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Regional models: A new approach for nonlinear system identification via clustering of the self-organizing map

Amauri H. Souza Júnior [a], Guilherme A. Barreto [a,*], Francesco Corona [b]

[a] Federal University of Ceará, Department of Teleinformatics Engineering, Av. Mister Hull, S/N, Center of Technology, Campus of Pici, CP 6005, CEP 60455-760 Fortaleza, Ceará, Brazil

[b] Aalto University, Department of Information and Computer Science, Konemiehentie 2, Espoo, Finland

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ABSTRACT

Global modelling consists in fitting a single regression model to the available data, using the whole set of input and output observations. On the other side of the spectrum stands the local modelling approach, in which the input space is segmented into several small partitions and a specialized regression model is fit to each partition. In this paper, we propose a novel approach, called Regional Models (RM), that stands in between the global and local modelling ones. The proposal extends the two-level clustering approach by Vesanto and Alhoniemi (2000 [1]) to regression problems, more specifically, to system identification. In this regard, we first partition the input space using the Self-Organizing Map (SOM), and then perform clustering over the prototypes of the trained SOM. Finally, regional regression models are built over the clusters (i.e. over the regions) of SOM prototypes, not over each SOM prototype as in local modelling. Under the proposed framework, we build regional linear and nonlinear regression models. For the linear case, we use autoregressive models with eXogenous (ARX) whose parameters are estimated using the ordinary least-squares (OLS) method. Regional nonlinear ARX (NARX) models are built using the Extreme Learning Machine network. Additionally, we develop robust variants of the proposed regional models through the use of *M*-estimation, a statistical framework for handling outliers, since the OLS is highly sensitive to them. Comprehensive performance evaluation of the proposed models on synthetic and real-world datasets is carried out and the results compared to those achieved by standard global and local models.

1. Introduction

System identification is concerned with the development of regression models that describe the dynamics of a system from measurements of its inputs and outputs. Knowing a model that describes the diversity of behaviors that a dynamical system can reveal, specially the nonlinear ones, is essential not only for theoretic or applied research fields, but also for the process or control engineer who is interested in understanding better the dynamics of the system he/she is dealing with. As an ultimate goal, the resulting model must approximate the actual system as faithfully as possible in order to be used for several purposes, such as predictive control or fault detection.

Modern industrial plants have been the source of challenging tasks in dynamical system identification and control [2]. Designing control systems to achieve the level of quality demanded by current industry standards requires building accurate models of the plant being controlled. Building accurate models requires reliable data, usually in the form of input and output time series. Once such data are available, they can be used for building direct and/or inverse models of nonlinear systems by means of computational intelligence methods, such as neural networks [3–6], Takagi–Sugeno–Kang fuzzy models [7–9] or hybrid systems [10–14], to mention just a few possibilities.

Although several techniques for nonlinear dynamical system identification have been proposed [15,16], they can be categorized into one of the two following approaches: global and local modelling. Global modelling involves the utilization of a single regression model, such as a feedforward or recurrent neural network model, that approximates the whole mapping between the input and the output of the system being identified. Global models constitute the mainstream in nonlinear system identification and control [17–20,5].

Local modelling utilizes instead multiple models to represent the input–output dynamics of the system of interest [21]. These approaches have been a source of much interest because they have the ability to fit to the local shape of an arbitrary surface (i.e. mapping). This feature is particularly important when the dynamical system

* Corresponding author.
E-mail addresses: amauriholanda@ifce.edu.br (A.H. Souza Júnior), gbarreto@ufc.br (G.A. Barreto), francesco.corona@aalto.fi (F. Corona).

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characteristics vary considerably throughout the input space. The input space is usually divided into smaller, localized regions, each one being associated with a simpler (usually linear) model. To estimate the system output at a given time, a single model is chosen from the pool of available local models according to some criteria defined on the current input data. In the neural network literature, multiple local models for system identification and control have been implemented mostly either by means of the Local Model Network (LMN) [22–27] or the Self-Organizing Map (SOM) [28–33,6,34,35].

The LMN uses basis functions to implement its localized nature, similar to the standard Radial Basis Function networks (RBFN). In LMNs, however, the output weights of a RBFN are replaced with local functions of the inputs. As a consequence, due to the better approximation properties of these local functions, the LMN model does not require as many basis functions as the standard RBFN to achieve the desired accuracy and, hence, the number of parameters is reduced dramatically.

The SOM is an unsupervised competitive learning algorithm which has been commonly applied to clustering, vector quantization and data visualization tasks [36–38]. When applied to multiple local modelling, the SOM is used to partition the input–output space into smaller regions, over which the local models are built. The results reported on the aforementioned studies are rather appealing, indicating that SOM-based local models can be feasible alternatives to global models based on supervised neural network architectures, such as the Multilayer Perceptron (MLP) and the Extreme Learning Machine (ELM) [39].

The main advantage of the local modelling approach over the global one relies on the fact that complex (e.g. nonlinear) dynamics of the input–output mapping can be represented by multiple simpler mappings. Another advantage is interpretability. Since multiple local models are used, one can easily associate IF–THEN rules with them in order to describe the current dynamics of the system.

However, there is no free-lunch in the realm of local modelling. The alleged flexibility of using multiple local models comes with some costs. One of the main problems is that it is not straightforward to select the appropriate number of local models beforehand, without any a priori information. An inappropriate selection may cause the over- or under-identification of the original system dynamics [40]. Another shortcoming is related to the occurrence of dead or interpolating neurons[1] after SOM training. In this case, it is impossible to associate a local model with this type of neuron, since there are no data points to estimate the parameters of the corresponding local model.

To handle these shortcomings, we propose a novel approach to system identification, called Regional Models (RM), that stands in between the global and local modelling approaches.[2] RM is motivated by the two-level clustering approach introduced by Vesanto and Alhoniemi [1] and can be thought as an extension of their work to regression problems, more specifically, to nonlinear dynamical system identification.

For this purpose, we first partition the input space using a single SOM network with C prototypes, and then perform clustering using the K-means algorithm [42] over the prototypes of the trained SOM in order to find an optimal number K_{opt} ($K_{opt} \ll C$) of clusters of SOM prototypes. The optimal number of clusters can be found by using any cluster validation technique, such as the Davies–Bouldin index [43,44]. A given cluster of SOM prototypes defines a region in the input space formed by merging the Voronoi cells of the prototypes belonging to that cluster. Finally, for each individual cluster of SOM prototypes we build a regional regression model using only the data vectors mapped to that specific cluster.

It is worth mentioning that by using K_{opt} regional models instead of C local models, the RM approach is much more parsimonious than the local modelling approach, i.e. few models are required to faithfully describe the dynamics of the system of interest (recall that $K_{opt} \ll C$). A second advantage is that the user has not to worry too much about the specification of the number C of SOM prototypes, since the subsequent application of K-means clustering to the SOM prototypes makes the number of regional models relatively decoupled from large variations in the value of C. In other words, large variations in C do not produce large variations in K_{opt}. Finally, a third advantage of the RM approach over local modelling is that regional models can be constructed even if dead/interpolating neurons occur after SOM training. This is possible because any of the existing dead/interpolating neurons will belong eventually to one out of the K_{opt} regions available.

Using the proposed RM framework, we develop regional linear and nonlinear regression models. For the linear case, we use autoregressive models with eXogenous (ARX) whose parameters are estimated using the ordinary least-squares (OLS) method. Regional nonlinear ARX (NARX) models are built using the Extreme Learning Machine. Additionally, we develop robust variants of the proposed regional models through the use of M-estimation, a robust statistics framework introduced by Huber [45,46] for handling outliers, since the OLS is highly sensitive to them. Comprehensive performance evaluation of the proposed models on synthetic and real-world datasets is carried out and the results compared to those achieved by standard global and local models.

The remainder of the paper is organized as follows. In Section 2, we briefly review the basics of dynamical system identification using ARX and NARX models. In Section 3, the SOM network and its learning process are described. In Section 4, the regional modelling framework for nonlinear system identification is introduced. In Section 5 we present the fundamentals of M-estimation and its use in the context of regional modelling. Computer simulations and results are presented and discussed in Section 6. The paper is concluded in Section 7.

2. Basics of dynamical system identification

Let us assume that the dynamical system we are dealing with can be described mathematically by the following ARX model [47]:

$$\begin{aligned} y(k) &= a_1 y(k-1) + \cdots + a_p y(k-p) + b_1 u(k-1) + \cdots + b_q u(k-q), \\ &= \sum_{j=1}^{p} a_j y(k-j) + \sum_{l=1}^{q} b_l u(k-l), \end{aligned} \tag{1}$$

where $u(k) \in \mathbb{R}$ and $y(k) \in \mathbb{R}$ denote, respectively, the input and output of the model at time step k, while $q \geq 1$ and $p \geq 1$ ($q \leq p$) are the input-memory and output-memory orders, respectively. The coefficients $a_j \in \mathbb{R}$, $j = 1, \ldots, p$ and $b_l \in \mathbb{R}$, $l = 1, \ldots, q$ are the parameters of the model to be estimated using the available data.

By defining the input vector $\mathbf{x}(k) \in \mathbb{R}^{p+q}$ at time step k and the vector of parameters $\boldsymbol{\theta} \in \mathbb{R}^{p+q}$ as

$$\mathbf{x}(k) = [y(k-1) \ \cdots \ y(k-p) | u(k-1) \ \cdots \ u(k-q)]^T, \tag{2}$$

$$\boldsymbol{\theta} = [a_1 \ \cdots \ a_p | b_1 \ \cdots \ b_q]^T, \tag{3}$$

we can write the output of the ARX model in Eq. (1) simply as

$$y(k) = \boldsymbol{\theta}^T \mathbf{x}(k). \tag{4}$$

The parameter vector $\boldsymbol{\theta}$ is commonly estimated by means of the ordinary least-squares (OLS) method, which leads to the following

[1] Neurons whose prototypes are located at positions without data points.

[2] A previous shorter version of this work [41] has been published in the 9th Workshop of Self-Organizing Maps (WSOM'2012), held in Santiago, Chile.