Neurocomputing 93 (2012) 1–9

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Existence of exponential periodic attractor of BAM neural networks with time-varying delays and impulses ☆

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ARTICLE INFO

Article history:
Received 17 July 2011
Received in revised form
12 January 2012
Accepted 24 March 2012
Communicated by H. Jiang
Available online 24 April 2012

Keywords:
BAM
Periodic solution
Exponential stability
Delays
Impulses

ABSTRACT

A class of BAM neural networks with time-varying delays and impulses is proposed in this paper. By using Mawhin's continuation theorem, some analysis techniques and Lyapunov functional, sufficient conditions ensuring the existence of exponential periodic attractor of this system are established. The main results are much different from previously known results [9,18,28]. Applications and an illustrative example are given to show the effectiveness of our main results.

1. Introduction

In real world, the dynamics of autonomous cellular neural networks with or without constant delays has been studied extensively because the properties of stability and convergence for equilibrium points or positive periodic solutions are important in various applications such as designing associative memory and solving optimization problems [1–3]. But in many cases, impulsive effects exist in a variety of evolutionary processes where states are changed abruptly at some certain moments. Neural networks are often subjected to impulsive perturbations which affect the dynamical behaviors just as delays. Hence impulsive neural networks represent more natural frameworks for mathematical modeling of neural networks [4–8]. In recent years, a class of two layers (the X-layer and Y-layer) heteroassociative networks called bi-directional associative memory (BAM) neural networks with impulsive effects or delays has been paid much attention, due to its applications in many fields such as pattern recognition, automatic control, signal processing and optimization. There have been many results for impulsive BAM neural networks with or without axonal signal transmission delays [9–15].

However, nonautonomous phenomena often occur in many systems. In practice, if long-term dynamical behaviors are considered, in view of the seasonality of changing environments, parameters of the system for mathematical modeling are often assumed to be periodic [16–19]. For example, authors [19] proposed the following nonautonomous neural networks with time varying delays:

$$x_i'(t) = -a_i(t)x_i(t) + \sum_{j=1}^{m} b_{ij}(t)f_j(y_j(t)) + \sum_{j=1}^{m} c_{ij}(t)f_j(y_j(t-\tau_{ij}(t))) + I_i(t).$$

Considering impulsive effects from environments, Shao [20] studied the following model:

$$\begin{cases} x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n}\sum_{l=1}^{m} a_{ijl}(t)f_{ijl}(x_j(t)) \\ \qquad + \sum_{j=1}^{n}\sum_{l=1}^{m} b_{ijl}(t)g_{ijl}(x_j(t-\tau_{ijl}(t))) + I_i(t), & t \neq t_k, \\ \Delta x_i(t_k) = e_{ik}(x_i(t_k)), & t = t_k. \end{cases}$$

The author studied the existence and stability of periodic solutions. Some other nonautonomous neural networks with impulses may refer to [21–24]. Due to the importance of the properties of periodic oscillatory solutions for BAM neural networks, some nonautonomous BAM neural networks with impulses and distributed delays have attracted more and more scholars and many nice results were obtained, see Refs. [25–30] and references cited therein. Then how about the dynamics of nonautonomous BAM neural networks with impulses and time-varying delays? It is interesting and necessary for us to investigate further.

☆ The paper is supported by National Natural Science Foundation of China (11161015) and Doctoral Foundation of Guilin University of Technology (01080).

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http://dx.doi.org/10.1016/j.neucom.2012.03.004

Motivated by the above discussion, in this paper, we study the existence and stability of periodic solution of the following BAM neural networks with time-varying delays and impulses:

$$\begin{cases} x_i'(t) = -a_i(t)x_i(t) + \sum_{j=1}^{m} b_{ij}(t)f_j(y_j(t)) \\ \qquad + \sum_{j=1}^{m} c_{ij}(t)f_j(y_j(t-\tau_{ij}(t))) + I_i(t), & t \neq t_k, \\ \Delta x_i(t) = I_{ik}(x_i(t)), & t = t_k, \\ y_j'(t) = -\tilde{a}_j(t)y_j(t) + \sum_{i=1}^{n} \tilde{b}_{ji}(t)g_i(x_i(t)) \\ \qquad + \sum_{i=1}^{n} \tilde{c}_{ji}(t)g_i(x_i(t-\sigma_{ji}(t))) + J_j(t), & t \neq t_k, \\ \Delta y_j(t) = J_{jk}(y_j(t)), & t = t_k, \end{cases} \tag{1.1}$$

where $x_i(t)$ and $y_j(t)$ are the activations of the ith neuron and the jth neuron at time t, respectively. Continuous functions $a_i(t)$, $\tilde{a}_j(t)$ denote the neuron charging times. $b_{ji}(t)$, $c_{ji}(t)$, $\tilde{b}_{ji}(t)$ and $\tilde{c}_{ji}(t)$ are the weights of the neuron interconnections. $\tau_{ij}(t)$ and $\sigma_{ji}(t)$ correspond to the finite speed of the axonal signal transmissions. Continuous functions $I_i(t)$ and $J_j(t)$ are the external inputs on the neurons. $\Delta x_i(t) = x_i(t^+) - x_i(t^-)$, $\Delta y_j(t) = y_j(t^+) - y_j(t^-)$ are the impulses at moment $t = t_k$ and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = \infty$.

System (1.1) is supplemented with initial values given by

$$x_i(s) = \phi_{x_i}(s), \quad s \in [-\sigma, 0], \quad y_j(s) = \phi_{y_j}(s),$$

$$s \in [-\tau, 0], i = 1,2,\ldots,n, \quad j = 1,\ldots,m,$$

where $\phi_{x_i}(s)$ and $\phi_{y_j}(s)$ are continuous ω-periodic functions defined on $[-\sigma, 0]$ and $[-\tau, 0]$, respectively, $\sigma = \max_{1 \le i \le n, 1 \le j \le m} \sigma_{ji}^+$, $\tau = \max_{1 \le i \le n, 1 \le j \le m} \tau_{ij}^+$, $\sigma_{ji}^+ = \max_{t \in [0,\omega]} \sigma_{ji}(t)$, $\tau_{ij}^+ = \max_{t \in [0,\omega]} \tau_{ij}(t)$.

As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \to (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T$, we assume that $(x_1(t_k^+), x_2(t_k^+), \ldots, x_n(t_k^+), y_1(t_k^+), y_2(t_k^+), \ldots, y_m(t_k^+))^T$ exists, $(x_1(t_k^-), x_2(t_k^-), \ldots, x_n(t_k^-), y_1(t_k^-), y_2(t_k^-), \ldots, y_m(t_k^-))^T \equiv (x_1(t_k), x_2(t_k), \ldots, x_n(t_k), y_1(t_k), y_2(t_k), \ldots, y_m(t_k))^T$. It is clear that there exist the limits $x_i'(t_k^+), x_i'(t_k^-)$, $y_j'(t_k^+)$ and $y_j'(t_k^-)$. According to the above convention, we assume that $x_i'(t_k^-) = x_i'(t_k)$, $y_j'(t_k^-) = y_j'(t_k)$.

The aim of this paper is, by using continuation theorem due to Gaines and Mawhin, some analysis techniques and constructing suitable Lyapunov functional, to derive the existence of exponential periodic attractor of system (1.1).

The organization of the rest is as follows. In Section 2, some preliminaries are introduced. In Section 3, by using Mawhin's continuation theorem, sufficient conditions ensuring the existence of periodic solution are established. In Section 4, by employing analysis techniques and Lyapunov functional method, the existence of exponential periodic attractor is derived. In Section 5, applications and an illustrative example are given to show the usefulness of the main results. Finally, simple conclusion is drawn in Section 6.

2. Preliminaries

Let PC be a class of function $\phi = (\phi_x^T, \phi_y^T)^T : ([-\sigma, 0], [-\tau, 0])^T \to (R^n, R^m)^T$ satisfying

(i) ϕ is piecewise continuous with first kind discontinuity at point t_k, and is left-continuous at t_k, $k = 1,2,\ldots$.

(ii) $\Delta x_i(t_k) = I_{ik}(x_i(t_k))$, $\Delta y_j(t_k) = J_{jk}(y_j(t_k))$ for $i = 1,\ldots,n$, $j = 1,\ldots,m$, $k = 1,2,\ldots$. For each $\phi = (\phi_x^T, \phi_y^T)^T \in PC$, we define

$$\|\phi\| = \sum_{i=1}^{n} \sup_{s \in [-\sigma,0]} |\phi_{x_i}(s)| + \sum_{j=1}^{m} \sup_{s \in [-\tau,0]} |\phi_{y_j}(s)|,$$

where $\phi_x = (\phi_{x_1}, \phi_{x_2}, \ldots, \phi_{x_n})^T$ and $\phi_y = (\phi_{y_1}, \phi_{y_2}, \ldots, \phi_{y_m})^T$.

Definition 2.1. A function $(x^T(t), y^T(t))^T : ([-\sigma, +\infty), [-\tau, +\infty))^T \to (R^n, R^m)^T$ is said to be a solution of impulsive system (1.1) with initial condition $\phi \in PC$, if

(i) $(x^T, y^T)^T$ is piecewise continuous with first kind discontinuity at point t_k and $(x^T, y^T)^T$ is left-continuous at t_k for $k = 1,2,\ldots$.
(ii) $(x^T, y^T)^T$ satisfies (1.1) for all $t > 0$ and $(x^T(s_1), y^T(s_2))^T = (\phi_x^T(s_1), \phi_y^T(s_2))^T$ for $s_1 \in [-\sigma, 0], s_2 \in [-\tau, 0]$.

For any $\phi \in PC$, we denote the solution of (1.1) through $(0, (\phi_x^T, \phi_y^T)^T)$ as $x(t,\phi) = (x_1(t,\phi), x_2(t,\phi), \ldots, x_n(t,\phi))^T$, $y(t,\phi) = (y_1(t,\phi), y_2(t,\phi), \ldots, y_m(t,\phi))^T$.

Definition 2.2 (*Huang and Xia [30]*). Impulsive BAM system (1.1) has an exponential periodic attractor if and only if there exists one ω-periodic solution $(x^T(t,\phi^*), y^T(t,\phi^*))^T$ with initial value $\phi^* \in PC$ and for any $\phi \in PC$, there exist positive constant α, β such that

$$\|(x^T(t,\phi^*), y^T(t,\phi^*))^T - (x^T(t,\phi), y^T(t,\phi))^T\| \le \alpha e^{-\beta t} \|\phi^* - \phi\|.$$

Definition 2.3 (*Varga [31]*). A real matrix $A = (a_{ij})_{n\times n}$ is said to be an M-matrix if $a_{ii} > 0, a_{ij} \le 0 (i,j = 1,2,\ldots,n, i \neq j)$ and successive principle minors of A are positive.

The following lemmas play an important role in the proofs of our main results, which may refer to Refs. [32–34].

Lemma 2.1 (*Fang et al. [32]*). *Let Q be an $n \times n$ matrix with non-positive off-diagonal elements. Then Q is an M-matrix if and only if one of the following conditions holds*:

(i) *There exists a vector $\xi > 0$ such that $Q\xi > 0$.*
(ii) *There exists a vector $\xi > 0$ such that $\xi^T Q > 0$.*

Lemma 2.2 (*Lu and Ge [33]*). *Suppose that $\tau(t) \in C^1(R, [0, +\infty))$ is an ω-periodic function and $\tau'(t) < 1, t \in [0, \omega]$. Then the function $t - \tau(t)$ has a unique inverse $u(t)$ satisfying $u(t) \in C(R,R)$ with $u(a + \omega) = u(a) + \omega$ for any $a \in R$.*

Lemma 2.3 (*Lu and Ge [33]*). *Let C_ω be a set of continuous ω-periodic functions for $t \in R$. If $g(t) \in C_\omega$, $\tau(t) \in C^1(R, [0, \infty))$ and $\tau'(t) < 1$ for any $t \in [0, \omega]$, then $g(u(t)) \in C_\omega$, where $u(t)$ is the inverse function of $t - \tau(t)$.*

Lemma 2.4 (*Gaines and Mawhin [34] (Mawhin's continuation theorem)*). *Let X and Y be two Banach spaces and $L : \mathrm{Dom}L \cap X \to Y$ be a Fredholm mapping of index zero. $\Omega \subset X$ be an open bounded set and $N : X \to Y$ be a continuous operator which is L-compact on $\overline{\Omega}$. Suppose that*

(a) *for each $\lambda \in (0,1), x \in \partial\Omega \cap \mathrm{DomL}, Lx \neq \lambda Nx$;*
(b) *for each $x \in \partial\Omega \cap \mathrm{KerL}, QNx \neq 0$;*
(c) $\deg\{JQN, \Omega \cap \mathrm{KerL}, 0\} \neq 0$. *Then Lx=Nx has at least one solution in $\overline{\Omega} \cap \mathrm{DomL}$.*