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Algorithm for faster computation of non-zero graph based invariants

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ABSTRACT

This paper presents a detailed study of the graph based algorithm used to generate geometric moment invariant functions. The graph based algorithm has been found to suffer from high computational complexity. One major cause of this problem is that the algorithm generates too many graphs that produce zero moment invariant functions. Hence, we propose an algorithm to determine and eliminate the zero moment invariant generating graphs and thereby generate non-zero moment invariant functions with reduced computational complexity. The correctness of the algorithm has been verified and discussed with suitable induction proofs and sample graphs. Asymptotic analysis has been presented to clearly illustrate the reduction in computational complexity achieved by the proposed algorithm. It has been found and illustrated with examples that the computational time for identifying non-zero invariants could be largely reduced with the help of our proposed algorithm.

1. Introduction

Image processing has been well known to be a vital area of research and development. This is attributed to the fact that images form a crucial part and parcel of human life. A large volume of information in today's world is presented as image rather than text. Image processing is a field of study that encompasses several stages such as Image acquisition, Image enhancement, Image restoration, Image representation, Image compression and Object recognition. Image acquisition as we know is accomplished via cameras and the same. An image $f(x,y)$ shall be acquired to be $g(u,v)$ due to the intrusion of a degradation operator D. Hence the relationship between the original image $f(x,y)$ and the acquired image $g(u,v)$ shall be written as in the following equation:

$$g(u,v) = f(x,y)h(x,y) + n(x,y), \tag{1}$$

where $(u,v) = T(x,y)$ is a spatial transformation undergone by the image. In Eq. (1), $h(x,y)$ represents the impulse response of the image acquisition system, and $n(x,y)$ is the induced random noise. Therefore the degradation operator D is a function of impulse response, noise and co-ordinate transformations. Although hardware and software technology today has advanced to acquire images with reduced degradation D, yet it cannot be totally eliminated. This phenomenon has caused object recognition to pose a heavy challenge to researchers. Numerous methods of object recognition such as edge detection and matching, gray scale matching, geometric hashing, Scale Invariant Feature Transform (SIFT) and Affine Scale Invariant Feature Transform (ASIFT) transforms have been proposed [1,2]. Moment invariant functions have been proving to be a viable alternative as they are applicable over a wide scope of research problems like character recognition, feature extraction, iris recognition and many more [3]. Invariance of the moment invariant functions to several transformations such as scaling, translation, rotation, and skew is a notable attribute for their success.

The moment invariant functions are constructed from raw moments of the form M_{pq}. The moment M_{pq} of an image $f(x,y)$ is defined as follows:

$$M_{pq} = \iint_D P_{pq}(x,y) f(x,y)\, dx\, dy, \tag{2}$$

where $r = p+q$ is the order of the moment and $p \geq 0,\ q \geq 0$. P_{pq} is a polynomial basis function. The polynomial basis function could be orthogonal or non-orthogonal. Non-orthogonal moment functions are also called as geometric moment functions. A number of non-orthogonal polynomial basis functions could be drawn. However

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in this paper we discuss about geometric affine moment invariants that are generated from the following function:

$$I(f) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{k,j=1}^{r} C_{kj}^{n_{kj}} \cdot \prod_{i=1}^{r} f(x_i, y_i)\, dx_i\, dy_i, \tag{3}$$

where the cross-product $C_{kj} = x_j y_k - y_k x_j$ involves three vertices $(x_k, y_k), (x_j, y_j), (0,0)$and n_{kj} are non–negative integers. We shall consider only $j > k$, because $C_{kj} = -C_{jk}$ and $C_{kk} = 0$. This research focuses mainly on identifying and eliminating the zero moment invariant generating graphs and determines the graphs that can be used to identify useful moment invariant functions of the form of Eq. (3). Further organization of the paper is as follows. Section 2 elaborates on the various commonly used orthogonal and non-orthogonal moment invariant functions in various areas of image processing. The predominant use of geometric invariants as observed in Section 2 motivates us to understand the working of graph based algorithm which is explained in Section 3. Section 4 summarizes the findings from the graph based algorithm and the need for improvising them. Section 5 presents the Proposed Graph Based Algorithm (P-GBA) as a solution to the findings in Section 4. Sections 6 and 7 present the experimental analysis and results that were used to evaluate the performance of the proposed solution in Section 5.

2. Related work

Moment invariant functions can be broadly classified into two major classes namely orthogonal and non-orthogonal or geometric moments. These two classes of moments have their own strengths and weaknesses [3,4]. Orthogonal moments are known for better performance at lower levels of precision as against geometric moments which require high precision computations for better object recognition [4]. Orthogonal moments enable straightforward reconstruction of images from their moments with a very low mean square error. On the contrary the geometric moments support image reconstruction in the Fourier domain, thereby making image reconstruction difficult. Geometric moments are at an advantage of being defined to work on pixel co-ordinates. This alleviates the need for transforming the images which are usually presented in two-dimensional co-ordinate space [5]. However the geometric moments are found to work on polar co-ordinates which require images to undergo transformation form Cartesian co-ordinates to polar co-ordinates thus causing overhead. From this comparative analysis we could infer that the ease of application of geometric moments has caused its wide-spread use over a wide number of applications. Despite ease of use, there are several successful works that have been accomplished by deploying geometric moments as explained in the following sections. This motivates us towards identifying better algorithms to generate higher order geometric moment invariant functions which can be used to accomplish better results in the field of image processing and object recognition.

Moment invariants have a long time history that dates back to the theory of algebraic invariants that was studied and explained by David Hilbert, a German mathematician [6]. Complex Zernike moment functions were one of the first orthogonal moments and invariant functions created from Zernike polynomials proposed in 1934. The complete set of orthogonal moments working over the interior of a unit circle was given by Zernike [7] which took the form given in Eq. (4). The applications of Zernike moments were mainly pioneered by Teague [8] in 1980. The image had to undergo transformation from the Cartesian form to polar form because the invariant function was complex. This is evitable from its formulation as in Eq. (4) where A_{mn} is the Zernike moment of mth order for n repetitions.

$$A_{mn} = \frac{(m+1)}{\Pi} \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} I(x,y)[V_{mn}(x,y)]^*$$
$$V_{mn}(x,y) = V_{mn}(r,\theta) = F(r)e^{jn\theta}$$
$$F(r) = \sum_{s=0}^{s=[m-|n|]/2} \frac{(m-s)!r^{m-2s}}{[s![((m+|n|)/2)-s]![(m+|n|/2)-s]!]}. \tag{4}$$

Zernike moments have been known to have minimum amount of information redundancy. Zernike moments were rotationally invariant. However they were not inherently invariant to translation or scale transformations. This invariance was always attained by normalizing the image. Moreover calculating the higher order moment invariants was computationally complex. Several works have been carried out to reduce the computational complexity of the Zernike moments and also improve their performance. Pseudo-Zernike moments were a result of one such attempt made by Bhatia and Wolf [9]. The radial function $F(r)$ is rewritten as follows:

$$F(r) = \sum_{s=0}^{s=[m-|n|]/2} \frac{(m-s)!r^{m-2s}}{[s![m+|n|+1-s]![(m-|n|)-s]!]}. \tag{5}$$

This redefinition of the radial functions caused the number of Pseudo-Zernike moments of nth order to be $(n+1)^2$ rather than the $(n+1)(n+2)/2$ number of Zernike moments. This is an advantage of Pseudo-Zernike moments. Pseudo-Zernike moments have also been tested and found to be robust against additive-noise.

Legendre moments are yet another set of orthogonal moments similar to Zernike moments which were constructed from Legendre polynomials [8]. Legendre moments are invariant functions calculated as in Eq. (6). They suffered a drawback of being a variant to rotation unlike Zernike moments. However they were invariant to rotation and scale. Moreover it was found that Legendre moments of higher orders were sensitive to noise. Non-orthogonal moment invariant functions have been found to be computationally easier when compared to the orthogonal functions.

$$\lambda_{mn} = \frac{(2m+1)(2n+1)}{4} \sum_x \sum_y P_m(x) P_n(y) P_{xy}. \tag{6}$$

Hu [10] presented the first set of seven geometric moment invariant functions composed of second and third order moments as in Eq. (7). The seven moment invariants of Hu were invariant to transformations like rotation, scaling and translation. A quantitative study on the skew invariance characteristic of the Hu moment invariants was explained in [11]. The study proved that the Hu moment invariants were sufficiently invariant to skew transformation.

$$\begin{aligned}
I_1 &= \eta_{20} + \eta_{02} \\
I_2 &= (\eta_{20} - \eta_{02})^2 + (2\eta_{11})^2 \\
I_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
I_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
I_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\
&\quad + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
I_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
&\quad + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\
I_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\
&\quad - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2].
\end{aligned} \tag{7}$$

Flusser and Suk explained that there is no general algorithm to extend the Hu moment invariant functions to involve higher order moments [12,13]. They also explained that the Hu moment invariant functions I_2 and I_3 as in Eq. (7) were polynomially dependent, thereby causing repetition of information. They also