Neurocomputing 149 (2015) 667–676

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Weak, modified and function projective synchronization of chaotic memristive neural networks with time delays ☆

Huaiqin Wu *, Ruoxia Li, Rong Yao, Xiaowei Zhang

College of Science, Yanshan University, Qinhuangdao 066001, China

ARTICLE INFO

Article history:
Received 17 December 2013
Received in revised form
8 July 2014
Accepted 8 August 2014
Communicated by R.W. Newcomb
Available online 16 September 2014

Keywords:
Memristor
Neural networks
Projective synchronization
Adaptive state-feedback controller
Time delays

ABSTRACT

This paper deals with the weak, modified and function projective synchronization issues for chaotic memristive neural networks with time delays. By applying the generalized Halanay inequality, a state-feedback controller is designed, and a novel condition is proposed to ensure the network be weak synchronization with certain error level. By means of Lyapunov–Krasovskii functional approach, the adaptive state-feedback controllers are designed and unknown control parameters are determined by adaptive updated laws to achieve function and modified projective synchronization. Several weak, modified and function projective synchronization conditions are addressed to ensure the synchronization goal. Finally, an illustrative example is given to demonstrate the effectiveness of the theoretical results.

1. Introduction

Memristor, the fourth basic circuital element along with resistor, inductor and capacitor, was first theoretically predicted by Chua in 1971 [1] (Fig. 1 [2]). In late 2008, an actual physical memristor was realized as a TiO_2 nanocomponent in Hewlett–Packard Laboratory [2]. This new passive electronic device has attracted unprecedented worldwide attention because of its potential applications in various fields, from new high-speed low-power processors [3], filters [4] to new biological models for associative memory [5], learning models of simple organisms [6], and in particular chaotic circuit [7–9], to which memristor has been introduced as new technology process nodes.

Recently, memristive neural networks (MNNs) have been designed based on primitive network by replacing resistors with memristors. Pershin and Di Ventra constructed a simple MNN composed of three electronic neurons connected by two memristor emulator synapses [5], it demonstrated the formation of the associative memory in this network. In [10], Itoh and Chua designed a memristic cellular automaton and discrete-time cellular neural network, which acts in quite a few applications, such as logical operations, image processing operations, complex behaviors, higher brain functions, and RSA algorithm. More recently, the nonlinear dynamics of MNNs with time delays have also been investigated in [11–19].

We note that a detailed analytical study of synchronization or anti-synchronization control of the neural network is necessary. Synchronization, which means the dynamical behaviors of coupled systems achieve spatial state at the same time, has been applied in secure communication, biology systems, linguistic networks, and information processing. As we know, synchronization exists in various types, such as complete synchronization [20], anti-synchronization [21], lag synchronization [22], generalized synchronization [23], projective synchronization [24–28], phase synchronization [29] and so on. In [31], Wen et al. considered the adaptive synchronization of memristor-based Chua's circuits and designed an adaptive controller to achieve the synchronization goal. Wu et al. investigated the exponential synchronization of memristor-based recurrent neural networks with time delays in [30,32]. It should be pointed out that (1) in [30–32], the following assumption

$$co\{\underline{a}_{ij},\overline{a}_{ij}\}f_j(x_j(t))-co\{\underline{a}_{ij},\overline{a}_{ij}\}f_j(y_j(t)) \subseteq co\{\underline{a}_{ij},\overline{a}_{ij}\}(f_j(x_j(t))-f_j(y_j(t))) \quad (1)$$

was used in the proof of the main results. It can be easily checked that this assumption holds only when $f_j(x_j(t))$ and $f_j(y_j(t))$ have different signs, or $f_j(x_j(t))=0$ or $f_j(y_j(t))=0$, where $co(Q)$ denotes the closure of the convex hull of Q. (2) The results in [30–32] are independent of the neuron state switching jumps $T_i, i=1,2,\dots,n$. From the view of mathematics, the results obtained in these works are meaningless to the theory research and application.

☆This work was supported by the Natural Science Foundation of Hebei Province of China (A2011203103).
* Corresponding author.
E-mail address: huaiqinwu@ysu.edu.cn (H. Wu).

http://dx.doi.org/10.1016/j.neucom.2014.08.008
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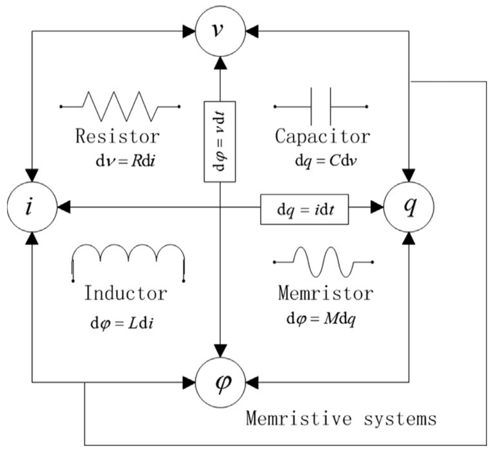

Fig. 1. The four fundamental two-terminal circuit elements: resistor, capacitor, inductor and memristor [2].

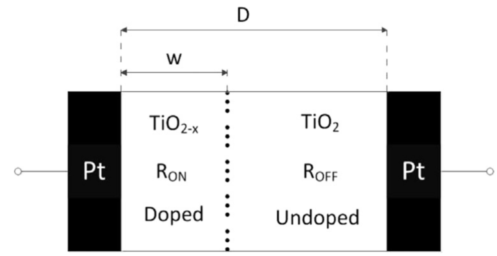

Fig. 2. Structure of the TiO_2 memristor [37].

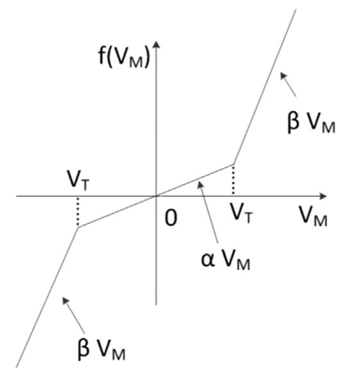

Fig. 3. The constitutive relation $f(V_M)$ of a piecewise-linear memristor.

In this paper, we treat the weak, modified and function projective synchronization issue of chaotic MNNs with time delays. The main novelty of our contribution lies in three aspects: (1) a novel state-feedback controller is designed, and a condition is presented to achieve weak projective synchronization with an error level; (2) an adaptive controller is proposed to achieve function projective synchronization. As a special case, an adaptive modified projective synchronization condition is also developed; (3) the assumption (1) is abandoned in this paper. More precisely, all the conclusions are related to the state switching jumps $T_i,\ i=1,2,\dots,n$.

The framework of this paper is organized as follows: In Section 2, the mathematics models of memristor and chaotic MNNs are described, and some preliminaries are introduced. In Section 3, a general scheme of weak projective synchronization for chaotic MNNs is presented. The adaptive modified and function projective synchronization are discussed in Section 4. In Section 5, a numerical example is presented to demonstrate the validity of the proposed results. Section 6 is the conclusions of this paper.

Notation: $\mathcal{R}$ denotes the set of real numbers, $\mathcal{R}^n$ denotes the n-dimensional Euclidean space, $\mathcal{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. For $r>0$, $\mathcal{C}([-r,0];\mathcal{R}^n)$ denotes the family of continuous function φ from $[-r,0]$ to $\mathcal{R}^n$ with the norm $\|\varphi\| = \sup_{-r\le s\le 0}|\varphi(s)|$. $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ denote the maximum eigenvalue and minimum eigenvalue of $P\in\mathcal{R}^{n\times n}$, respectively. P^T, P^{-1} represent the transpose and inverse of $P\in\mathcal{R}^{n\times n}$, respectively. The Euclidian norm of the square matrix P is denoted by $\|P\|$, where $\|P\| = \sqrt{\lambda_{\max}(PP^T)}$.

2. Model description and preliminaries

2.1. Mathematics model of memristor

In the TiO_2 memristor, as shown in Fig. 2, a thin undoped titanium dioxide (TiO_2) layer and a thin oxygen-deficient doped titanium dioxide (TiO_{2-x}) layer are sandwiched between two platinum electrodes. When a voltage (or current) is applied to the device, the width of the TiO_2 and TiO_{2-x} layer changes as a function of the applied voltage (or current). As a result, the resistance between the two electrodes is altered.

Let D and ω denote the thickness of the sandwiched area and the doped area (oxygen deficient area) in the TiO_2 memristor, respectively, and let R_{ON} and R_{OFF} be the resistances at high ($\omega/D=0$) and low dopant ($\omega/D=1$) concentration areas, respectively.

Then, the voltage current relationship is given as

$$v(t) = \left(R_{ON}\frac{\omega(t)}{D} + R_{OFF}\left(1-\frac{\omega(t)}{D}\right)\right)i(t),$$

and the state variable ω is defined by

$$\frac{d\omega(t)}{D} = \mu_v\frac{R_{ON}}{D}i(t),$$

where μ_v is the dopant mobility. This model is called a linear drift model.

In order to better understand the memristor, we implemented a threshold model of voltage-controlled memristor previously suggested in [36]. In this model, the following equations were used:

$$f(V_M) = \beta V_M + 0.5(\alpha-\beta)[|V_M+V_T| - |V_M-V_T|],$$
$$\dot{x} = f(V_M)\theta(x-R_1)\theta(R_2-x),$$

where the memristance rate of change is determined by the function $f(V_M)$, x is the state variable of memristor, $\theta(\cdot)$ is the step function, α and β characterize the rate of memristance change at $|V_M|\le V_T$ and $|V_M|>V_T$, respectively, V_M is a voltage drop on memristor, V_T is a threshold voltage, R_1 and R_2 are limiting values of memristance. In this equation, the θ-functions symbolically show that the memristance can change only between R_1 and R_2 with different rates α and β below and above the threshold voltage, as shown in Fig. 3.

2.2. Model of MNNs

Memristor-based recurrent neural network can be implemented by very large-scale integration (VLSI) circuits as shown in Fig. 4, and the connection weights are implemented by the memristors. For the i-th subsystem, $x_i(t)$ is the voltage of the capacitor C_i, $f_j(x_j(t))$, $g_j(x_j(t-\tau))$ are the functions about $x_i(t)$ without and with time delays, respectively, τ is the time delay with $\tau\ge 0$. M_{fij} is the memristor between the feedback function $f_j(x_j(t))$ and $x_i(t)$, M_{gij} is the memristor between the feedback function $g_j(x_j(t-\tau))$ and $x_i(t)$, $W_{fij}(x_j(t))$ and $W_{gij}(x_j(t))$ are the