Neurocomputing 135 (2014) 262–267

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Letters

On pinning group consensus for dynamical multi-agent networks with general connected topology

Xiaofeng Liao [a,*], Lianghao Ji [b]

[a] College of Electronics and Information Engineering, Southwest University, Chongqing 400715, PR China
[b] Chongqing Key Laboratory of Computational Intelligence (Chongqing University of Posts and Telecommunications), Chongqing 400065, PR China

ARTICLE INFO

Article history:
Received 29 May 2013
Received in revised form
8 October 2013
Accepted 13 December 2013
Communicated by H. Jiang
Available online 15 January 2014

Keywords:
Group consensus
Pinning control
Multi-agent networks
General connected coupling topology

ABSTRACT

In this paper, we focus on investigating group consensus of dynamical multi-agent networks via pinning scheme. The topology of the network is a general digraph, which needs neither being symmetric nor containing a spanning directed tree, and some criteria are proposed to guarantee the realization of group consensus instead of relying on other conservative assumptions presented by majority of the relevant research works, such as in-degree balance. In addition, it is interesting to find that the nodes with zero in-degree should be pinned first based on the property of M-matrix. Furthermore, an adaptive pinning control approach is developed to obtain the appropriate control gains. Finally, the effectiveness and correctness of our theoretical findings are verified by some numerical simulations.

1. Introduction

Consensus problem is one type of critical problem of distributed coordinative control of multi-agent networks (MANs), and is aimed at designing suitable protocols and control strategies such that all agents converge to a consistent state. Evidently, it could be regarded as a special case of synchronization of coupled complex networks. Thanks to their wide applications, consensus problem of MANs has attracted increasing attention from researchers in various fields, such as physics, mathematics, engineering, biology, and sociology. Among the various consensus problems, there is a particular consensus phenomenon called group consensus which requires all agents in each group reaching a consistent state while there is no consensus among different groups. The consensus problems of MANs have been extensively investigated recently, as seen in Refs. [1–16] and the references therein.

Usually, the whole network cannot reach consensus or synchronization by itself. Therefore, some appropriate controllers may be designed and applied to drive the network to achieve consensus. Of course, it is costly and literally impractical to add controllers to all nodes. In order to reduce number of controlled nodes, some local feedback controllers may be adopted to a small fraction of the network nodes which is known as pinning control. Hitherto, some works for pinning group consensus in MANs or cluster synchronization in complex networks have been published [17–28]. From the research results in [17], Li et al. showed that the nodes with direct connections between groups must be pinned first in order to achieve cluster synchronization. In [18], Wu et al. obtained some sufficient conditions such that the cluster synchronization was achieved by means of introducing a single controller for each cluster. Lu et al. [19] investigated cluster consensus of second-order multi-agent systems, and showed that cluster consensus was guaranteed when the nodes with direct connections between sub-clusters and those which are the root of either isolated sub-clusters or sub-clusters containing connecter components were controlled. By using the intermittent pinning control method, Liu et al. [20] proposed a centralized adaptive control approach to realize cluster synchronization and pointed out that enlarging the couplings of nodes in the same cluster played a key role in achieving cluster synchronization. In [21], Sun et al. derived some criteria to reach multi-group consensus for the networks with fixed and switching topology, and discussed the relation between the number of equilibriums and Laplacian matrix. Feng et al. [22] addressed a method about cluster synchronization of nonlinearly coupled complex networks and some sufficient conditions and a pinning scheme requiring the node with largest degree to be pinned were derived analytically. By adopting pinning controller, Li et al. [23] investigated cluster synchronization for coupled stochastic neural networks with delays. And in [24], Wang et al. discussed cluster synchronization for continuous/discrete-time Lur'e dynamical networks, and some sufficient conditions were obtained. Hu et al. [25] proposed a pinning strategy by

* Corresponding author.
E-mail address: xfliao@yahoo.com (X. Liao).

0925-2312/$ - see front matter
http://dx.doi.org/10.1016/j.neucom.2013.12.024

means of viewing each community as a whole and according to the information of each community. The community whose in-degrees were bigger than the out-degrees should be preferentially considered to be controlled. In [26], Wu et al. obtained several sufficient conditions for the network to achieve cluster synchronization by using a linear pinning scheme in which the nodes in one cluster having direct connections to the nodes in other cluster should be controlled. Recently, Su et al. [27] proposed a novel decentralized adaptive pinning-control scheme for cluster synchronization of undirected networks by using a local adaptive strategy on both coupling strengths and feedback gains, and drew a conclusion that at least one node in each cluster was selected to be pinned in order to reach cluster synchronization. Ma et al. [28] further investigated the complex networks with directed and weakly connected topology, and got some simple criteria to guarantee the realization of cluster synchronization, and the corresponding pinning scheme was addressed which is that the inter-act nodes and intra-act nodes with zero in-degree should be pinned by using the property of the nodes.

Therefore, we have to point out that some conservative assumptions on coupling matrix of the network were given to ensure the realization of group consensus (or synchronization) in the aforementioned papers. Some of them are requiring the coupling matrix to be symmetric [19,26–27], irreducible [17,22,24], and assuming that the influence from any other group to the node is equal to zero [5,21], i.e., in-degree of the node is balanced to other groups. These conditions are so special that they are not suitable for investigating the common problems. Meanwhile, they are very conservative in practice.

Inspired by the relevant works, in this paper, by introducing a novel pinning scheme, we investigate the problem of group consensus for MANs with general coupling topology. The main contribution of this paper will be presented in the following aspects. Firstly, an original protocol which fully takes into account the interaction to a node from other subgroups is designed. Therefore the conservative condition existing in some relevant works such as in-degree balance is not needed here. Secondly, some useful criteria have been derived analytically which can guarantee the dynamical MANs with general coupling topology to achieve group consensus. To our knowledge, there are few research papers dealing with this issue. Thirdly, we propose a novel pinning scheme which is that the node with zero in-degree should be pinned first based on the property of M-matrix. As far as we know, the pinning strategy we addressed is different from all the schemes mentioned in the relevant literature. Finally, in order to obtain appropriate control gains, an adaptive pinning strategy is addressed.

The rest of this paper is organized as follows. In Section 2, some preliminaries are briefly outlined. And the main results of pinning group consensus are addressed in Section 3. In Section 4, some numerical simulation examples are provided to validate our theoretical results. And conclusions are finally drawn in Section 5.

Notation: Throughout this paper, let $\Re$ and C denote the sets of real and complex numbers, respectively. For $z \in C$, Re(z) and Im(z) represent its real and imaginary part, respectively. Let I_n be an n-dimensional identity matrix. For a real matrix $A \in \Re^{n\times n}$, let A^T be its transpose and $A_s = (A + A^T)/2$ be its symmetric part. And for a square matrix A, denote its inverse by A^{-1}. For a real symmetric matrix A, denote its ith eigenvalue by $\lambda_i(A)$, and denote $A>0 (A<0)$, if A is positive (negative) definite.$\otimes$ Denotes the Kronecker product.

2. Preliminaries and problem statement

This section provides some supporting lemmas and preliminaries to obtain the main results of the paper.

2.1. Graph theory

In MANs, each agent can be represented as a node, and the information exchange among the agents can be described by an interaction graph. Let $G = \{V, \varepsilon, A\}$ be a diagraph (or weighted directed graph), in which $V = \{1, \dots, N\}$ is the node set, $\varepsilon \subseteq V \times V$ is the edge set, and $A = (a_{ij}) \in \Re^{N\times N}$ is the weighted adjacency matrix which can represent the coupling configuration of the network. The elements of matrix A are defined as follows: if there is a directed link from node j to node i $(i \neq j)$ which means that node i can receive information from node j (i.e.,$e_{ji} \in \varepsilon$), then $a_{ij} = 1$; otherwise $a_{ij} = 0$. In this paper, the graph is always supposed to be simple (without self-loops and multiple edges), i.e., $a_{ii} = 0$ for all $i \in V$. The in-degree and out-degree of the node i can be defined as follows:

$$\deg_{in}(i) = \sum_{j=1, j\neq i}^{N} a_{ij}, \tag{1}$$

$$\deg_{out}(i) = \sum_{j=1, j\neq i}^{N} a_{ji}. \tag{2}$$

The Laplacian matrix $L = (l_{ij}) \in \Re^{N\times N}$ associated with the weighted adjacency matrix A is defined as follows:

$$l_{ij} = -a_{ij}, \quad i \neq j, \tag{3}$$

$$l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}, \tag{4}$$

which ensures that $\sum_{j=1}^{N} l_{ij} = 0$. In view of (1)–(4), we can easily know that

$$\deg_{in}(i) = l_{ii}, \tag{5}$$

$$\deg_{out}(i) = -\sum_{j=1, j\neq i}^{N} l_{ji}. \tag{6}$$

2.2. Problem statement

Consider a network G consisting of N linearly and diffusively coupled identical nodes described by an n-dimensional dynamical system:

$$\dot{x}_i(t) = c \sum_{j=1, j\neq i}^{N} a_{ij} \Gamma [x_j(t) - x_i(t)], \quad i = 1, \dots, N, \tag{7}$$

where $x_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \Re^n$ is the state vector of the node i; $c>0$ is the coupling strength; $\Gamma > 0 \in \Re^{n\times n}$ describes the inner-coupling between the subsystems. For simplicity, we can assume that Γ is a diagonal matrix with positive elements; $A = (a_{ij}) \in \Re^{N\times N}$ is the coupling matrix representing the topological structure of the networks, a_{ij} is the (i, j)th entry of the adjacency matrix of A.

Suppose that the network has m groups with $m \geq 2$. If node i belongs to the jth group, let $\sigma_i = j$. Let ϕ_i denote the set of all the nodes in the ith group and $\phi_p \cap \phi_q = \phi (p \neq q)$. So we can easily know that $\sum_{i=1}^{m} \phi_i = N$.

Definition 1. A network G is said to realize m-group consensus if for any initial values, it follows that

$$\begin{aligned} &\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| = 0, \quad \sigma_i = \sigma_j, \\ &\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| \neq 0, \quad \sigma_i \neq \sigma_j. \end{aligned} \tag{8}$$

Let x_{σ_i} be the desired equilibrium state of the group that the node i exists in. So the network has m groups and can own at most m desired equilibriums, such as $x_{\sigma_i}, 1 \leq i \leq N, 1 \leq \sigma_i \leq m$. In particular, if $x_{\sigma_i} = x_{\sigma_j}$ holds for $\forall i \neq j, i, j \in [1, N]$, it means that the