Neurocomputing 94 (2012) 13–21

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Chaos and hyperchaos in fractional-order cellular neural networks ☆

Xia Huang [a], Zhao Zhao [a], Zhen Wang [b], Yuxia Li [a,*]

[a] Shandong Key Laboratory of Robotics and Intelligent Technology, College of Information and Electrical Engineering, Shandong University of Science and Technology, Qingdao 266590, China
[b] College of Information Science and Engineering, Shandong University of Science and Technology, Qingdao 266590, China

ARTICLE INFO

Article history:
Received 15 June 2011
Received in revised form
24 October 2011
Accepted 7 January 2012
Communicated by H. Jiang
Available online 12 April 2012

Keywords:
Hyperchaos
Fractional-order
Cellular neural network
Lyapunov exponents
Poincaré section
Bifurcation diagram

ABSTRACT

In this paper, a fractional-order four-cell cellular neural network is proposed and the complex dynamical behaviors of such a network are investigated by means of numerical simulations. Several varieties of interesting dynamical behaviors, such as periodic, chaotic and hyperchaotic motions, are displayed. In addition, it can be found that the network does exhibit hyperchaotic phenomena over a wide range of values of some specified parameter. The existence of chaotic and hyperchaotic attractors is verified with the related Lyapunov exponent spectrum, bifurcation diagram and phase portraits. Meanwhile, the Lyapunov exponents and Poincaré sections are calculated for some typical parameters, respectively.

1. Introduction

Hyperchaos, which is defined as a chaotic attractor with more than one positive Lyapunov exponents, was firstly reported by Rössler in 1979 [1]. Compared with chaotic attractor, it displays much richer dynamical behaviors, and therefore is considered as a powerful tool in secure communication for its capability of improving the security of chaotic communication systems [2]. During the past few decades, a considerable amount of research has been devoted to the development of efficient methods and techniques for the generation and circuit implementation of hyperchaos [3–5]. However, as we all know, one of the necessary conditions which ensure the existence of hyperchaotic attractors is that the minimal dimension of the phase space of an autonomous system is at least four. Evidently, if one wants to generate a more complicated hyperchaos (i.e., a hyperchaotic attractor with more than two positive Lyapunov exponents), it is unavoidable to increase the dimension of the autonomous system and this restricts the applications of the developed hyperchaotic system.

Fractional calculus, which deals with derivatives and integrals of arbitrary order, was firstly introduced 300 years ago. However, it is only in recent decades that fractional calculus is applied to physics, applied mathematics and engineering [6–8]. Fractional-order models have been proven to be an excellent instrument for the description of memory and hereditary properties of various materials and processes in comparison with classical integer-order models [6–8]. Nowadays, study on the complex dynamical behaviors of fractional-order systems, including chaos and hyperchaos, has become a very hot research topic. Compared with integer-order chaotic systems, fractional-order chaotic systems show higher nonlinearity and more degrees of freedom in the models due to the existence of the fractional derivatives. Moreover, the derivative orders can be used as secret keys as well [9]. It has been found that hyperchaotic behavior of an integer-order nonlinear system is preserved when the order becomes fractional [10]. Obviously, this allows us to generate a fractional-order hyperchaotic system from its integer-order counterpart without changing the numbers of the system equations. For instance, in [11], it is shown that the fractional-order Rössler hyperchaotic equation can still behave in a hyperchaotic manner with order as low as 3.8. In [12], the lowest fractional order for the hyperchaotic system to yield hyperchaos is approximately 3.7. In [13], a fractional-order hyperchaotic system with order 3.8 is proposed. In [14], the lowest order for the proposed system to generate hyperchaos is determined as 2.88.

Cellular neural networks (CNNs) have been extensively investigated both in theory and applications since it was proposed by Chua and Yang in [15,16]. It has been demonstrated that the classical integer-order CNNs can exhibit quite rich dynamical behaviors. For instance, bifurcation phenomena and chaotic behaviors for CNNs with or without delay are investigated in [17,18].

☆ This work was supported by the National Natural Science Foundation of China (Nos. 61004078, 60971022), the Natural Science Foundation of Shandong Province (Nos. ZR2009GQ009, ZR2009GM005), China Postdoctoral Science Foundation funded project and Special Funds for Postdoctoral Innovative Projects of Shandong Province.

* Corresponding author.
E-mail address: yuxiali2004@yahoo.com.cn (Y. Li).

0925-2312/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2012.01.011

Hyperchaotic behaviors in CNNs or Hopfield-type neural networks are displayed in [19,20]. In [21], a new four-dimensional CNN is designed, which can generate complex dynamics, including periodic solutions, chaos and hyperchaos. Furthermore, efforts have been made to study the complex dynamics of fractional-order cellular neural networks in recent years. In [22], Arena et al. firstly introduced a new class of CNN with fractional order cells. In [23], Petráš presented a fractional three-cell CNN which exhibits limit cycle and stable orbit for different values of parameters and the total order of such a system is 2.7. A fractional two-cell chaotic CNN was reported in [24] and the corresponding strange attractor was shown. It is worth noting that in view of the significance of fractional-order CNNs and the approximation methods, we refer the readers to [23] and references cited therein for more details.

However, to the best of our knowledge, there is no effort being made in the literature to study the hyperchaotic dynamics of fractional-order CNNs so far. Motivated by the above-mentioned reasons, we introduce fractional derivative to the four-cell CNN proposed in [21] and present a new fractional-order CNN system. The main aim of this paper is to investigate the hyperchaotic phenomena in the newly proposed fractional-order CNN, which may provide potential applications in secure communication. As will be shown below, when the parameters of the network are fixed, the lowest order for chaos to exist is determined. Moreover, hyperchaotic attractors are found within a certain range of some specified parameter. Besides, the existence of chaos and hyperchaos is verified with Lyapunov exponent spectrum, bifurcation diagram, Poincaré sections, and phase portraits, respectively.

The rest of this paper is organized as follows: in Section 2, basic definition in fractional calculus and a lemma to testify the stability of fractional-order linear systems are presented. In Section 3, the fractional-order four-cell CNN is proposed and the lowest order for chaos to exist is determined. Bifurcation analysis is made in Section 4 and the ranges of the specified parameter are determined for different dynamical behaviors, respectively. Finally, some concluding remarks are reported in Section 5.

2. Basic definition and preliminaries

There are three most frequently used definitions for fractional derivatives [6], that is, Riemann–Liouville, Grünwald–Letnikov and Caputo definitions. In pure mathematics, Riemann–Liouville derivative is more commonly used than Caputo derivative. However, the main advantage of Caputo definition is that the initial conditions for fractional-order differential equations with Caputo derivatives take on the same form as for integer-order differential equations [6]. And therefore, Caputo definition is used in this paper.

Definition 1 (*Podlubny [6]*). The Caputo fractional derivative of order α of a continuous function $f: R^+ \to R$ is defined as follows:

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_0^t \dfrac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}\, d\tau, & n-1 < \alpha < n, \\ \dfrac{d^n}{dt^n} f(t), & \alpha = n, \end{cases}$$

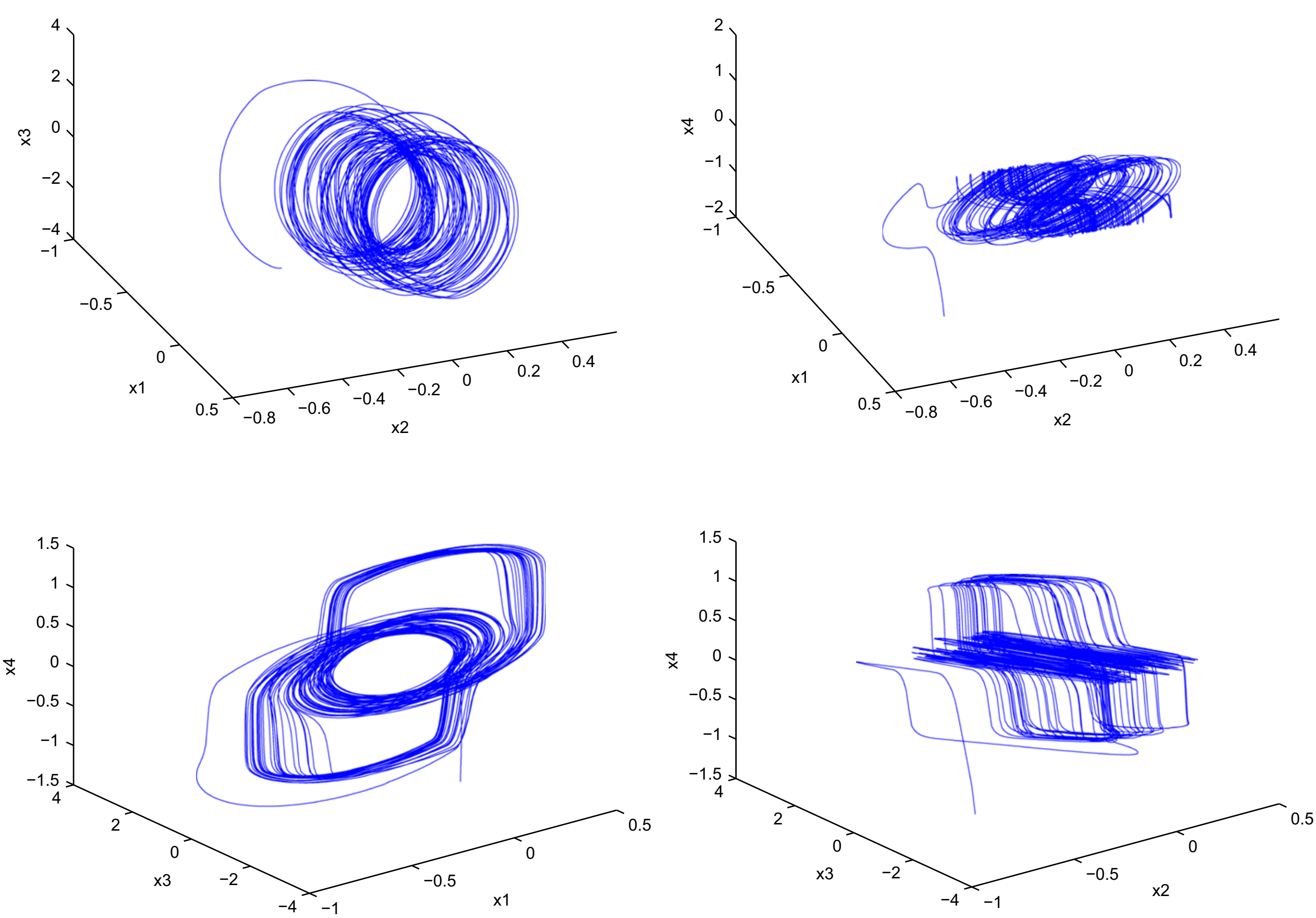

Fig. 1. Phase portraits of system (2) with $\alpha = 1$ and $w=0$.