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Global attracting set for non-autonomous neutral type neural networks with distributed delays

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ABSTRACT

This paper deals with the asymptotic properties of a class of nonlinear and non-autonomous neutral type neural networks with distributed delays. By developing a new integral inequality and using the properties of nonnegative matrix, we obtain the sufficient conditions for the existence of the global attracting set of the considered delayed neural networks. The results have extended and improved the related reports in the literature and the algebra criteria are easy to check and apply in practice.

1. Introduction

The study of the stability and asymptotic properties of neural networks have attracted the interest of a great number of researchers in the past years, and many significant results have been obtained [1–5,17–19]. However, the equilibrium point sometimes does not exist in many real physical systems. Therefore, an interesting subject is to discuss the attracting sets of the neural networks with delays [6–10]. As is well known, inequality technique is an important researching tool, see [11–16]. However, the inequalities mentioned above are ineffective for studying the asymptotic behavior of a class of nonlinear and non-autonomous neutral type neural network with distributed delays. Motivated by the above discussions, in this paper, we will improve the inequality established in [11] such that it is effective for the considered delayed neural networks. Combining with the properties of nonnegative matrix, some sufficient conditions ensuring the global attracting set for a class of nonlinear and non-autonomous neutral type neural network with distributed delays are obtained. The results extend the earlier publications.

2. Preliminaries

In this section, we introduce some notations and recall some basic definitions. E denotes unit matrix, $\mathbb{R}$ is the set of real numbers and $\mathbb{R}_+ = [0,+\infty)$. $A \le B (A < B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\le (<)$". Especially, A is called a nonnegative matrix if $A \ge 0$.

$C(X,Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C \triangleq C((-\infty,t_0],\mathbb{R}^n)$ with $\phi \in C$ is bounded on $(-\infty,t_0]$.

For $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $\varphi \in C$, $\tau(t) \in C(\mathbb{R},\mathbb{R}_+)$, we define $[x]^+ = (|x_1|,|x_2|,\ldots,|x_n|)^T$, $[A]^+ = (|a_{ij}|)_{n\times n}$, $[x(t)]^+_{\tau(t)} = (\|x_1(t)\|_{\tau(t)}, \|x_2(t)\|_{\tau(t)},\ldots,\|x_n(t)\|_{\tau(t)})^T$, $\|x_i(t)\|_{\tau(t)} = \sup_{0\le s\le \tau(t)} |x_i(t-s)|$, $i=1,2,\ldots,n$.

We consider the following non-autonomous neural networks with distributed delays

$$\begin{cases} \dfrac{d}{dt}\left[x_i(t) - \sum_{j=1}^n h_{ij}(t)x_j(t-\tau(t))\right] \\ \quad = -a_i(t)x_i(t) + \sum_{j=1}^n b_{ij}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^n \int_{-\infty}^t w_{ij}(t-s)f_j(x_j(s))\,ds + I_i(t), \quad t \ge t_0, \\ x_i(t) = \varphi_i(t), \; -\infty < t \le t_0, \; i=1,\cdots,n, \end{cases} \tag{1}$$

where $x_i(t)$ is the state of the ith unit at time t; $a_i(t) \ge 0$ denotes the passive decay rate; $b_{ij}(t), w_{ij}(t)$ and $h_{ij}(t)$ represent the weight coefficients of the neurons; $I_i, i=1,\ldots,n$ is the external inputs; $f_j, j=1,\ldots,n$ are activation functions; $\tau(t) \in C(\mathbb{R},\mathbb{R}_+)$ is the transmission delays, $\lim_{t\to+\infty}(t-\tau(t)) = +\infty$. We always assume that for any $\varphi \in C$, the system (1) has at least one solution through (t_0,φ) denoted by $x(t,t_0,\varphi)$ or simply $x(t)$ if no confusion should occur.

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Definition 2.1 (*Xu* [11]). $f(t,s)\in UC_t$ means that $f\in C(\mathbb{R}_+\times\mathbb{R},\mathbb{R}_+)$ and for any given α and any $\varepsilon>0$ there exist positive numbers B,T and A satisfying

$$\int_\alpha^t f(t,s)\,ds\le B,\ \int_\alpha^{t-T} f(t,s)\,ds<\varepsilon,\quad \forall t\ge A. \tag{2}$$

Especially, $f\in UC_t$ if $f(t,s)=f(t-s)$ and $\int_0^\infty f(u)\,du<\infty$.

Definition 2.2. The set $S\subset\mathbb{R}^n$ is called a global attracting set of (1), if for any initial value $\varphi\in C$, the solution $x(t,t_0,\varphi)$ converges to S as $t\to+\infty$. That is,

$$\mathrm{dist}(x(t),S)\to 0\quad \text{as } t\to+\infty,$$

where $\mathrm{dist}(\phi,S)=\inf_{\psi\in S}d(\phi,\psi)$, $d(\phi,\psi)$ is the distance of ϕ to ψ in $\mathbb{R}^n$.

For a nonnegative matrix $A\in\mathbb{R}^{n\times n}$, let $\rho(A)$ denotes the spectral radius of A.

Lemma 2.1 (*Lasalle* [20]). *If $A\ge 0$ and $\rho(A)<1$, then $(E-A)^{-1}\ge 0$.*

3. Main results

In order to obtain the asymptotic properties of Eq.(1), we first establish the following integral inequality with delays.

Lemma 3.1. *Let $y(t)\in C(\mathbb{R},\mathbb{R}^n_+)$ be a solution of the delay integral inequality*

$$y(t)\le G(t,t_0)+B[y(t)]^+_{\tau(t)}+\int_{\alpha_1}^t Q(t,s)[y(s)]^+_{\tau(s)}\,ds + \int_{\alpha_2}^t \Psi(t,s)\int_{\alpha_3}^s \zeta(s,v)[y(v)]^+_{\tau(v)}\,dv\,ds+J,\quad t\ge t_0, \tag{3}$$

$$y(t)\le\varphi(t),\quad \forall t\in(-\infty,t_0], \tag{4}$$

where $G(t,t_0)\in C(\mathbb{R}\times\mathbb{R},\mathbb{R}^n_+)$, $B\in\mathbb{R}^{n\times n}_+$, $Q(t,u)\in C(\mathbb{R}\times\mathbb{R},\mathbb{R}^{n\times n}_+)$, $J=(j_1,\dots,j_n)^T\ge 0$, $\varphi(t)\in C((-\infty,t_0],\mathbb{R}^n_+)$, α_i is constant, $i=1,2,3$. Assume that the following conditions are satisfied:

($\mathbb{A}_1$) *$G\triangleq\sup_{t_0\le s<+\infty}G(s,t_0)<+\infty$ and there exists constant matrix $\Pi_1\ge 0$ such that*

$$\int_{\alpha_1}^t Q(t,s)\,ds+\int_{\alpha_2}^t \Psi(t,s)\int_{\alpha_3}^s \zeta(s,v)\,dv\,ds\le \Pi_1\quad \text{for } \forall t\ge t_0. \tag{5}$$

($\mathbb{A}_2$) *Let $\Pi=\Pi_1+B$, $\rho(\Pi)<1$.*

Then there is a constant vector $K>0$ such that

$$y(t)<(E-\Pi)^{-1}(K+J)\quad \text{for } t\ge t_0. \tag{6}$$

Proof. From $\rho(\Pi)<1$ and Lemma 2.1, $(E-\Pi)^{-1}$ exists and $(E-\Pi)^{-1}\ge 0$. Then there is a constant vector $K>G$ such that

$$\varphi(t)<(E-\Pi)^{-1}K,\quad \forall t\in(-\infty,t_0]. \tag{7}$$

Combining with the continuity of $y(t)$, if (6) is not true, then there must be a constant $t_1>t_0$ and some integer i such that

$$y_i(t_1)=\{(E-\Pi)^{-1}(K+J)\}_i, \tag{8}$$

$$y(t)\le(E-\Pi)^{-1}(K+J)\quad \text{for } t\le t_1, \tag{9}$$

where $\{\cdot\}_i$ denotes the ith component of vector $\{\cdot\}$.

Using (3), (5) and (9) and $G(t_1,t_0)<K$, we obtain that

$$\begin{aligned} y_i(t_1)&\le\Big\{G(t_1,t_0)+B[y(t_1)]^+_{\tau(t_1)}+\int_{\alpha_1}^{t_1} Q(t_1,s)[y(s)]^+_{\tau(s)}\,ds + \int_{\alpha_2}^{t_1}\Psi(t_1,s)\int_{\alpha_3}^s\zeta(s,v)[y(v)]^+_{\tau(v)}\,dv\,ds+J\Big\}_i\\ &<\Big\{K+\Big[B+\int_{\alpha_1}^{t_1}Q(t_1,s)\,ds+\int_{\alpha_2}^{t_1}\Psi(t_1,s)\int_{\alpha_3}^s\zeta(s,v)\,dv\,ds\Big](E-\Pi)^{-1}(K+J)+J\Big\}_i\\ &\le\{K+\Pi(E-\Pi)^{-1}(K+J)+J\}_i\\ &=\{[\Pi(E-\Pi)^{-1}+E](K+J)\}_i\\ &=\{(E-\Pi)^{-1}(K+J)\}_i. \end{aligned} \tag{10}$$

This contradicts the equality in (8), and so (6) holds. The proof is complete. □

For convenience, we denote

$A(t)=\mathrm{diag}\{a_1(t),\dots,a_n(t)\}$,
$B(t)=(|b_{ij}(t)|)_{n\times n}$,
$W(t,s)=(|w_{ij}(t,s)|)_{n\times n}$,

$H(t)=(h_{ij}(t))_{n\times n}$, $\hat{I}(t)=(|I_1(t)|,\dots,|I_n(t)|)^T$.

For (1), we suppose the following:

($\mathbb{B}_1$): For any $x_j\in\mathbb{R}$, $j\in N$, there exist nonnegative constants $\hat{h}_{ij}$ and l_j such that $|h_{ij}(t)|\le\hat{h}_{ij}$, $|f_j(x_j)|\le l_j|x_j|$, $\hat{H}=(\hat{h}_{ij})_{n\times n}$, $L=\mathrm{diag}\{l_1,\dots,l_n\}$.

($\mathbb{B}_2$): For $\forall t\ge t_0$, there exist constant matrices $\Pi_1,\Pi_2\ge 0$, and a constant vector $J\ge 0$ such that

$$\int_{t_0}^t e^{-\int_s^t A(v)\,dv}\{A(s)\hat{H}+B(s)L\}\,ds\le\Pi_1,$$
$$\int_{t_0}^t e^{-\int_s^t A(v)dv}\int_{-\infty}^s W(s-v)L\,dv\,ds\le\Pi_2,$$

$$\int_{t_0}^t e^{-\int_s^t A(v)dv}\hat{I}(s)\,ds\le J.$$

($\mathbb{B}_3$): let $\Pi=\Pi_1+\Pi_2+\hat{H}$, $\rho(\Pi)<1$.

($\mathbb{B}_4$): $k_i\triangleq\inf_{t_0\le s\le t}\int_s^{s+\delta}a_i(v)\,dv>0$ for some $\delta>0$, $i=1,2,\dots,n$. There exist nonnegative constants q_{ij} such that $|b_{ij}(t)|\le q_{ij}a_i(t)$, $i,j=1,2,\dots,n$; $\int_0^{+\infty}W(s)\,ds<+\infty$.

Theorem 3.1. *Assume that ($\mathbb{B}_1$)–($\mathbb{B}_3$) hold. Then (1) is uniformly bounded.*

Proof. By the variation of parameters formula and combining with ($\mathbb{B}_1$), we can get for $\forall t\ge t_0$

$$\begin{aligned} |x_i(t)|&\le e^{-\int_{t_0}^t a_i(v)\,dv}\Big|\varphi_i(t_0)-\sum_{j=1}^n h_{ij}(t_0)\varphi_j(t_0-\tau(t_0))\Big|\\ &+\sum_{j=1}^n\hat{h}_{ij}\|x_j(t)\|_{\tau(t)}+\int_{t_0}^t e^{-\int_s^t a_i(v)\,dv}\Big\{\sum_{j=1}^n a_i(s)\hat{h}_{ij}\|x_j(s)\|_{\tau(s)} +\sum_{j=1}^n|b_{ij}(s)|l_j\|x_j(s)\|_{\tau(s)}\Big\}ds\\ &+\int_{t_0}^t e^{-\int_s^t a_i(v)\,dv}\sum_{j=1}^n\int_{-\infty}^s|w_{ij}(s-v)|l_j\|x_j(s)\|_{\tau(s)}\,dv\,ds\\ &+\int_{t_0}^t e^{-\int_s^t a_i(v)dv}\hat{I}_i(s)\,ds. \end{aligned} \tag{11}$$

That is,

$$[x(t)]^+\le e^{-\int_{t_0}^t A(v)\,dv}[\varphi(t_0)-H(t_0)\varphi(t_0-\tau(t_0))]^+ +\hat{H}[x(t)]^+_{\tau(t)} +\int_{t_0}^t e^{-\int_s^t A(v)\,dv}\{A(s)\hat{H}+B(s)L\}[x(s)]^+_{\tau(s)}\,ds$$