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Adaptive neural control for a general class of pure-feedback stochastic nonlinear systems

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ABSTRACT

In this paper, the problem of the adaptive neural control is considered for a class of pure-feedback stochastic nonlinear systems. Based on the radial basis function (RBF) neural networks′ universal approximation property, an adaptive neural controller is developed via backstepping technique. It is shown that the proposed controller can guarantee that all the signals in the closed-loop system are bounded in the sense of mean quartic value. Compared with the existing results on adaptive control of stochastic pure-feedback nonlinear systems, the main novelty of this note is that a systematic design procedure is presented for a class of pure-feedback stochastic nonlinear systems with a more general form of the diffusion term. Simulation results are presented to demonstrate the effectiveness of the proposed scheme.

1. Introduction

During the past decades, many control methods have been developed for control of nonlinear systems, such as adaptive control [1], backstepping control [2] and fault tolerant control [3,4]. Specifically, adaptive backstepping technique provides a systematic design approach for nonlinear strict-feedback systems without satisfying matching condition. So far, some interesting results on backstepping-based adaptive control of strict-feedback systems have been obtained in [2,5–7] for deterministic nonlinear systems and in [8–15] for stochastic cases. However, these control schemes are only suitable for those nonlinear systems with the nonlinear dynamic models known exactly or with the unknown parameters appearing linearly with respect to known nonlinear functions [19]. When the system nonlinear functions are not available, these classical adaptive backstepping controllers cannot be applied. Neural networks and fuzzy logic systems are mostly used as approximation models for the unknown nonlinearities due to their inherent approximation capabilities. With the help of neural networks (or fuzzy logical systems) approximation, it is not necessary to spend much effort on system modeling which might be very difficult in some cases. So, the control design methods by combining adaptive backstepping with neural networks and fuzzy logical systems have been found to be a particularly useful tool for the control of nonlinear strict-feedback systems. The advantages of adaptive neural backstepping control approach lie in the fact that a priori knowledge of the system nonlinear functions is not necessary, and the matching condition may be unsatisfied. Generally, adaptive neural (or fuzzy) backstepping control provides a systematic methodology for the tracking or regulation problems of nonlinear systems with unknown functions. Until now, many valuable research results on the strict-feedback nonlinear systems have been obtained, for example, see [16–40]and the reference therein. The works in [17–30] developed adaptive neural or fuzzy control approaches for the deterministic nonlinear systems with or without time delays. Correspondingly, in [31–40], the problem of approximation-based adaptive backstepping control was considered for uncertain stochastic nonlinear strict-feedback systems. In the aforementioned papers [16–40], however, the obtained results required that the unknown nonlinear functions were in the affine forms, i.e., the systems were characterized by input appearing linearly in the system state equation.

The nonaffine pure-feedback systems, which represent a more general class of lower-triangular systems, have no affine appearance of the variables to be utilized as virtual control inputs. This makes the control of pure-feedback nonlinear systems difficult and challenging. In addition, many practical systems are of non-affine structure, such as mechanical systems [41], and biochemical processes [2]. Therefore, the investigation on the control design of

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nonaffine pure-feedback nonlinear systems is a meaningful issue and has received considerable attention in the control community in recent years [42–47]. In [42,43], by combining the backstepping methodology with adaptive neural design, several special cases of pure-feedback systems where the last one or two equations are assumed to be in affine form were investigated. Furthermore, some elegant control approaches were developed for completely pure-feedback systems [44] and other cases, such as pure-feedback systems with time-delay [45], disturbances [46] and dead-zone [47]. More recently, the study on stability analysis and control design of stochastic pure-feedback nonlinear systems has attracted increasing attention. In [48], an adaptive fuzzy control scheme was proposed for pure-feedback stochastic nonlinear systems. Nevertheless, the work in [48] requires that the first $n-1$ subsystems have to be in affine form whereas the last differential equation is in non-affine form only. Furthermore, in [49], the problem of adaptive neural tracking control was considered for a class of completely pure-feedback stochastic nonlinear systems where the number of adaption laws is the order of the considered systems. In [51], an adaptive fuzzy output-feedback control scheme was developed for a class of pure-feedback stochastic nonlinear systems in which the number of adaptive law depends on the order of the original system and the weight vector dimensions of fuzzy logical systems. If the order of the considered systems or the weight vector dimensions increases, the number of adaptive parameters will increase correspondingly, and as a result, the online learning time may be very long. In addition, for the ease of the controller design, the diffusion terms $\psi_i(\cdot)$ in the existing results [48–51] are assumed to be the function of the previous state variables $\bar{x}_i=[x_1,x_2,\ldots,x_i]^T$. Apparently, if the diffusion terms in the considered stochastic pure-feedback systems are in a more general form, the aforementioned control schemes may be invalid.

Motivated by the above observation, an adaptive neural control approach is developed for a class of stochastic pure-feedback nonlinear systems based on the backstepping technique. It is shown that the proposed controller can ensure boundedness of all the signals in the closed-loop system in the sense of mean quartic value. The main contribution of this study is that a backstepping-based adaptive neural control methodology is systematically developed for a class of pure-feedback stochastic nonlinear systems in a more general form. In addition, by estimating the maximum value of the norm of weight vector of neural networks rather than the weight vector elements themselves, only one adaptive parameter needs to be estimated online. As a result, the computational burden is alleviated significantly, which could render this control design more suitable for practical applications.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section 2. An adaptive neural control scheme is presented in Section 3. The simulation examples are given in Section 4, and followed by Section 5 which concludes the work.

2. Problem formulation and preliminaries

In this paper, consider a class of stochastic pure-feedback nonlinear system as follows:

$$\begin{cases} dx_i=f_i(\bar{x}_i,x_{i+1})\,dt+\psi_i^T(\bar{x}_i,x_{i+1})\,dw, \quad 1\le i\le n-1, \\ dx_n=f_n(\bar{x}_n,u)\,dt+\psi_n^T(\bar{x}_n)\,dw, \\ y=x_1, \end{cases} \tag{1}$$

where $x=[x_1,x_2,\ldots,x_n]^T\in R^n$, $u\in R$ and $y\in R$ are the state vector, system input, and system output, respectively, $\bar{x}_i=[x_1,x_2,\ldots,x_i]^T \in R^i$, w is a r-dimensional standard Brownian motion defined on the complete probability space (Ω,F,P) with Ω being a sample space, F being a σ-field, $\{F_t\}_{t\ge 0}$ being a filtration, and P being a probability measure. The drifting terms $f_i(\cdot):R^{i+1}\to R$ and the diffusion terms $\psi_i(\cdot):R^{i+1}\to R^r$, $(i=1,2,\ldots,n)$ are unknown smooth nonlinear functions.

Remark 1. It is noted that the pure-feedback nonlinear systems investigated in [42–47] do not contain stochastic disturbance, and the diffusion terms $\psi_i(\cdot)$ in [49,48,51] are assumed to be the function of the previous state variables $\bar{x}_i=[x_1,x_2,\ldots,x_i]^T$. Apparently, the system considered in (1) is in a more general form.

The objective of this note is to design an adaptive neural controller u for the system (1) under some assumptions such that all the signals in the closed-loop remain bounded in the sense of mean quartic value.

According to mean the value theorem [52], the following equations can be obtained

$$\begin{aligned} &f_i(\bar{x}_i,x_{i+1})=f_i(\bar{x}_i,x_{i+1}^0)+g_{\mu_i}(x_{i+1}-x_{i+1}^0), \quad (1\le i\le n-1), \\ &f_n(\bar{x}_n,u)=f_n(\bar{x}_n,u^0)+g_{\mu_n}(u-u^0), \end{aligned} \tag{2}$$

where smooth function $f_i(\cdot)$ is explicitly analyzed between $f_i(\bar{x}_i,x_{i+1})$ and $f_i(\bar{x}_i,x_{i+1}^0)$, $g_{\mu_i}:=g_i(\bar{x}_i,x_{\mu_i})=(\partial f_i(\bar{x}_i,x_{i+1})/\partial x_{i+1})|_{x_{i+1}=x_{\mu_i}}$, $i=1,2,\ldots,n$, $x_{n+1}=u$, $x_{\mu_i}=\mu_i x_{i+1}+(1-\mu_i)x_{i+1}^0$, $0<\mu_i<1$. Furthermore, by substituting (2) into (1), and choosing $x_{i+1}^0=0$, $u^0=0$, it follows

$$\begin{cases} dx_i=(f_i(\bar{x}_i,0)+g_{\mu_i}x_{i+1})\,dt+\psi_i^T(\bar{x}_{i+1})\,dw, \quad 1\le i\le n-1, \\ dx_n=(f_n(\bar{x}_n,0)+g_{\mu_n}u)\,dt+\psi_n^T(\bar{x}_n)\,dw, \\ y=x_1. \end{cases} \tag{3}$$

To facilitate the control design in Section 3, the following assumptions are required.

Assumption 1 (*Chen et al. [25]*). The signs of $g_{\mu_i}, i=1,2,\ldots,n$, are known, and there exist unknown constants b_m and b_M such that, for $1\le i\le n$

$$0<b_m\le |g_{\mu_i}|\le b_M<\infty, \quad \forall(\bar{x}_i,x_{i+1})\in R^i\times R. \tag{4}$$

Assumption 2. There exist strict increasing smooth functions $\rho_i(\cdot):R^+\to R^+$ with $\rho_i(0)=0$ such that for $i=1,2,\ldots,n$

$$\|\psi_i(\bar{x}_i)\|\le\rho_i(\|\bar{x}_i\|).$$

Remark 2. The increasing property of $\rho_i(\cdot)$ means that if $a_k\ge 0$, for $k=1,2,\ldots,n$, then $\rho_i(\sum_{k=1}^n a_k)\le\sum_{k=1}^n\rho_i(na_k)$. Note that $\rho_i(s)$ is a smooth function with $\rho_i(0)=0$, so there exists a smooth function $\eta_i(s)$ such that $\rho_i(s)=s\eta_i(s)$, which results in

$$\rho_i\left(\sum_{k=1}^n a_k\right)\le\sum_{k=1}^n na_k\eta_i(na_k). \tag{5}$$

To introduce some useful definitions and lemmas, we first consider the following stochastic system:

$$dx=f(x,t)\,dt+h(x,t)\,dw, \quad \forall x\in R^n \tag{6}$$

where $x\in R^n$ is the state of the system, w is a r-dimensional independent standard Brownian motion defined on the complete probability space (Ω,F,P), and $f(\cdot)$, $h(\cdot)$ are locally Lipschitz functions in x and satisfy $f(0)=g(0)=0$.

Definition 1. For any given $V(x,t)\in C^{2,1}$, associated with the stochastic differential equation (6) define the differential operator