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New results on robust finite-time boundedness of uncertain switched neural networks with time-varying delays

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ABSTRACT

This paper investigates the finite-time boundedness (FTB) problem for a class of uncertain switched neural networks with time-varying delays. By exploring the mode-dependent properties of each subsystem, all the subsystems could be categorized into stable and unstable ones under the Lyapunov-like function framework. The sufficient conditions and a set of unified switching signals with average dwell time (ADT) are first derived with a known limit to the total activation time ratio between unstable and stable subsystems. Then, the obtained results are extended to a new switching approach with mode-dependent average dwell time (MDADT). Compared with general results, our proposed approach distinguishes the stable and unstable subsystems rather than viewing all subsystems as being unstable, thus getting less conservative switching criteria. Finally, a numerical example is provided to show the validity and the advantages of the finding techniques.

1. Introduction

The last few decades have witnessed the development of neural networks (NNs) with the extension applications in signal processing, pattern recognition, associative memory and other related areas [1–6]. As a main research field, switched NNs have drawn much attention for modeling the phenomenon of information latching, abrupt phenomenon and changes of interconnections between different subsystems, and remarkable achievements were obtained in the existing literature, see [7–11] and references therein. It is well known that most published results of NNs have focused on the Lyapunov asymptotic stability, which is defined in an infinite-time interval, see for example, [12,13]. Generally speaking, the Lyapunov asymptotic stability is enough for practical applications, however, there are some cases that large values are not permitted [14]. To this end, finite-time boundedness (FTB) could be exerted. Specifically, a system is said to be FTB if its state (norm) holds below some given threshold in a fixed finite-time interval [15]. Researches on FTB of switched NNs have also been available so far with regarding all the subsystems as being unstable under the Lyapunov-like function framework [14,16].

In fact, the investigation on the issue of the switched systems with both stable and unstable subsystems is not rare. To mention a few, by limiting the maximum activation time ratio between unstable and stable subsystems, the stability of switched systems is demonstrated with the average dwell time (ADT) approach [12,17–19]. Besides, assuming the maximum allowable mismatch duration of a certain plant and the corresponding controller is known a priori, the asynchronous switching problems could be dealt with by denoting the match and mismatch interval, respectively, as the stable and unstable subsystems [7,20]. All the results above are addressed with respect to infinite-time interval. For the analysis of FTB, however, most existing results have cursorily addressed all the subsystems being unstable with a rising upper bound under the Lyapunov-like framework [14,16]. In spite of that any stable subsystems could be dealt as unstable ones, however, if we adequately explore the mode-dependent information and further differentiate the stable ones from all the unstable subsystems or reintroducing the new stable subsystems, the admissible

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switching signals will be more available. Additionally, time delays phenomena are commonly encountered in practical NNs due to the limited switching speed of data processing and the inherent communication of neurons, which may degrade the system performance, cause oscillation and even leads to instability [12]. Therefore, the studies of the switched NNs with both stable and unstable time-delay subsystems are of great importance.

In addition, as an important feature, switching signals distinguish the so-called switched systems from general time-variant systems. Currently, plenty of studies on switched systems could be roughly categorized into two classes: (a) Markov jump systems, in which the transitions among different modes are governed by a Markov chain; (b) switched systems with a class of time-dependent switching signals, for instance the ADT switching approach. Extensive studies have been done on Markov jump NNs, including the cases of transition rates being exactly known [21–23] and partly known [13,24]. However, few results have been published for the switched NNs with ADT switching approach, which motivates us for the studies on this topic. Moreover, a more available mode-dependent average dwell time (MDADT) approach will be further proposed based on the distinction of stable and unstable subsystems to reduce the conservativeness of switching signals with ADT [17,25].

In this paper, we aim at addressing the FTB problem for a class of the uncertain switched NNs with stable and unstable time-delay subsystems. The mode-dependent properties between stable and unstable time-delay subsystems will be fully utilized to obtain less conservative switching signals with ADT and MDADT. The main contributions of this paper are highlighted as follows. First, the considered switched NNs with both stable and unstable subsystems are more general and applicable than the existing FTB results of switched NNs only with unstable subsystems. Second, sufficient conditions of switching signals under ADT and MDADT approaches are respectively obtained. The existence criteria are formulated in terms of a set of linear matrix inequalities. The remainder of this paper is organized as follows. In Section 2, the switched time-delay NNs considered are given and the objectives are declared. The existence conditions are derived and the minimal ADT and MDADT criteria are obtained in Section 3. A numerical example is analyzed to illustrate the effectiveness and the advantages of the developed findings in Section 4. The final conclusions are presented in the last section.

Notation: Throughout this paper, the notations used are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ refer to, respectively, the n dimensional Euclidean space and the set of all $m\times n$ real matrices; $\mathbb{N}^+$ denotes the sets of positive integers. The notation $P>0$ (≥ 0) means P is real symmetric positive (semi-positive) definite and the superscript "T" stands for the transpose for vectors or matrices. In addition, in symmetric block matrices or long matrix expressions, * is used as the ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ represents the block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Preliminaries

Consider the following n-neuron uncertain switched neural networks (NNs) with time-varying delays, defined in a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{aligned}\dot{\delta}(t) = &-(A_{\sigma(t)}+\Delta A_{\sigma(t)}(t))\delta(t)+(B_{\sigma(t)}+\Delta B_{\sigma(t)}(t))h(t,\delta(t))\\ &+(C_{\sigma(t)}+\Delta C_{\sigma(t)}(t))h(t,\delta(t-d(t)))+J,\end{aligned} \tag{1}$$

where $\delta(t)=(\delta_1(t),\delta_2(t),\ldots,\delta_n(t))^T\in\mathbb{R}^n$ denotes the state vector associated with the n neurons; $\sigma(t):[0,\infty)\to\mathcal{I}\triangleq\{1,\ldots,l\}$ is a piecewise constant function which is deterministic and right continuous, specifying the index of the active subsystem, i.e., $\sigma(t)\in\mathcal{I}$ for $t\in[t_k,t_{k+1})$, where t_k is the k-th switching instant, and l is the number of subsystems; $h(t,\delta(t))=(h_1(t,\delta(t)), h_2(t,\delta(t)),\ldots,h_n(t,\delta(t)))^T\in\mathbb{R}^n$ denotes the nonlinear neuron activation function. $\delta(\theta)=\epsilon(\theta)$, $\forall\theta\in[-d_M,0]$ and $\epsilon(\theta)$ is the initial condition. $A_{\sigma(t)}=diag\{a_{1,\sigma(t)},a_{2,\sigma(t)},\ldots,a_{n,\sigma(t)}\}$ is a known mode-dependent diagonal matric with positive entries $a_{i,\sigma(t)}>0$, $\forall i=1,2,\ldots,n$. The mode-dependent matrices $B_{\sigma(t)}$ and $C_{\sigma(t)}$ are respectively the connection weight matrix and the delayed connection weight matrix. $J=[J_1\ J_2\ \cdots\ J_n]^T$ is a constant external input vector. $d(t)$ is an unknown time-varying delay function satisfying

$$\begin{cases}0\leq d(t)\leq d_M,\\ \dot{d}(t)\leq d<1,\end{cases}$$

where d and d_M are known a priori.

Throughout this paper, the following assumptions are assumed to be satisfied.

Assumption 2.1. The parameters $\Delta A_{\sigma(t)}(t)$, $\Delta B_{\sigma(t)}(t)$ and $\Delta C_{\sigma(t)}(t)$ in NNs (1) are time-varying but norm bounded, which admit

$$[\Delta A_{\sigma(t)}(t),\Delta B_{\sigma(t)}(t),\Delta C_{\sigma(t)}(t)]=H_{\sigma(t)}F_{\sigma(t)}(t)[M_{1,\sigma(t)}\ M_{2,\sigma(t)}\ M_{3,\sigma(t)}],$$

where $M_{1,\sigma(t)}$ $M_{2,\sigma(t)}$ $M_{3,\sigma(t)}$ and $H_{\sigma(t)}$ are known mode-dependent real constant matrices, $F_{\sigma(t)}(t)$ is an unknown real matrix with appropriate dimension and Lebesgue norm measurable elements, satisfying

$$F^T_{\sigma(t)}(t)F_{\sigma(t)}(t)\leq I.$$

Assumption 2.2 (*Lou and Cui [26]*). The nonlinear neuron activation function $h(t,\delta(t))$ is bounded with $L=diag\{\ell_1,\ell_2,\ldots,\ell_n\}$, such that

$$0\leq\frac{h_j(s_1)-h_j(s_2)}{s_1-s_2}\leq\ell_j,\qquad\forall j=1,\ldots,n, \tag{2}$$

or

$$|h_j(s_1)-h_j(s_2)|\leq\ell_j|s_1-s_2|,\quad\forall j=1,\ldots,n, \tag{3}$$

where s_1, $s_2\in\mathbb{R}$, $s_1\neq s_2$ and ℓ_j is a known positive scalar.

Note that condition (3) is less conservative than (2), and both assumptions are commonly existed in some conventional NNs, such as the standard sigmoidal functions and piecewise linear function [26].

Moreover, according to the celebrated Brouwer's fixed-pointed theorem [26], there exists at least one equilibrium point for the switched NNs (1). Thus, denote $\delta^*=(\delta_1^*,\delta_2^*,\ldots,\delta_n^*)^T$ as one of the equilibrium points for switched NNs (1), shifting δ^* to the origin via the following approach:

$$x_j(t)=\delta_j(t)-\delta_j^*,\quad f_j(t,x(t))=h_j(t,\delta_j(t))-h_j(\delta_j^*),\quad\forall j=1,\ldots,n$$

then the switched time-delay NNs (1) could be rewritten as

$$\begin{aligned}\dot{x}(t)=&-(A_{\sigma(t)}+\Delta A_{\sigma(t)}(t))x(t)+(B_{\sigma(t)}+\Delta B_{\sigma(t)}(t))f(t,x(t))\\ &+(C_{\sigma(t)}+\Delta C_{\sigma(t)}(t))f(t,x(t-d(t))).\end{aligned} \tag{4}$$

To proceed further, the issue to be investigated in this note is formulated as follows.

Definition 2.3 (*He and Liu [16]*). Given the positive scalars c_1, c_2, T_f with $c_1<c_2$ and $T_f>0$, a positive matrix Φ and a switching signal $\sigma(t)$, switched NNs (1) is said to be finite-time bounded (FTB) with respect to $(c_1,c_2,\Phi,\sigma,T_f)$, if $\forall t\in[0,T_f]$

$$\sup_{-d_M\leq\theta\leq 0}\{x^T(\theta)\Phi x(\theta),\dot{x}^T(\theta)\Phi\dot{x}(\theta)\}\leq c_1\overset{\sigma(t)}{\Longrightarrow}x^T(t)\Phi x(t)\leq c_2.$$

Remark 2.4. FTB holds the evaluation criterion $x^T(t)\Phi x(t)$ below the assigned upper bound c_2 in a fixed interval of $[0,T_f]$, which is