dealt with the networks with space-invariant coefficients.

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Motivated by the above analysis, this paper addresses the problem of exponential synchronization for a class of linear complex spatio-temporal networks with space-varying coefficients, where each nodes' dynamics is modeled by coupled parabolic PDEs. Distributed proportional-spatial derivative (P-sD) are proposed to ensure the exponential synchronization of the complex PDE network. By using Lyapunov's direct method, the main result of this paper is presented in term of spatial differential linear matrix inequality (SDLMI) in space as a result of the existing space-varying coefficients in the complex spatio-temporal network. It has been pointed out in [27] that this feasibility problem can be approximated by the finite difference method and the standard LMI optimization [28,29]. A numerical example is presented to demonstrate the effectiveness of the proposed method.

The main contribution and novelty of this paper can be summarized as follows: (i) The problem of a class of linear complex spatio-temporal networks with space-varying coefficients is formulated, which is actually modeled by an array of coupled PDEs. (ii) An SDLMI based approach to the distributed P-sD controller design of exponential synchronization is developed via Lyapunov's direct method, the technique of integration by parts and the Kronecker product for matrices for the complex spatio-temporal network with space-varying coefficients.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section 2. Section 3 presents a distributed P-sD control design for exponential synchronization of the complex PDE network in terms of SDLMI. An example to illustrate the effectiveness of the proposed method is presented in Section 4 and Section 5 offers some concluding remarks.

Notations: The following notations will be used throughout this paper. $\mathfrak{R}$, $\mathfrak{R}^n$ and $\mathfrak{R}^{m\times n}$ denote the set of all real numbers, n-dimensional Euclidean space and the set of all $m\times n$ matrices, respectively. $\|\cdot\|$ and $\langle\cdot,\cdot\rangle_{\mathfrak{R}^n}$ denote the Euclidean norm and inner product for vectors, respectively. $\otimes$ means the Kronecker product for matrices. Identity matrix, of appropriate dimensions, will be denoted by $\boldsymbol{I}$. For a symmetric matrix $\boldsymbol{R}$, $\boldsymbol{R}>0$ and $\boldsymbol{R}<0$ respectively means that it is positive definite and negative definite. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ stand for the minimum and maximum eigenvalues of a square matrix, respectively. $\mathscr{L}_2([l_1,l_2];\mathfrak{R}^n)$ is a Hilbert space of n-dimensional square integrable vector functions $\boldsymbol{\omega}(x)\in\mathfrak{R}^n$, $x\in[l_1,l_2]\subset\mathfrak{R}$ with the inner product and norm:

$$\langle\boldsymbol{\omega}_1,\boldsymbol{\omega}_2\rangle=\int_{l_1}^{l_2}\langle\boldsymbol{\omega}_1(x),\boldsymbol{\omega}_2(x)\rangle_{\mathfrak{R}^n}dx \quad\text{and}\quad \|\boldsymbol{\omega}_1\|_2=\langle\boldsymbol{\omega}_1,\boldsymbol{\omega}_1\rangle^{1/2},$$

where $\boldsymbol{\omega}_1,\boldsymbol{\omega}_2\in\mathscr{L}_2([l_1,l_2];\mathfrak{R}^n)$. The superscript "$T$" is used for the transpose of a vector or a matrix. The symbol "$*$" is used as an ellipsis in matrix expressions that are induced by symmetry, e.g.,

$$\begin{bmatrix}\boldsymbol{R}+[\boldsymbol{M}+\boldsymbol{N}+*] & \boldsymbol{X}\\ * & \boldsymbol{Y}\end{bmatrix}\triangleq\begin{bmatrix}\boldsymbol{R}+[\boldsymbol{M}+\boldsymbol{N}+\boldsymbol{M}^T+\boldsymbol{N}^T] & \boldsymbol{X}\\ \boldsymbol{X}^T & \boldsymbol{Y}\end{bmatrix}.$$

2. Preliminaries and problem formulation

Consider a class of complex spatio-temporal networks described by coupled parabolic PDEs with space-varying coefficients of the following form:

$$\begin{cases}\frac{\partial\boldsymbol{y}_i(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(\boldsymbol{\Theta}_1(x)\frac{\partial\boldsymbol{y}_i(x,t)}{\partial x}\right)+\boldsymbol{\Theta}_2(x)\frac{\partial\boldsymbol{y}_i(x,t)}{\partial x}+\boldsymbol{A}(x)\boldsymbol{y}_i(x,t)+c\sum_{j=1}^{N}g_{ij}(x)\boldsymbol{y}_j(x,t)\\ +\boldsymbol{H}(x)\boldsymbol{u}_i(x,t)\\ \left.\frac{\partial\boldsymbol{y}_i(x,t)}{\partial x}\right|_{x=l_1}=\left.\frac{\partial\boldsymbol{y}_i(x,t)}{\partial x}\right|_{x=l_2}=0\\ \boldsymbol{y}_i(x,0)=\boldsymbol{y}_{i,0}(x),\quad i\in\{1,2,\ldots,N\}\end{cases} \tag{1}$$

where $\boldsymbol{y}_i(x,t)\in\mathfrak{R}^n$ and $\boldsymbol{u}_i(x,t)\in\mathfrak{R}^m$, $i\in\{1,2,\ldots,N\}$, are the state and distributed control input of the i-th node, respectively. $\boldsymbol{\Theta}_1(x)\in\mathfrak{R}^{n\times n}$, $\boldsymbol{\Theta}_2(x)\in\mathfrak{R}^{n\times n}$, $\boldsymbol{A}(x)\in\mathfrak{R}^{n\times n}$, $\boldsymbol{H}(x)\in\mathfrak{R}^{n\times m}$, $x\in[l_1,l_2]$ are known matrix functions of x, $x\in[l_1,l_2]$. c is a known scalar describing the coupling strength. $x\in[l_1,l_2]\subset\mathfrak{R}$ and $t\in[0,\infty)$ are the spatial position and time, respectively. $\boldsymbol{y}_{i,0}(x)$ means the initial value of the i-th node. $\boldsymbol{G}(x)=(g_{ij}(x))_{N\times N}$, $x\in[l_1,l_2]$ is the coupling configuration matrix function of x, which represents the topological structure of the complex spatio-temporal network, where $g_{ii}(x)=-\sum_{j=1,j\neq i}^{N}g_{ij}(x)$, $i,j\in\{1,2,\ldots,N\}$. Moreover, $\boldsymbol{G}(x)$ is not required to be symmetric or irreduciblein this paper.

Assume that there is an isolated node $\boldsymbol{s}(x,t)$ satisfies

$$\begin{cases}\frac{\partial\boldsymbol{s}(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(\boldsymbol{\Theta}_1(x)\frac{\partial\boldsymbol{s}(x,t)}{\partial x}\right)+\boldsymbol{\Theta}_2(x)\frac{\partial\boldsymbol{s}(x,t)}{\partial x}+\boldsymbol{A}(x)\boldsymbol{s}(x,t)\\ \left.\frac{\partial\boldsymbol{s}(x,t)}{\partial x}\right|_{x=l_1}=\left.\frac{\partial\boldsymbol{s}(x,t)}{\partial x}\right|_{x=l_2}=0\\ \boldsymbol{s}(x,0)=\boldsymbol{s}_0(x)\end{cases} \tag{2}$$

Define the synchronization error of the i-th node as

$$\boldsymbol{e}_i(x,t)\triangleq\boldsymbol{y}_i(x,t)-\boldsymbol{s}(x,t). \tag{3}$$

Hence the synchronization error system of the complex PDE network can be represented by

$$\begin{cases}\frac{\partial\boldsymbol{e}_i(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(\boldsymbol{\Theta}_1(x)\frac{\partial\boldsymbol{e}_i(x,t)}{\partial x}\right)+\boldsymbol{\Theta}_2(x)\frac{\partial\boldsymbol{e}_i(x,t)}{\partial x}+\boldsymbol{A}(x)\boldsymbol{e}_i(x,t)\\ +c\sum_{j=1}^{N}g_{ij}(x)\boldsymbol{e}_j(x,t)+\boldsymbol{H}(x)\boldsymbol{u}_i(x,t)\\ \left.\frac{\partial\boldsymbol{e}_i(x,t)}{\partial x}\right|_{x=l_1}=\left.\frac{\partial\boldsymbol{e}_i(x,t)}{\partial x}\right|_{x=l_2}=0\\ \boldsymbol{e}_i(x,0)=\boldsymbol{e}_{i,0}(x),\quad i\in\{1,2,\ldots,N\}\end{cases} \tag{4}$$

where

$$\boldsymbol{e}_{i,0}(x)\triangleq\boldsymbol{y}_{i,0}(x)-\boldsymbol{s}_0(x).$$

To achieve the synchronization of the complex PDE network (1), this paper considers the following identical distributed P-sD feedback controllers [27,31,32]:

$$\boldsymbol{u}_i(x,t)=\boldsymbol{K}(x)\boldsymbol{e}_i(x,t)+\boldsymbol{L}(x)\frac{\partial\boldsymbol{e}_i(x,t)}{\partial x},\quad i\in\{1,2,\ldots,N\} \tag{5}$$

in which $\boldsymbol{K}(x)$ and $\boldsymbol{L}(x)$, $x\in[l_1,l_2]$ are real continuous $m\times n$ matrix functions to be determined. The controller structure in this paper is shown in Fig. 1, where the notation "$\partial/\partial x$" represents a first-order spatial differentiator.

Remark 1. It must be pointed out that the implementation of the distributed P-sD feedback controllers (5) needs, which is normally recognized as a critical drawback. However, with recent advances in the filed of technological developments especially in micro-electro-mechanical systems, it has become feasible to manufacture large arrays of micro-sensors and actuators with integrated control circuitry, which can help the controllers (5) to be implemented in some practical applications (see [27,30–32]). The signal $(\partial\boldsymbol{e}_i(x,t))/\partial x$ in the controllers (5) can be obtained via the finite difference method. Additionally, it has been pointed out in

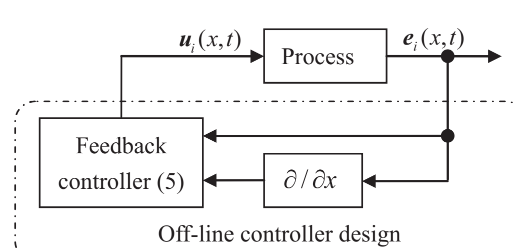

Fig. 1. The structure of distributed P-sD state-feedback controllers.