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Leader-following consensus of linear multi-agent systems with randomly occurring nonlinearities and uncertainties and stochastic disturbances

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ABSTRACT

For the multi-agent community, one of the key challenges is the agents operating in open environments, in which stochastic noises affect the dynamics of agents. This paper deals with the leader-following consensus problem for a class of linear multi-agent systems model with randomly occurring nonlinearities and uncertainties, and stochastic disturbances. The communication topology is assumed to be undirected and fixed. The stochastic Brownian motions are used to describe the source of extrinsic disturbances. The randomly occurring nonlinearities are introduced to describe the nonlinear intrinsic dynamics occurring in a probabilistic way. The randomly occurring uncertainties are adopted to reflect more realistic dynamical behaviors of the agent systems that are caused by noisy environment. Sufficient conditions are derived to make all follower agents asymptotically reach the state of leader in the mean square sense. Simulation results illustrate the theoretical results.

1. Introduction

The problem of consensus for multi-agent systems has attracted compelling attention over the last ten years due to its extensive applications in real-world distributed computation, rendezvous tasks, spacecraft formation flying, biological systems, sensor networks, cooperative surveillance and so on [1–4]. Consensus means the agreement of a group of agents on their common states via local interactions. Consensus of multi-agents or synchronization of complex dynamical networks [5–8], which is tightly related to the Lyapunov stability theory, has been attracting great interest in the scientific community. Some other relevant topics for multi-agent systems have also been addressed, such as group consensus [9], finite-time consensus [10], consensus controllability, observability [11], consensus of multi-agent systems on time scales [12], and consensus of fractional-order heterogeneous multi-agent systems [13].

A particularly interesting topic is the consensus of a group of agents with a leader, where the leader is a special agent that specifies the objective for the whole group. Such a problem is commonly called leader-following consensus problem [14–20]. Compared with its counterpart-leaderless cases, leader-following configuration is an energy saving mechanism, which was found in many biology systems, and it can also enhance the communication and orientation of the flock [16].

Many works on consensus problem focus on the case where the dynamics of each agent is single-integrator or double-integrator [10,21–25], or even a more general case: an nth order linear system [16,26–28] described by the differential equations:

$$\frac{dx_i(t)}{dt} = Ax_i(t) + Bu_i(t), \tag{1}$$

where x_i is the state of agent i and u_i is the control input. Two matrices A and B with compatible dimensions are assumed that the pair (A, B) is stabilizable. However, in reality, mobile agents may be governed by more complicated intrinsic dynamics. In recent five years, some authors have considered the consensus problems with nonlinear agent dynamics [22–25]. The authors [22] investigated the consensus problem for the second order multi-agent systems by introducing a nonlinear term describing the intrinsic dynamics of each agent based on algebraic graph theory, matrix theory, and Lyapunov control approach. The work of [22] was extended to the leader-following case via pinning control in [23]. Both local and global consensus are investigated for cooperative agents with nonlinear dynamics in a directed network [24]. The design of distributed control gains for leader-following consensus in multi-agent systems with second-order nonlinear dynamics is proposed in [25].

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In the networked world nowadays, the nonlinear intrinsic dynamics may be subject to random changes in environmental disturbances, for instance, network-induced random failures and repairs of components and sudden environmental disturbances. Therefore, nonlinear dynamics may occur in a probabilistic way with certain types and intensity, which is particularly true in a networked environment. Very recently, the randomly occurring nonlinearities (RONs) have been introduced into the model of complex networks for synchronization [29,30], state estimation [31], stability [32] and control problem [33]. However, RONs are not taken into account when modeling multi-agent networks for the consensus problem. On the other hand, the agents that operate in open environments will have to cope with a tremendous amount of uncertainty, due to limited computational and communication resources. Kim et al. [28] studied the output consensus of linear multi-agent systems (1) with the introduction of parameter uncertainties in matrix A. A new type of uncertainties called as randomly occurring uncertainties (ROUs) has been proposed by Hu et al. [33] due to the fact that the uncertainties may be subject to random changes in environmental circumstances. To the best of our knowledge, ROUs have not yet attracted adequate research attention in multi-agent systems.

Recently, some initial works have been made on the second-order consensus problem for multi-agent systems with exogenous disturbance [34,35]. The deterministic disturbances are generated by a linear exogenous system [34], or from some nonlinear exogenous systems [35]. In these two works, the external disturbance is assumed to be exactly known a priori or precisely observable, which seems not to be practical. In order to avoid this limitation, we use stochastic disturbances in this paper to model the external disturbances which are widely existed in practical processes.

Motivated by the above discussion, the principal focus of the present study is to propose a new type of nth order linear multi-agent systems model with RONs, ROUs and stochastic disturbances and then to study its leader-following consensus problem. The rest of this paper is organized as follows. In Section 2, some basic notations and useful results of the graph theory are reviewed. In Section 3, we show that the leader-following consensus for the linear multi-agent systems with RONs, ROUs and stochastic disturbances can be achieved via the proposed control protocol. Simulation results are given in Section 4 to validate our control laws and a brief conclusion is made in Section 5.

2. Preliminaries and problem formulation

2.1. Basic definitions and notations

We use a graph to represent the communication topology or information flow between agents and the leader. The interaction topology of information flow between N agents is described by a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{1,2,\dots,N\}$ is the set of vertices representing N follower agents and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ is the set of ordered edges of the graph. An ordered edge of $\mathcal{G}$ is denoted by (i,j), representing that agent i send information to agent j. The graph is undirected, that is, the edges (i,j) and (j,i) in $\mathcal{E}$ are considered to be the same. Neighbors of node i are the set $\mathcal{N}_i=\{j:(j,i)\in\mathcal{E}\}$. A path is a sequence of edges of the form $(i_1,i_2),(i_2,i_3),\dots$. A graph is connected if any two distinct nodes of the graph are connected through a path that follows the direction of the edges of digraph [2]. The leader is represented by vertex 0 sending the information to the follower agents which are neighbors of the leader. Then, we have a graph $\overline{\mathcal{G}}$, which consists of graph $\mathcal{G}$, vertex 0 and edges between the leader 0 and its neighbors. Throughout this paper, graph $\mathcal{G}$ is assumed to be undirected and the edges $(0,i)$ of $\overline{\mathcal{G}}$ are directed.

The adjacency matrix $\mathcal{A}$ of a graph $\mathcal{G}$ on vertex $\{1,2,\dots,N\}$ is an $N\times N$ matrix, whose (ij)th entry a_{ij} is defined as $a_{ii}=0$, $a_{ij}=a_{ji}=1$ if $(i,j)\in\mathcal{E}$ and $a_{ij}=a_{ji}=0$ otherwise. The Laplacian matrix $\mathcal{L}=(\mathcal{L}_{ij})_{N\times N}$ of $\mathcal{G}$ is defined by $\mathcal{L}_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$ and $\mathcal{L}_{ij}=-a_{ij}$ for $i\neq j$. The structure of $\overline{\mathcal{G}}$ is described by a matrix $\mathcal{H}=\mathcal{L}+\mathcal{D}$, where $\mathcal{D}=\text{diag}\{b_1,b_2,\dots,b_n\}$ with $b_i=1$ if $(0,i)$ is an edge of $\overline{\mathcal{G}}$, and with $b_i=0$ otherwise, and $\mathcal{H}=(h_{ij})_{N\times N}$ with

$$h_{ij}=\begin{cases}\mathcal{L}_{ii}+b_i, & i=j,\\ \mathcal{L}_{ij}, & i\neq j.\end{cases} \tag{2}$$

For more details, one can see [11,16,27].

Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n dimensional Euclidean space and the set of all $m\times n$ real matrices. $P>0$ means that matrix P is symmetric, real and positive definite. I and 0 denote the identity matrix and the zero matrix with compatible dimensions, respectively. diag$\{\cdots\}$ stands for a block-diagonal matrix and tr$\{A\}$ denotes the trace of matrix A. $\lambda_{max}(A)$ and $\lambda_{min}(A)$ mean the largest and the smallest eigenvalue of matrix A, respectively. The superscript "T" denotes matrix transposition and the asterisk "$*$" in a matrix is used to represent the term that is induced by symmetry. The Kronecker product of matrices $A\in\mathbb{R}^{m\times n}$ and $B\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $A\otimes B$. $(\Omega,\mathcal{F},\mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all $\mathcal{P}-$null sets and is right continuous) and $\mathbb{E}\{\cdot\}$ stands for the mathematical exception operator with respect to the given probability measure $\mathcal{P}$. Sometimes, the arguments of a function will be omitted in the analysis when no confusion arises.

2.2. Models and consensus algorithms

Taking RONs, ROUs and stochastic disturbances into consideration, the agent model (1) can be described by

$$\frac{dx_i(t)}{dt}=A(t)x_i(t)+\alpha(t)f(t,x_i(t))+Bu_i(t)+\sigma_i(t,x_i(t))n(t),\quad i=1,2,\dots,N \tag{3}$$

where N is the number of follower agent, $x_i\in\mathbb{R}^n$ is the agent i's state, and $u_i\in\mathbb{R}^q$ is agent i's input which can only use local information from its neighbor agents. $\sigma_i(\cdot,\cdot):\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ is the noise intensity function vector. $n(t)$ is a scalar zero mean Gaussian white noise process. Recall that the time derivative of a Wiener process (Brownian motions) is a white noise process. We have $dw(t)=n(t)\,dt$, where $w(t)$ is a one-dimensional Wiener process defined on $(\Omega,\mathcal{F},\mathcal{P})$ satisfying

$$\mathbb{E}\{w(t)\}=0,\quad \mathbb{E}\{[w(t)]^2\}=dt. \tag{4}$$

Hence, system (3) can be rewritten as the following stochastic differential equations:

$$dx_i(t)=[A(t)x_i(t)+\alpha(t)f(t,x_i(t))+Bu_i(t)]\,dt+\sigma_i(t,x_i(t))\,dw(t). \tag{5}$$

In this paper, we suppose that the leader, labeled as $i=0$, has the following dynamics:

$$dx_0(t)=[A(t)x_0(t)+\alpha(t)f(t,x_0(t))]\,dt+\sigma_0(t,x_0(t))\,dw(t), \tag{6}$$

where $x_0\in\mathbb{R}^n$ is the state of the leader, $\sigma_0(t,x_0(t)):\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ is the noise intensity and $w(t)$ satisfies (4). In Eqs. (3) and (6), the matrix

$$A(t)=A+\beta(t)\Delta A(t), \tag{7}$$

and

$$\Delta A(t)=MF(t)H. \tag{8}$$

The real-valued matrix $\Delta A(t)$ represents the norm-bounded parameter uncertainties of the structure (8) with M and H are known real constant matrices with appropriate dimensions, which