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Stability analysis for a class of neutral-type neural networks with Markovian jumping parameters and mode-dependent mixed delays ☆

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ABSTRACT

This paper is concerned with the stability problem for a class of Markovian jumping neutral-type neural networks with mode-dependent mixed time-delays. The mixed time-delays are composed of discrete and distributed delays, both of which are mode-dependent. In addition, the distributed time-delays are characterized by the upper and lower bounds, both of which are mode-dependent. By constructing new Lyapunov–Krasovskii functionals, a unified framework is established to derive sufficient conditions for the concerned systems to be globally exponentially stable in mean square. A simulation example is provided to demonstrate the usefulness of the main results obtained.

1. Introduction

In the past few decades, the successful applications of recurrent neural networks (RNNs) in a variety of areas (e.g. pattern recognition, associative memory and combinational optimization [1,26]) have aroused a surge of research interests in the dynamical behaviors of the RNNs. Among various behaviors, the stability has proven to be the most important one that has received considerable research attention. For instance, if a neural network is employed to solve some optimization problems, it is highly desirable for the neural network to have a unique globally stable equilibrium, and it is not surprising that the stability analysis of neural networks has been an ever hot research topic resulting in enormous stability conditions reported in the literature, see e.g. [1,6,7,10,12,13,16,17,21–23,34–36].

Time delay results from the signal transmission lags between neurons and is now a well-known source for causing instability and poor performances of neural networks (see e.g. [1,14,15]). So far, in the context of neural networks, two categories of time-delays, namely, discrete and distributed time-delays, have been thoroughly investigated in the literature. Discrete time-delays have clear practical significance that can be easily detected in reality. The stability analysis for RNNs with discrete delays has been an attractive subject of research in the past few years. Various sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the global asymptotic or exponential stability for the RNNs, see e.g. [6,29,31] for some recent publications. Distributed delays, on the other hand, reflect the spatial nature of the neural networks because of the presence of an amount of parallel pathways of a variety of axon sizes and lengths, which provides a neural network with a spatial nature [25,30]. Recently, the so-called mixed time-delays (also called *discrete and distributed delays*) have received an increasing research attention and many relevant results have been reported in the literature, see e.g. [31,34,36] and the references therein.

In practice, the RNNs often exhibit the behavior of finite state representations (also called clusters, patterns, or modes) which are referred to as the information latching problems [4]. In this case, the network states may switch (or jump) between different RNN modes according to a *Markovian chain*, and this gives rise to the so-called Markovian jumping recurrent neural networks (MJRNNs). The MJRNNs have the advantage of modeling the RNNs subject to information latching, abrupt variation in the network structures, sudden environmental disturbance, changing neuronal interconnections, etc. Recently, the dynamics analysis problem of MJRNNs has attracted a great deal of research interest [27,32,37,38]. For example, in [32], the exponential stability problem has been first

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addressed for a class of delayed recurrent neural networks with the Markovian jumping parameters. In [38], the problem of exponential stability has been investigated for a class of stochastic neural networks with both Markovian jump parameters and mixed time delays. In [27], a noise-induced stabilization method has been proposed for RNNs with mixed time-varying delays and Markovian switching parameters. In [34], the passivity analysis has been conducted for discrete-time stochastic neural networks with both Markovian jumping parameters and mixed time delays.

It is quite common in engineering systems that the time-delays occur not only in the system states (or outputs) but also in the derivatives of system states [24]. Examples of such kind of neutral delay systems include chemical reactors, transmission lines, partial element equivalent circuits in VLSI systems, and Lotka–Volterra systems [8]. Accordingly, RNNs with neutral terms have gained much research interests due to the fact that the neutral delays could exist during the implementation process of RNNs in VLSI circuits. The stability analysis issue of neutral RNNs has recently received considerable research attention and a rich body of results has been reported, see e.g. [8,9,18]. Naturally, MJRNNs of neutral type should be more capable of modeling both the Markovian switching behavior and the derivative-dependent delay for the RNNs, and the corresponding stability analysis issue should be of theoretical significance. Nevertheless, a literature search reveals that there have been very few results on dynamics analysis of *Markovian jumping neutral-type neural networks with mode-dependent mixed time-delays*. It is worth mentioning that, in two recent papers [2,3], the passivity and stability analysis problems have been addressed for neural networks of neutral type with Markovian jumping parameters and time delays, where the time-delays are not mode-dependent.

So far, to the best of the authors' knowledge, the stability analysis problem has not yet been investigated for Markovian jumping neural networks of neutral type where *all discrete, distributed and neutral delays are mode-dependent with lower and upper bounds on the distributed delays*. The major challenges are as follows: (1) when the neutral delay term is dependent on the Markovian jumping mode, the conventional stochastic analysis tool no longer applies and some novel analysis technique needs to be developed; (2) when the *lower and upper bounds* of the distributed time-delays are both subject to the Markovian switching (i.e., mode-dependent), the corresponding stability analysis becomes more complicated since a new Lyapunov functional is required to reflect the Markovian jumps of the delay bounds; and (3) it is non-trivial to establish a unified framework to handle the Markovian jumping parameters, neutral terms and mixed time-delays. It is, therefore, the main purpose of this paper to make the first attempt to handle the listed challenges.

In this paper, we consider the stability analysis problem for a new class of continuous-time neural networks of neutral-type with Markovian jumping parameters as well as mode-dependent mixed time-delays. Note that the mixed time-delays comprise both the discrete and distributed delays that are all dependent on the Markovian jumping mode. We first develop a special matrix inequality to account for the mixed and neutral time-delays, and then a novel Lyapunov–Krasovskii functional is proposed to reflect the mode-dependent nature of the time-delays. A matrix inequality approach is utilized to derive sufficient conditions guaranteeing the stochastic stability of the considered neural networks. A numerical example is presented to illustrate the usefulness and effectiveness of the main results obtained.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the matrix transposition and the notation $X \geq Y$ (respectively, $X>Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). diag$\{\cdots\}$ denotes a block-diagonal matrix, I_n is the $n\times n$ identity matrix, and $|\cdot|$ denote the Euclidean norm in $\mathbb{R}^n$. If A is a square matrix, denote by $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) the largest (respectively, smallest) eigenvalue of A. In symmetric block matrices, an asterisk "$*$" is used to represent a term that is induced by symmetry. $\mathbb{E}[x]$ and $\mathbb{E}[x|y]$ will, respectively, mean the expectation of x and the expectation of x conditional on y. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Let $r(t)$ $(t\geq 0)$ be a right-continuous Markovian chain taking values in a finite state space $\mathcal{N}=\{1,2,\ldots,n_0\}$ with generator $\Pi=\{\pi_{ij}\}$ given by

$$P\{r(t+\Delta)=j|r(t)=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta) & \text{if } i\neq j,\\ 1+\pi_{ij}\Delta+o(\Delta) & \text{if } i=j.\end{cases}$$

Here $\Delta>0$, and $\pi_{ij}\geq 0$ is the transition rate from i to j if $j\neq i$ while

$$\pi_{ii}=-\sum_{j\neq i}\pi_{ij}.$$

Consider the following neutral-type neural networks with mixed time delays and Markovian jumping parameters:

$$\begin{aligned}\dot{x}(t)&=E(r(t))\dot{x}(t-\tau_{1,r(t)})-A(r(t))x(t)+B(r(t))f(x(t))\\&\quad+C(r(t))g(x(t-\tau_{2,r(t)}))+D(r(t))\int_{t-\tau_{3,r(t)}}^{t-\tau_{4,r(t)}}h(x(s))\,ds,\end{aligned}\tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T$ is the state vector of n neurons; $\{r(t),t>0\}$ is the continuous-time Markov chain which describes the evolution of the mode of system (1) at time t; $E(r(t))=[e_{ij}(r(t))]$, $B(r(t))=[b_{ij}(r(t))]$, $C(r(t))=[c_{ij}(r(t))]$, $D(r(t))=[d_{ij}(r(t))]$ denote the connection weight matrices representing the interconnection structure between the neurons, and $A(r(t))=\mathrm{diag}\{a_1(r(t)),a_2(r(t)),\ldots,a_n(r(t))\}>0$ represents the passive decay rate of the state. The vectors $f(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\ldots f_n(x_n(t))]^T$, $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\ldots,g_n(x_n(t))]^T$ and $h(x(t))=[h_1(x_1(t)),h_2(x_2(t)),\ldots,h_n(x_n(t))]^T$ are the activation function vectors of neurons; $\tau_{1,r(t)}$ and $\tau_{2,r(t)}$ denote the discrete time-delays of the network in the mode $r(t)$, while $\tau_{3,r(t)}$ and $\tau_{4,r(t)}$ characterize the mode-dependent upper and lower bounds of distributed time-delay with $\tau_{4,j}\leq\tau_{3,j}$ $(1\leq j\leq n_0)$.

Remark 1. The neural network model (1) is quite general and includes many well-studied models as special cases, for example, neural networks of neutral type with discrete time-delays [2,8,18,32], neural networks with mixed time delays [13–15,19,31,36], and neural networks with both Markovian jumping parameters and mixed time delays [34,38].

Denote

$$\overline{\tau}_1=\max_{1\leq j\leq n_0}\{\tau_{1,j}\},\quad \overline{\tau}_2=\max_{1\leq j\leq n_0}\{\tau_{2,j}\},\quad \overline{\tau}_3=\max_{1\leq j\leq n_0}\{\tau_{3,j}\},$$
$$\overline{\tau}_4=\max_{1\leq j\leq n_0}\{\tau_{4,j}\},\quad \tau=\max\{\overline{\tau}_1,\overline{\tau}_2,\overline{\tau}_3\},$$

$$\underline{\tau}_1=\min_{1\leq j\leq n_0}\{\tau_{1,j}\},\quad \underline{\tau}_2=\min_{1\leq j\leq n_0}\{\tau_{2,j}\},\quad \underline{\tau}_3=\min_{1\leq j\leq n_0}\{\tau_{3,j}\},$$
$$\underline{\tau}_4=\min_{1\leq j\leq n_0}\{\tau_{4,j}\},\quad \overline{\pi}=\max_{1\leq i\leq n_0}\{|\pi_{ii}|\}.$$

For neuron activation functions, we make the following assumptions.

Assumption 1 (*Liu et al. [19,20]*). For the activation functions $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$, there exist constants λ_i^-, λ_i^+, σ_i^-, σ_i^+, υ_i^- and $\upsilon_i^+ (1\leq i\leq n)$ such that

$$\lambda_i^-\leq\frac{f_i(s_1)-f_i(s_2)}{s_1-s_2}\leq\lambda_i^+,\tag{2}$$