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Design of controller on synchronization of memristor-based neural networks with time-varying delays

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ABSTRACT

In this paper, synchronization of memristor-based neural networks (MNNs) with time-varying delays is investigated. By employing the Newton–Leibniz formulation and inequality technique, the controller with state or output coupling is designed to obtain global exponential synchronization of MNNs. The obtained delay-dependent conditions can be checked easily and they also enrich and improve the results in earlier publications. Finally, one numerical example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

The practical memristor device was realized by scientists at Hewlett–Packard Laboratories and the finding was published in 2008 [1] since memristor was originally theorized by Chua in 1971 [2]. Memristor was predicted as the fourth circuit element (the other three are resistor, capacitor and inductor) and it could play the role as resistor in circuit system. In the past few years, memristor has received increasing research attention for its potential applications in the next generation computer and powerful brainlike neural computers [3]. In addition, it has been shown that memristors are proposed to work as synaptic weights in artificial neural networks [4,18]. Due to this feature, the model of memristor-based neural networks (MNNs) can be built to emulate the human brain where synapses are implemented with memristors.

As is well known, synchronization of neural networks is significant and it has received great attention due to their potential applications in many different areas such as secure communication [19-23], information science [24-27, 29-31], and biological system [32-35]. In addition, synchronization control has been used to investigate the dynamic properties of neural networks. Moreover, the results in [17] show that memristor-based nonlinear hybrid system plays an important role in the security of secure communication due to the special feature of memristor. Therefore, it is significant to study synchronization of MNNs.

Motivated by the above discussion, in this paper, we focus our attention on the design of the controller for the synchronization of MNNs. The contributions of this paper are as follows. Firstly, by using the nonsmooth analysis of control theory, the synchronization of MNNs with discontinuous right-hand side is investigated. Different from continuous neural networks, the system of MNNs is discontinuous since the parameters concerning memristors change according to its state. The classical solution is not applicable, so the existence of solutions for MNNs is a delicate problem. Also, this problem brings challenges to investigate the synchronization of MNNs. Secondly, a general controller with state or output coupling is proposed. In [9,12], the synchronization of MNNs was obtained with memoryless controller. But the controller considered in our paper contains the information of the size of $\tau(t)$. Thirdly, the coupling matrix in [12,13] is required to be symmetrical while the coupling matrices in our paper are random and can be easily solved by using the MATLAB tool boxes. Finally, by using a new lemma and the transform scaling, our results are true for MNNs which in terms of differential inclusion.

The organization of this paper is as follows. The system and some preliminaries are introduced in Section 2. In Section 3, a delay-dependent controller is designed to obtain the sufficient conditions for the synchronization of MNNs. Then, numerical simulations are given to demonstrate the effectiveness of the obtained results in Section 4. Finally, conclusions are drawn in Section 5.

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2. System description and preliminaries

In this paper, we consider the memristor-based neural networks as follows:

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t))g_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_j(t-\tau_{ij}(t))) \times g_j(x_j(t-\tau_{ij}(t))), \quad t \geq 0, \ i = 1, 2, \ldots, n, \tag{1}$$

where

$$a_{ij}(x_j(t)) = \begin{cases} a_{ij}^{*}, & x_j(t) < 0, \\ a_{ij}^{**}, & x_j(t) \geq 0, \end{cases}$$

$$b_{ij}(x_j(t-\tau_{ij}(t))) = \begin{cases} b_{ij}^{*}, & x_j(t-\tau_{ij}(t)) < 0, \\ b_{ij}^{**}, & x_j(t-\tau_{ij}(t)) \geq 0, \end{cases}$$

$x_i(t)$ is the state variable of the i-th neuron. $(a_{ij}(x_j(t)))$ and $(b_{ij}(x_j(t-\tau_{ij}(t))))$ denote the feedback connection weight and delayed feedback connection weight, respectively. $g_j : R \to R$ is bounded continuous function, $\tau_{ij}(t)$ corresponds to the transmission delay, $i, j = 1, 2, \ldots, n$. $a_{ij}^{*}, a_{ij}^{**}, b_{ij}^{*}, b_{ij}^{**}, i, j = 1, 2, \ldots, n$ are all constant numbers. The initial condition of system (1) is $x(s) = \phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$.

Obviously, system (1) is a discontinuous system, then its solution is different from the classic solution and cannot be defined in the conventional sense. In order to obtain the solution of system (1), some definitions and lemmas are given.

Definition 1. For a system with discontinuous right-hand side:

$$\frac{dx}{dt} = F(x), \quad x(0) = x_0, \quad x \in R^n, \ t \geq 0, \tag{2}$$

where $F(x) : R^n \longrightarrow R^n$ is discontinuous. A set-valued map is defined as

$$\Phi(x) = \bigcap_{\delta > 0} \bigcap_{\mu(N) = 0} \overline{co}[F(B(x, \delta) \backslash N],$$

where $\overline{co}[E]$ is the closure of the convex hull of set E, $E \subset R^n$, $B(x, \delta) = \{y : \|y - x\| < \delta, x, y \in R^n, \delta \in R^{+}\}$, and $N \subset R^n$, $\mu(N)$ is Lebesgue measure of set N.

A solution in Filippov's sense [5] of system (2) with the initial condition $x(0) = x_0 \in R^n$ is an absolutely continuous function $x(t), t \in [0, T], T > 0$, which satisfy $x(0) = x_0$ and differential inclusion:

$$\frac{dx}{dt} \in \Phi(x) \quad \text{for a.a. } t \in [0, T].$$

If $F(x)$ is bounded, then the set-valued function $\Phi(x)$ is nonempty, bounded and closed, convex, and it is upper semicontinuous [5], then the solution $x(t)$ of system (2) with the initial condition exists and it can be extended to the interval $[0, +\infty)$ in the sense of Filippov.

By applying the theories of set-valued maps and differential inclusions [5–7], then system (1) can be rewritten as the following differential inclusion:

$$\dot{x}_i(t) \in -x_i(t) + \sum_{j=1}^{n} co[a_{ij}(x_j(t))]g_j(x_j(t)) + \sum_{j=1}^{n} co[b_{ij}(x_i(t-\tau_{ij}(t)))]g_j(x_j(t-\tau_{ij}(t))) \quad \text{for a.a. } t \geq 0, \ i = 1, 2, \ldots, n, \tag{3}$$

where

$$co[a_{ij}(x_j(t))] = \begin{cases} a_{ij}^{*}, & x_j(t) < 0, \\ co\{a_{ij}^{*}, a_{ij}^{**}\}, & x_j(t) = 0, \\ a_{ij}^{**}, & x_j(t) > 0, \end{cases}$$

$$co[b_{ij}(x_j(t-\tau_{ij}(t)))] = \begin{cases} b_{ij}^{*}, & x_j(t-\tau_{ij}(t)) < 0, \\ co\{b_{ij}^{*}, b_{ij}^{**}\}, & x_j(t-\tau_{ij}(t)) = 0, \\ b_{ij}^{**}, & x_j(t-\tau_{ij}(t)) > 0. \end{cases}$$

Throughout this paper, we consider system (3) as the drive system. Then the corresponding response system is as follows:

$$\dot{y}_i(t) \in -y_i(t) + \sum_{j=1}^{n} co[a_{ij}(y_j(t))]g_j(y_j(t)) + \sum_{j=1}^{n} co[b_{ij}(y_j(t-\tau_{ij}(t)))]g_j(y_j(t-\tau_{ij}(t))) + u_i(t), \quad \text{for a.a. } t \geq 0, \ i = 1, 2, \ldots, n, \tag{4}$$

where $y_i(t)$ is the state variable of the i-th neuron, $i = 1, 2, \ldots, n$, $u_i(t)$ is the appropriate control input to obtain a certain control objective, other parameters are the same as in systems (3). The initial condition of (4) is $y(s) = \varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$.

Definition 2. A function $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is a solution of (1), with the initial condition $x(s) = \phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$, if $x(t)$ is an absolutely continuous function and satisfies the differential inclusion (3).

Throughout this paper, the following assumptions are given for system (1).

(H1) For $j \in 1, 2, \ldots, n, g_j$ is bounded and there exists constant $l_j > 0$ such that

$$0 \leq \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \leq l_j, \quad g_j(0) = 0,$$

for all $s_1, s_2 \in R, s_1 \neq s_2$.

(H2) The transmission delay $\tau_{ij}(t)$ is a differential function and there exist $\tau, \mu > 0$ such that

$$0 \leq \tau_{ij}(t) \leq \tau, \quad \dot{\tau}_{ij}(t) \leq \mu,$$

for all $t \geq 0, i, j = 1, 2, \ldots, n$.

Lemma 1. *Suppose that the assumption (H 1) holds, then solution $x(t)$ with the initial condition $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$ of (1) exists and it can be extended to the interval $[0, +\infty)$ in the sense of Filippov.*

Define the synchronization error as $\epsilon(t) = (\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_n(t))^T$ where $\epsilon_i(t) = y_i(t) - x_i(t)$ for all $i = 1, 2, \ldots, n$. Then based on theories of set-valued maps and differential inclusions, we can obtain the following error system:

$$\dot{\epsilon}_i(t) \in -\epsilon_i(t) + \sum_{j=1}^{n} \{co[a_{ij}(y_j(t))]g_j(y_j(t)) - co[a_{ij}(x_j(t))]g_j(x_j(t))\} + \sum_{j=1}^{n} \{(co[b_{ij}(y_j(t-\tau_{ij}(t)))]g_j(y_j(t-\tau_{ij}(t))) - co[b_{ij}(x_j(t-\tau_{ij}(t)))]g_j(x_j(t-\tau_{ij}(t))))\} + u_i(t), \quad \textit{for a.a. } t \geq 0, \ i = 1, 2, \ldots, n. \tag{5}$$

The aim of this paper is to design a controller $u(t)$ to let the response system (4) synchronize with the drive system (3). Since the information on the size of $\tau(t)$ is available, the controller in the following form is considered:

$$u(t) = K_1 \epsilon(t) + K_2 \epsilon(t - \tau(t)) \tag{6}$$