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Dual-stage impulsive control for synchronization of memristive chaotic neural networks with discrete and continuously distributed delays

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ABSTRACT

This paper focus on the exponential synchronization problem of a class of memristive chaotic neural networks with discrete, continuously distributed delays and different parametric uncertainties using a novel impulsive control scheme (dual-stage impulsive control). By using the Lyapunov method combining with the comparison principle, a global synchronization error bound together with new sufficient conditions in the form of linear matrix inequalities (LMI) are derived in order to guarantee that the synchronization error dynamics can converge to the predetermined level. Simulation results further demonstrate the validation of the proposed results.

1. Introduction

The memristor was firstly proposed by Chua and demonstrated the realization of the memristor by Hewlett-Packard (HP) research team, see [1,2]. Since then, it has been widely studied both in theory and applications [3–7]. The memristor has been successfully applied in integrated circuits design and artificial neural networks design. The memristive neural networks in the integrated circuit based on memristor was firstly introduced in [5] where a memristive neural network is made of bio-inspired neuron oscillatory circuits with nanoscale memristors. It is expected that the memristive neural networks will help us to build a brain-like machine to implement the synapses of biological brains because the conventional neural networks have only limited success. Hence, it is necessary and important to develop the memristive neural networks.

In the past two decades, memristive neural networks have received increasing research due to their widely applications in many areas such as signal processing, associative memory and pattern recognition. In [8–12], some novel stability criteria were proposed to guarantee the asymptotically or exponentially stable of the memristive neural networks. The synchronization problem have been addressed of memristive neural networks in [13–16]. As we know, synchronization of dynamic networks is an important topic which has drawn a great deal of attentions [13–30,36–39,10,41–47]. Synchronization phenomena has been found in different forms both in nature and in man-made systems, such as fireflies in the forest, applause, description of hearts, distributed computing systems, routing messages in the internet, chaos-based communication network, and so on. On the other hand, when designing the neural networks or implementation of neural systems, time delay is almost inevitable. Therefore, it is necessary to both consider investigate effects of both discrete and continuously distributed delays. To the best of the author's knowledge, the synchronization problem for memristive neural networks with discrete and continuously distributed delays is still an open yet challenging issue.

It is widely recognized that there are many synchronization strategies such as coupling control [31], time delay feedback control [32], adaptive control [33] and impulsive control method [34,35] to synchronize two identical or different chaotic systems. It is worth noting that the impulsive control method is effective and robust in the investigation of chaos synchronization, since it can achieve synchronization of chaotic systems by using only small impulses generated by the nodes of the state variables of the drive and the response chaotic systems at discrete time instants. Such kind of controllers, which are discontinuous, usually have a simple structure. Moreover, when the synchronization impulses are sent to the receiving systems at discrete time instants, it can greatly decrease the information redundancy in the transmitted signal and increase robustness against the disturbances. Generally speaking, to achieve the synchronization between different chaotic

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systems or identical ones with different parametric uncertainties, complicated continuous controllers are often required to compensate for the structure mismatch [36,38,39].

Motivated by the above discussion, in this paper, the memristive neural networks model which characterize the discrete, continuously distributed delays and impulsive effects is firstly presented. Then, a practical issue of using the dual-stage impulsive control method to synchronization a class of chaotic memristive neural networks with different time-varying parametric uncertainties and discrete and continuous distributed delays, which has not been investigated in the open literatures is studied. By using the Lyapunov method combining with the comparison principle, some easy-to-check sufficient conditions in the form of LMI are derived, The main contribution of this paper can be listed as follows: (1) The general memristive neural networks model which concerning the impulsive effects is established. (2) The Lyapunov method combining with the comparison principle is used to deal with the synchronization problem of the considered model.

The rest of this paper is organized as follows. In the next section, some necessary notations, definitions, and preliminaries are presented. In Section 3, several sufficient conditions are derived to ensure the robust exponentially synchronize of considered model via dual-stage impulsive control. In Section 4, two numerical examples and their computer simulations are presented to further demonstrate the effectiveness of the previous results. Finally, some conclusions are drawn in Section 5.

2. Model formulation and preliminaries

Different from the most of the previous works [5,8–16,23], in this paper, we will deal with the impulsive synchronization problem for the memristive uncertain chaotic neural networks with discrete and distributed delays described by

$$\begin{aligned}\dot{x}_i(t) = &-c_i(x_i(t))x_i(t) + \sum_{j=1}^{n}(a_{ij}(x_i(t)) + \Delta a_{ij})f_j(x_j(t))\\ &+\sum_{j=1}^{n}(b_{ij}(x_i(t)) + \Delta b_{ij})f_j(x_j(t-\tau(t))\\ &+\sum_{j=1}^{n}(w_{ij}(x_i(t)) + \Delta w_{ij})\int_0^{\infty} k_{ij}(s)f_j(x_j(t-s))\,ds + I_i(t)\end{aligned} \tag{1}$$

where

$$a_{ij}(x_i(t)) = \begin{cases}\hat{a}_{ij} & |x_i(t)| < T_i\\ \check{a}_{ij} & |x_i(t)| > T_i\end{cases} \quad b_{ij}(x_i(t)) = \begin{cases}\hat{b}_{ij} & |x_i(t)| < T_i\\ \check{b}_{ij} & |x_i(t)| > T_i\end{cases}$$

$$c_i(x_i(t)) = \begin{cases}\hat{c}_i & |x_i(t)| < T_i\\ \check{c}_i & |x_i(t)| > T_i\end{cases} \quad w_{ij}(x_i(t)) = \begin{cases}\hat{w}_{ij} & |x_i(t)| < T_i\\ \check{w}_{ij} & |x_i(t)| > T_i\end{cases}$$

with switching jumps $T_i > 0, \hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}, \hat{w}_{ij}, \check{w}_{ij}, \hat{c}_i, \check{c}_i, i, j = 1, 2, \ldots, n$ are all constant numbers; $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is the state vector of the neurons; $f_j(\cdot)$ represents the neuron activation function; $I_i(t)$ denotes the external inputs from outside the neural networks at time t; $\tau(t)$ is the discrete transmission time-varying delay satisfying $0 \le \tau(t) \le \overline{\tau}$, where $\overline{\tau}$ is a real constant; k_{ij} is the delay kernel function; $\Delta A = (\Delta a_{ij})_{n\times n}$, $\Delta B = (\Delta b_{ij})_{n\times n}$, $\Delta W = (\Delta w_{ij})_{n\times n}$ denotes the parameter uncertainty.

Remark 1. Model (1) includes the unbounded distributed delays and the parametric uncertain, which is the extension of the previous models studied in [5,8–16,23]. To be more specific, if the connection weight is deterministic, i. e., $c_i(x_i(t)) = c_i$, $a_{ij}(x_i(t)) = a_{ij}$, $b_{ij}(x_i(t)) = b_{ij}$ and $k_{ij}(t) = 1$ for $t \in [0, \theta]$, where θ is a positive constant, then (1) reduces to the model considered in [39]. And if $k_{ij}(t) = 1$, then the model (1) turns into the model considered in [5,8–16,23]. Hence, the model here is general than the open literatures.

To state conveniently, in this paper, let $\overline{c}_i = \min\{\hat{c}_i, \check{c}_i\}$, $\underline{c}_i = \max\{\hat{c}_i, \check{c}_i\}$, $\overline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\overline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\overline{w}_{ij} = \min\{\hat{w}_{ij}, \check{w}_{ij}\}$, $\underline{w}_{ij} = \max\{\hat{w}_{ij}, \check{w}_{ij}\}$. Based on the theory of differential inclusions and the definition of Filippov solution [40], the model (1) can be rewrite as the following differential inclusion:

$$\begin{aligned}\dot{x}_i(t) \in &-[\overline{c}_i, \underline{c}_i]x_i(t) + \sum_{j=1}^{n}([\overline{a}_{ij}, \underline{a}_{ij}] + \Delta a_{ij})f_j(x_j(t))\\ &+\sum_{j=1}^{n}([\overline{b}_{ij}, \underline{b}_{ij}] + \Delta b_{ij})f_j(x_j(t-\tau(t)))\\ &+\sum_{j=1}^{n}([\overline{w}_{ij}, \underline{w}_{ij}] + \Delta w_{ij})\int_0^{\infty} k_{ij}(s)f_j(x_j(t-s))\,ds + I_i(t)\end{aligned} \tag{2}$$

Transform system (2) into the vector form as

$$\begin{aligned}\dot{x}(t) \in &-[\overline{C}, \underline{C}]x(t) + [\overline{A}, \underline{A}] + \Delta A)f(x(t)) + ([\overline{B}, \underline{B}] + \Delta B)f(x(t-\tau(t)))\\ &+([\overline{W}, \underline{W}] + \Delta W)\int_0^{\infty} K(s)f(x(t-s))\,ds + I(t)\end{aligned} \tag{3}$$

where

$\overline{A} = (\overline{a}_{ij})_{n\times n}, \overline{B} = (\overline{b}_{ij})_{n\times n}, \overline{W} = (\overline{w}_{ij})_{n\times n}, \overline{C} = \mathrm{diag}(\overline{c}_1, \overline{c}_2, \ldots, \overline{c}_n)$
$\underline{A} = (\underline{a}_{ij})_{n\times n}, \underline{B} = (\underline{b}_{ij})_{n\times n}, \underline{W} = (\underline{w}_{ij})_{n\times n}, \underline{C} = \mathrm{diag}(\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_n)$
$\Delta A = (\Delta a_{ij})_{n\times n}, \Delta B = (\Delta b_{ij})_{n\times n}, \Delta W = (\Delta w_{ij})_{n\times n},$
$I(t) = [I_1(t), I_2(t), \ldots, I_n(t)]^{\mathrm{T}}$

Taking system (1) or (2) as the drive system, and the response system is given by

$$\begin{aligned}\dot{y}_i(t) \in &-[\overline{c}_i, \underline{c}_i]xy_i(t) + \sum_{j=1}^{n}([\overline{a}_{ij}, \underline{a}_{ij}] + \Delta\tilde{a}_{ij})f_j(y_j(t))\\ &+\sum_{j=1}^{n}([\overline{b}_{ij}, \underline{b}_{ij}] + \Delta\tilde{b}_{ij})f_j(y_j(t-\tau(t)))\\ &+\sum_{j=1}^{n}([\overline{w}_{ij}, \underline{w}_{ij}] + \Delta\tilde{w}_{ij})\int_0^{\infty} k_{ij}(s)f_j(y_j(t-s))\,ds + I_i(t)\end{aligned} \tag{4}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^{\mathrm{T}}$ is the state vector of the response system; $\Delta\tilde{a}_{ij}$, $\Delta\tilde{b}_{ij}$, $\Delta\tilde{w}_{ij}$ are the parameter uncertainty of the response system. Moreover, it is different with the parameter uncertainty of the drive system.

Consider the impulsive effects in system (4), we have the following model:

$$\begin{cases}\dot{y}_i(t) \in -[\overline{c}_i, \underline{c}_i]y_i(t) + \sum_{j=1}^{n}([\overline{a}_{ij}, \underline{a}_{ij}] + \Delta\tilde{a}_{ij})f_j(y_j(t))\\ \quad +\sum_{j=1}^{n}([\overline{b}_{ij}, \underline{b}_{ij}] + \Delta\tilde{b}_{ij})f_j(y_j(t-\tau(t)))\\ \quad +\sum_{j=1}^{n}([\overline{w}_{ij}, \underline{w}_{ij}] + \Delta\tilde{w}_{ij})\int_0^{\infty} k_{ij}(s)f_j(y_j(t-s))\,ds + I_i(t), & t \ne t_k\\ y_i(t_k) = y_i(t_k^-) + (\mu_{ik} - 1)e_i(t_k^-) & t = t_k, k = 1, 2, \ldots, \mathbb{N}\end{cases} \tag{5}$$

where μ_{ik} denotes the impulsive perturbations of the ith neuron at time t_k; $e_i(t) = y_i(t) - x_i(t)$ is the synchronization error. Suppose that the discrete instant set t_k satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$. We also suppose that $\lim_{k\to\infty} t_k = \infty$, $\lim_{h\to 0^+} y(t_k - h) = x(t_k^-)$ and $\lim_{h\to 0^+} y(t_k + h) = y(t_k^+)$, which imply that $y(t)$ is right continuous at t_k.

For notational simplicity, similar as the method used in Ref [14], we assume that

$[\underline{C}, \overline{C}]x(t) - [\underline{C}, \overline{C}]y(t) \subseteq [\underline{C}, \overline{C}][x(t) - y(t)],$
$[\underline{A}, \overline{A}]f(x(t)) - [\underline{A}, \overline{A}]f(y(t)) \subseteq [\underline{A}, \overline{A}][f(x(t)) - f(y(t))],$
$[\underline{B}, \overline{B}]f(x(t-\tau(t))) - [\underline{B}, \overline{B}]f(y(t-\tau(t))) \subseteq [\underline{B}, \overline{B}][f(x(t-\tau(t))) - f(y(t-\tau(t)))],$