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Robust passivity analysis of neural networks with discrete and distributed delays

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ABSTRACT

This paper focuses on the problem of passivity of neural networks in the presence of discrete and distributed delay. By constructing an augmented Lyapunov functional and combining a new integral inequality with the reciprocally convex approach to estimate the derivative of the Lyapunov–Krasovskii functional, sufficient conditions are established to ensure the passivity of the considered neural networks, in which some useful information on the neuron activation function ignored in the existing literature is taken into account. Three numerical examples are provided to demonstrate the effectiveness and the merits of the proposed method.

1. Introduction

During the last few decades, neural networks have received great attention due to their extensive applications in various fields such as signal processing, pattern recognition, fixed-point computation and other scientific areas (see, e.g., [1–3]). Time delays are inevitable in the implementation of artificial neural networks as a result of the finite switching speed of amplifier. On the other hand, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, i.e., the conduction velocities along these pathways are distributed. Thus, there is a distribution of propagation delays. As is well known, the manifestation of time delays in neurons may lead to some undesirable dynamic network behaviors such as oscillation, instability or other poor performances. Therefore, various issues of neural networks with time delays have been addressed, and many results have been achieved in the literature (see, e.g., [4–26] and the references therein).

It is well known that the dissipativity theory plays an important role in the stability analysis of dynamical systems, nonlinear control and other areas (see, e.g., [27–30]). Passivity, as a special case of dissipativity, tells more than just stability, which relates the input and output to the storage function, and hence defines a set of useful input–output properties. The delay-independent passivity of neural networks was addressed in [31,32]. In [33,34,40], delay-dependent passivity conditions were obtained for uncertain continuous-time neural networks with discrete delay. Recently, a complete delay-decomposing approach was employed to study the passivity of neural networks with time-varying delay in [35]. For neural networks with both discrete and distributed delays, the problem of passivity analysis was addressed in [36–38]. By employing an augmented Lyapunov functional, improved passivity conditions were proposed in [39]. Nevertheless, some useful information on the activation function was ignored in [39] and thus the results can be further improved. In addition, to bound the integral term in derivative of Lyapunov functional [36–39], unexceptionally employ Jensen's inequality-based integral inequalities or the free matrix approach. Very recently, a new integral inequality based on Wirtinger's inequality was proposed in [42], which is less conservative than other integral inequalities derived by Jensen's inequality. Therefore, the results in [36–39] are still conservative and need to be further investigated.

This paper investigates the delay-dependent passivity of neural networks with both discrete and distributed delays. By constructing an augmented Lyapunov functional and utilizing a tighter integral inequality to handle the integral term in derivative of Lyapunov functional, some improved passivity conditions are obtained, which are formulated in terms of linear matrix inequalities (LMIs) and can be readily checked by existing convex optimization algorithms. The effectiveness is verified by three illustrating examples.

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Notations: Through this paper, N^T and N^{-1} stand for the transpose and the inverse of the matrix N, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $P>0$ $(P\geq 0)$ means that the matrix P is symmetric and positive definite (semi-positive definite); diag $\{\cdots\}$ denotes a block-diagonal matrix; $\|z\|$ is the Euclidean norm of z; I and 0 represent the identity matrix and a zero matrix, respectively. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. System description

Consider the following neural network with discrete and distributed delay:

$$\begin{cases} \dot{x}(t)=-Ax(t)+Wg(x(t))+W_1g(x(t-\tau(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\,\mathrm{d}s+u(t)\\ y(t)=g(x(t))\\ x(t)=\phi(t),\quad -\overline{\tau}\leq t\leq 0\end{cases} \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in\mathbb{R}^n$ is the neuron state vector; $u(t)$ and $y(t)$ are the input and output vectors, respectively; $g(\cdot)=[g_1(\cdot),g_2(\cdot),\ldots,g_n(\cdot)]^T$ denotes the neuron activation functions; $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)>0$, W, W_1 and W_2 are known interconnection weight matrices, and the delay, $\tau(t)$, is a time-varying function with $0\leq\tau(t)\leq\overline{\tau}$, $\dot{\tau}(t)\leq\mu$. $\phi(t)$ is the initial condition. The neuron activation function is assumed to satisfy the following assumption.

Assumption 1 (*Liu et al.* [19]). The function $g_i(\cdot)$ in (1) is continuous and satisfies

$$F_i^-\leq\frac{g_i(\alpha_1)-g_i(\alpha_2)}{\alpha_1-\alpha_2}\leq F_i^+,\quad i=1,2,\ldots,n \tag{2}$$

where $g_i(0)=0$, α_1, $\alpha_2\in\mathbb{R}$, $\alpha_1\neq\alpha_2$, and F_i^- and F_i^+ are known real scalars.

Remark 1. The above assumption on the activation function was originally proposed in [19], which is more general than [33,34,36,37] since F_i^- and F_i^+ may be positive, zero or negative, that is to say, the activation function under Assumption 1 may be non-monotonic, non-differentiable, and unbounded. Thus, the passivity condition developed in this paper is less restrictive than [33,34,36,37].

Now, we are in a position to introduce the following definition and lemmas, which are indispensable to derive the main result of the this paper.

Definition 1 (*Li and Liao* [32]). The neural network (1) is said to be passive if there exists a scalar $\gamma\geq 0$ such that for all $t_f\geq 0$

$$2\int_0^{t_f}y(s)^Tu(s)\,\mathrm{d}s\geq-\gamma\int_0^{t_f}u(s)^Tu(s)\,\mathrm{d}s \tag{3}$$

under the zero initial condition.

Lemma 1 (*Gu* [45]). *For any symmetric positive definite matrix $M>0$, a scalar $\gamma>0$ and a vector function $\omega:[0,\gamma]\rightarrow\mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^{\gamma}\omega(s)\,\mathrm{d}s\right)^TM\left(\int_0^{\gamma}\omega(s)\,\mathrm{d}s\right)\leq\gamma\left(\int_0^{\gamma}\omega^T(s)M\omega(s)\,\mathrm{d}s\right). \tag{4}$$

Lemma 2 (*Park et al.* [41]). *Given positive integers m and n, a scalar $\beta\in(0,1)$, a given $R>0$, and two matrices W_1, $W_2\in\mathbb{R}^{n\times m}$, define, for all vector $\xi\in\mathbb{R}^m$, the function $\Theta(\beta,R)$ described by*

$$\Theta(\beta,R)=\frac{1}{\beta}\xi^TW_1^TRW_1\xi+\frac{1}{1-\beta}\xi^TW_2^TRW_2\xi. \tag{5}$$

Then, if there exists a matrix $X\in\mathbb{R}^{n\times n}$ such that $\begin{bmatrix}R & X\\ X^T & R\end{bmatrix}>0$, the following inequality holds:

$$\min_{\beta\in(0,1)}\Theta(\beta,R)\geq\begin{bmatrix}W_1\xi\\ W_2\xi\end{bmatrix}^T\begin{bmatrix}R & X\\ X^T & R\end{bmatrix}\begin{bmatrix}W_1\xi\\ W_2\xi\end{bmatrix}. \tag{6}$$

Lemma 3 (*Seuret and Gouaisbaut* [42]). *For any positive matrix Z and for differentiable signal x in $[\alpha,\beta]\rightarrow\mathbb{R}^n$, the following inequality holds:*

$$\int_{\alpha}^{\beta}\dot{x}^T(u)Z\dot{x}(u)\,\mathrm{d}u\geq\frac{1}{\beta-\alpha}\hat{\Omega}^T\hat{Z}\hat{\Omega} \tag{7}$$

where $\hat{Z}=\mathrm{diag}\{Z,3Z\}$ *and*

$$\hat{\Omega}=\begin{bmatrix}x(\beta)-x(\alpha)\\ x(\beta)+x(\alpha)-\frac{2}{\beta-\alpha}\int_{\alpha}^{\beta}x(u)\,\mathrm{d}u\end{bmatrix}.$$

Lemma 4 (*Petersen and Hollot* [43]). *Let H, E, and F(t) be real matrices of appropriate dimensions with F(t) satisfying $F^T(t)F(t)\leq I$. Then, for any scalar $\varepsilon>0$,*

$$HF(t)E+(HF(t)E)^T\leq\varepsilon^{-1}HH^T+\varepsilon E^TE.$$

3. Main results

Firstly, for simplicity of vector and matrix representation, the following is denoted as

$$\eta_1(t)=\left[x^T(t)\quad\int_{t-\overline{\tau}}^{t}x^T(s)\,\mathrm{d}s\right]^T,$$
$$\eta_2(t)=[x^T(t)\ \ g^T(x(t))]^T,$$
$$\eta_3(t)=[x^T(t)\ \ x^T(t-\tau(t))\ \ x^T(t-\overline{\tau})]^T,$$
$$\eta_4(t)=\left[g^T(x(t))\ \ g^T(x(t-\tau(t)))\quad\int_{t-\tau(t)}^{t}g^T(x(s))\,\mathrm{d}s\ \ u^T(t)\right]^T,$$
$$\xi(t)=\Big[\eta_3^T(t)\ \ \eta_4^T(t)\ \ \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}x^T(s)\,\mathrm{d}s\ \ \frac{1}{\overline{\tau}-\tau(t)}\int_{t-\overline{\tau}}^{t-\tau(t)}x^T(s)\,\mathrm{d}s$$
$$\int_{t-\overline{\tau}}^{t-\tau(t)}g^T(x(s))\,\mathrm{d}s\ \ \dot{x}^T(t)\Big]^T,$$
$$K_1=\mathrm{diag}\{F_1^+,F_2^+,\ldots,F_n^+\},$$
$$K_2=\mathrm{diag}\{F_1^-,F_2^-,\ldots,F_n^-\},$$
$$e_i=[0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(11-i)n}],\quad i=1,2,\ldots,11.$$

Based on the Lyapunov–Krasovskii functional approach, the following result can be obtained.

Theorem 1. *The neural network (1) is passive if there exist $2n\times 2n$-matrices $P_a>0$, $Q_a>0$, $S_a>0$, X, Y, $n\times n$-matrices $R>0$, $Z>0$, U_1, U_2, diagonal matrices $\Lambda_1>0$, $\Lambda_2>0$, $\Lambda_3>0$, $D_1>0$, $D_2>0$, and scalars $\gamma\geq 0$, such that the following LMIs are satisfied for $\tau(t)\in\{0,\overline{\tau}\}$:*

$$\Xi<0, \tag{8}$$

$$\Phi_1=\begin{bmatrix}S_a & X\\ X^T & S_a\end{bmatrix}>0, \tag{9}$$

$$\Phi_2=\begin{bmatrix}\overline{Z} & Y\\ Y^T & \overline{Z}\end{bmatrix}>0, \tag{10}$$

where

$$\begin{aligned}\Xi=&\,\Pi_1^TP_a\Pi_2+\Pi_2^TP_a\Pi_1+e_4^T(D_1-D_2)e_{11}+e_{11}^T(D_1-D_2)e_4\\&+e_1^T(K_1D_2-K_2D_1)e_{11}+e_{11}^T(K_1D_2-K_2D_1)e_1+\Pi_3^TQ_a\Pi_3\\&-(1-\mu)\Pi_4^TQ_a\Pi_4+e_1^TRe_1-e_3^TRe_3+\overline{\tau}^2\Pi_3^TS_a\Pi_3\\&+\overline{\tau}^2e_{11}^TZe_{11}-\Pi_5^T\Phi_1\Pi_5-\Pi_6^T\Phi_2\Pi_6+\Pi_7^T\Pi_8+\Pi_8^T\Pi_7\end{aligned}$$