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Further improvement in delay-dependent stability criteria for continuous-time systems with time-varying delays

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ABSTRACT

This study is concerned with the issue of delay-dependent stability criteria for continuous-time systems with time-varying delays. By introducing an appropriate Lyapunov–Krasovskii functional and the improved reciprocally convex technique, a new less conservative and simplified stability criterion is obtained in terms of linear matrix inequalities (LMIs). At last, numerical examples are also designated to demonstrate the effectiveness and reduced conservatism of the developed results.

1. Introduction

It is well known that time delay is always encountered in the practical systems and usually results in instability and poor performance of dynamical systems, it is very significant to examine the dynamical systems with time delay. It should be noted that in many practical systems, time delay is always varying and has been successful applied to various areas such as networked control systems, biological systems and chemical processes systems. Up to now, various research issues on dynamical systems with time-varying delay have been studied [1–5]. Major of the existing results are classified into the following two cases: delay-independent criteria [6] and delay-dependent criteria [7–15]. Compared with delay-independent stability criteria, the delay-dependent ones always receive less conservative. Therefore, major attentions have been concentrated with the derivation of delay-dependent ones.

With the development of microelectronic technology, neural networks have drawn a lot of attention due to their extensive applications in various fields, for instance, pattern recognition, combinatorial optimization and signal processing. In recent years, there exists the finite speed in the processing of information, the neural networks with time-varying delays have been extensively improved in terms of LMIs via different analysis techniques [8–34]. In [8], the stability criterion for neural networks was introduced by using the technique of free-weighting matrix. In [9], the stability result was reported by employing the method of convex combination. The reciprocally convex technique was proposed in [10]. The improved bounding method was discussed in [11]. Recently, several seeks also have been devoted by choosing an augmented Lyapunov–Krasokii functional in [12,13] and formula of Newton–Leibniz was also introduced in [14]. As a forceful tool to get the efficient condition, Jensen's inequality has also been considered to overcome the conservativeness in [16]. In [17], using the Lyapunov functionals of delay partition, improved conditions are given. Therefore, it is very important to choose an appropriate technique with Lyapunov–Krasovskii functional and obtain an upper bound of time delay.

In this paper, the delay-dependent stability criteria for continuous-time systems with time-varying delays are revisited. The integral inequalities are employed to ensure the positiveness of the Lyapunov–Krasovskii functional and using the formula of Newton–Leibniz, less conservative stability result is obtained. At last, numerical examples are also given to demonstrate the reduced conservatism and the effectiveness of the proposed method.

2. Preliminaries

Consider a class of neural network with continuous-time neural networks with time-varying delays as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau(t)) + J \\ x(t) = \phi(t), \quad t \in [-\tau_M, 0), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^\top$ denotes the neural state vector of the continuous-time neural network, $\phi(t)$ is the initial condition

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and continuously differentiable; $J=[J_1,J_2,\ldots,J_n]^\top$ represents a constant external input vector. A and B are the known real constant matrices. $\tau(t)$ is the time-varying function and satisfies

$$\tau_0 \le \tau(t) \le \tau_M, \tag{2}$$

$$0 \le \dot{\tau}(t) \le \tau, \tag{3}$$

where τ and $0<\tau_0\le\tau_M$ are the constant scalars. It should be noted that two cases will be considered in this paper. Firstly, the time-varying delay $\tau(t)$ is not differentiable. Secondly, the time-varying delay $\tau(t)$ is differentiable and has upper bound τ.

It follows from Brouwer's fixed-point theorem, for the neural networks, there exists an equilibrium point. In general, assuming $x^*=[x_1^*,x_2^*,\ldots,x_n^*]^\top$ is the equilibrium point of the system (1). With the coordinate change $\eta(\cdot)=x(\cdot)-x^*$, the system (1) is transformed as follows:

$$\dot{\eta}(t)=A\eta(t)+B\eta(t-\tau(t)) \tag{4}$$

where $\eta(t)=[\eta_1(t),\eta_2(t),\ldots,\eta_n(t)]^\top$ denotes the state vector of the system (1).

In order to simplicity of matrix representation in this paper, χ_i, $i=10$, are limited as the block entry matrices. For example, $\chi_2^\top=[0\ I\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^\top$. The other representations are defined as follows:

$$\begin{aligned}\xi^\top(t)=\Big[&\eta^\top(t),\ \eta^\top(t-\tau(t)),\ \eta^\top(t-\alpha\tau(t)),\ \eta^\top(t-\tau_0),\ \eta^\top(t-\tau_M),\\ &\int_{t-\tau_0}^{t}\eta(s)\,ds,\ \int_{t-\tau(t)}^{t-\tau_0}\eta(s)\,ds,\ \int_{t-\tau_M}^{t-\tau(t)}\eta(s)\,ds,\\ &\int_{t-\alpha\tau(t)}^{t}\eta(s)\,ds,\ \int_{t-\tau(t)}^{t-\alpha\tau(t)}\eta(s)\,ds\Big],\end{aligned}$$

$$\begin{aligned}\varsigma^\top(t)=\Big[&\eta^\top(t)\ \int_{t-\tau_0}^{t}\eta(s)\,ds\ \int_{t-\tau(t)}^{t-\tau_0}\eta^\top(s)\,ds\ \int_{t-\tau_M}^{t-\tau(t)}\eta^\top(s)\,ds\\ &\times\int_{t-\alpha\tau(t)}^{t}\eta^\top(s)\,ds\ \int_{t-\tau(t)}^{t-\alpha\tau(t)}\eta^\top(s)\,ds\Big].\end{aligned}$$

Lemma 2.1. *For any constant matrix $M=M^\top>0$ and scalars $\tau_M>\tau_0\ge 0$ such that the following integrations are well defined, then*

$$\int_{t-\tau_M}^{t-\tau_0}\eta^\top(s)M\eta(s)\,ds\ge\frac{1}{\tau_M-\tau_0}\left[\int_{t-\tau_M}^{t-\tau_0}\eta(s)\,ds\right]^\top M\left[\int_{t-\tau_M}^{t-\tau_0}\eta(s)\,ds\right],$$

$$\begin{aligned}\int_{-\tau_M}^{-\tau_0}\int_{t+\theta}^{t}\eta^\top(s)M\eta(s)\,ds\,d\theta\ge\frac{2}{\tau_M^2-\tau_0^2}&\left[\int_{-\tau_M}^{-\tau_0}\int_{t+\theta}^{t}\eta(s)\,ds\,d\theta\right]^\top\\ &\times M\left[\int_{-\tau_M}^{-\tau_0}\int_{t+\theta}^{t}\eta(s)\,ds\,d\theta\right].\end{aligned}$$

3. Main results

In this section, one method would be introduced to analyze the asymptotically stable of for continuous-time systems with time-varying delays.

Theorem 3.1. *Given the constant scalars $\tau_M>\tau_0\ge 0$, $1>\alpha>0$ and $k>0$, the neural networks (1) are asymptotically stable if there exist mode-dependent symmetric matrices $\mathcal{P}$, $Q_i>0$, $i=1,2,3$, $Z_j>0$, $j=1,2,3$, any appropriately dimensioned matrices M_l, $l=1,2,3$, such that the following matrix inequalities hold:*

$$\begin{aligned}\Omega_1=\ &\Gamma\mathcal{P}\Gamma^\top+e^{-2k\tau_0}\frac{1}{\tau_0}\chi_6Q_1\chi_6^\top+e^{-2k\tau_M}\frac{1}{\tau_M}(\chi_8+\chi_{10}+\chi_9)Q_2(\chi_8+\chi_{10}+\chi_9)^\top\\ &+e^{-2k\alpha\tau_M}e^9Q_3e_9^\top+e^{-2k\tau_0}\frac{2}{\tau_0^2}(\tau_0\chi_1-\chi_6)W_1(\tau_0\chi_1-\chi_6)^\top\\ &+e^{-2k\tau_M}\frac{2}{\tau_M^2-\tau_0^2}((\tau_M-\tau_0)e_1-e_8-e_7)W_2((\tau_M-\tau_0)e_1-e_8-e_7)^\top\\ &+e^{-2k\tau_M}\frac{2}{\tau_M^2}(\tau_Me_1-e_8-e_{10}-e_9)W_3(\tau_Me_1-e_8-e_{10}-e_9)^\top>0,\end{aligned}\tag{5}$$

$$\begin{bmatrix}\Omega_2 & e^{-k\tau_M}M_1 & e^{-k\tau_M}M_2 & e^{-k\tau_M}M_3\\ * & -\frac{1}{\alpha\tau_0}W_3 & 0 & 0\\ * & * & -\frac{1}{(1-\alpha)\tau_0}W_3 & 0\\ * & * & * & -\frac{1}{\tau_M-\tau_0}W_3\end{bmatrix}<0,\tag{6}$$

$$\begin{bmatrix}\Omega_2 & e^{-k\tau_M}M_1 & \\ * & -\frac{1}{\alpha\tau_M}W_3 & 0\\ * & * & -\frac{1}{(1-\alpha)\tau_M}W_3\end{bmatrix}<0,\tag{7}$$

where

$$\begin{aligned}&\Gamma=[\chi_1,\chi_6,\chi_7,\chi_8,\chi_9,\chi_{10}],\\ &\Delta=[\chi_1A^\top+\chi_2B^\top,\chi_1-\chi_4,\chi_4-(1-\tau)\chi_2,(1-\tau)\chi_2-\chi_5,\\ &\qquad\chi_1-(1-\alpha\tau)\chi_3,(1-\alpha\tau)\chi_3-(1-\tau)\chi_2],\end{aligned}$$

$$\begin{aligned}\Omega_2=\ &\Gamma\mathcal{P}\Delta^\top+\Delta\mathcal{P}\Gamma^\top+2\Gamma^\top\mathcal{P}\Gamma+\chi_1(Q_1+Q_2+Q_3)\chi_1^\top\\ &-e^{-2k\tau_0}\chi_4Q_1\chi_4^\top-e^{-2k\tau_M}\chi_5Q_2\chi_5^\top-(1-\alpha\tau)e^{-2\alpha k\tau_0}\chi_3Q_3\chi_3^\top\\ &+(\chi_1A^\top+\chi_2B^\top)(\tau_0W_1+(\tau_M-\tau_0)W_2+\tau_MW_3)(\chi_1A^\top+\chi_2B^\top)^\top\\ &-e^{-2k\tau_0}\frac{1}{\tau_0}\chi_1W_1\chi_1^\top+e^{-2k\tau_0}\frac{1}{\tau_0}\chi_4W_1\chi_4^\top\\ &-e^{-2k\tau_M}\frac{1}{\tau_M-\tau_0}\left\{\begin{bmatrix}\chi_2-\chi_5\\ \chi_4-\chi_2\end{bmatrix}\begin{bmatrix}W_2 & R\\ * & W_2\end{bmatrix}\begin{bmatrix}\chi_2-\chi_5\\ \chi_4-\chi_2\end{bmatrix}^\top\right\}\\ &+(e_1A^\top+e_2B^\top)\left(\frac{\tau_0^2}{2}Z_1+\frac{\tau_M^2-\tau_0^2}{2}Z_2+\frac{\tau_M^2}{2}Z_3\right)(e_1A^\top+e_2B^\top)^\top\\ &-e^{-2k\tau_0}\frac{2}{\tau_0^2}(\tau_0\chi_1-\chi_6)Z_1(\tau_0\chi_1-\chi_6)^\top\\ &-e^{-2k\tau_M}\frac{2}{\tau_M^2-\tau_0^2}((\tau_M-\tau_0)\chi_1-\chi_8-\chi_7)Z_2((\tau_M-\tau_0)\chi_1-\chi_8-\chi_7)^\top\\ &-e^{-2k\tau_M}\frac{2}{\tau_M^2}(\tau_M\chi_1-\chi_8-\chi_{10}-\chi_9)Z_3(\tau_M\chi_1-\chi_8-\chi_{10}-\chi_9)^\top\\ &+2e^{-2k\tau_M}M_1(\chi_1^\top-\chi_3^\top)+2e^{-2k\tau_M}M_2(\chi_3^\top-\chi_2^\top)\\ &+2e^{-2k\tau_M}M_3(\chi_2^\top-\chi_5^\top)\end{aligned}$$

Proof. In this paper, we construct the Lyapunov functional with the following form:

$$V(\eta(t))=V_1(\eta(t))+V_2(\eta(t))+V_3(\eta(t))+V_4(\eta(t)),\tag{8}$$

$$V_1(\eta(t))=e^{2kt}\varsigma^\top(t)\mathcal{P}\varsigma(t),$$

$$\begin{aligned}V_2(\eta(t))=\ &\int_{t-\tau_0}^{t}e^{2k\theta}\eta^\top(\theta)Q_1\eta(\theta)\,d\theta+\int_{t-\tau_M}^{t}e^{2k\theta}\eta^\top(\theta)Q_2\eta(\theta)\,d\theta\\ &+\int_{t-\alpha\tau(t)}^{t}e^{2k\theta}\eta^\top(\theta)Q_3\eta(\theta)\,d\theta,\end{aligned}$$

$$\begin{aligned}V_3(\eta(t))=\ &\int_{-\tau_0}^{0}\int_{t+s}^{t}e^{2k\theta}\dot{\eta}^\top(\theta)W_1\dot{\eta}(\theta)\,d\theta\,ds\\ &+\int_{-\tau_M}^{-\tau_0}\int_{t+s}^{t}e^{2k\theta}\dot{\eta}^\top(\theta)W_2\dot{\eta}(\theta)\,d\theta\,ds\\ &+\int_{-\tau_M}^{0}\int_{t+s}^{t}e^{2k\theta}\dot{\eta}^\top(\theta)W_3\dot{\eta}(\theta)\,d\theta\,ds,\end{aligned}$$

$$\begin{aligned}V_4(\eta(t))=\ &\int_{-\tau_0}^{0}\int_{s}^{0}\int_{t+\lambda}^{t}e^{2k\theta}\dot{\eta}^\top(\theta)Z_1\dot{\eta}(\theta)\,d\theta\,d\lambda\,ds\\ &+\int_{-\tau_M}^{-\tau_0}\int_{s}^{0}\int_{t+\lambda}^{t}e^{2k\theta}\dot{\eta}^\top(\theta)Z_2\dot{\eta}(\theta)\,d\theta\,d\lambda\,ds\end{aligned}$$