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Stability and Hopf bifurcation analysis of a tri-neuron BAM neural network with distributed delay

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ABSTRACT

In this paper, a tri-neuron BAM neural network with distributed delay is considered. The distributed delay is regarded as the bifurcating parameter to study the dynamic behaviors in terms of local asymptotical stability and local Hopf bifurcation. By analyzing the associated characteristic equation, Hopf bifurcation occurs when the delay passes through a sequence of critical values. The direction and stability of bifurcating periodic solutions are also derived by the normal form theory and the center manifold theorem. Finally, an illustrative example is also given to support the theoretical results.

1. Introduction

Since the pioneering works of Kosko and Gopalsamy [1–3], the bidirectional associative memory (BAM) neural networks have attracted considerable attention, due to its bidirectional structure, BAM neural networks have practical applications in storing paired patterns or memories and possess the ability of searching the desired patterns via both directions: forward and backward [4]. Thus, BAM neural networks have been widely studied by many authors and various interesting results have been reported, see [5–9]. However, most literatures focus on establishing the local and global stability. It is known that stability is not the unique dynamic behavior, there are many other dynamic behaviors such as periodic oscillation, bifurcation, chaos and so on. Therefore, it has a wider significance to study the bifurcation of neural networks.

In the past a few decades, there are extensive literatures on bifurcation analysis of some special BAM neural networks with delays, see [4,10–12,14–16]. In [4], the authors investigated a two-layer BAM neural network with discrete delays where Hopf bifurcation occurs when the sum of the delays passes through a sequence of critical values. In [10,11], different types of neural networks with discrete delays were studied where all the variables of the network could be regarded as the bifurcating parameter. In [12], a class of four-neuron BAM neural networks with discrete delays was studied, the sum of the delays was regarded as the bifurcating parameter.

It is well known that neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, and hence there is a distribution of propagation delays over a period of time, so that the distributed delays should be incorporated in the model [13]. In [14,15], the authors studied the neural network model with infinite distributed delays, the mean delay was regarded as the bifurcating parameter, respectively. In [16], the authors studied a two-neuron BAM neural network with finite distributed delay where the interconnection weights were regarded as the bifurcating parameter. However, if the distributed delay was regarded as the bifurcating parameter, the methods developed in [16] may be difficult to be applied, and it is therefore necessary to further investigate the bifurcation problem by regarding distributed delay as the bifurcating parameter. To the best of our knowledge, when neural networks with finite distributed delay, the bifurcation problem is seldom discussed by taking delay as the bifurcating parameter, which remains important and challenging. On the other hand, as it is pointed out in [4], when the number

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of neurons is larger, the neural network model can reflect the really large neural networks more closely. Thus, it is necessary to further investigate the larger neural network models.

Motivated by the above discussions, in this paper, we consider a simplified two-layer BAM network described by the following differential equations with finite distributed time delay as follows:

$$\begin{cases} \dfrac{dx(t)}{dt} = -\mu_1 x(t) + \alpha_{21} \int_0^\tau f(y_1(t-\theta))\, d\theta + \alpha_{31} \int_0^\tau f(y_2(t-\theta))\, d\theta, \\ \dfrac{dy_1(t)}{dt} = -\mu_2 y_1(t) + \alpha_{12} \int_0^\tau f(x(t-\theta))\, d\theta, \\ \dfrac{dy_2(t)}{dt} = -\mu_3 y_2(t) + \alpha_{13} \int_0^\tau f(x(t-\theta))\, d\theta \end{cases} \tag{1}$$

with initial conditions

$$x(\theta) = \phi_1(\theta), \quad y_1(\theta) = \phi_2(\theta), \quad y_2(\theta) = \phi_3(\theta), \quad \theta \in [-\tau, 0],$$

where $\phi_1(\theta)$, $\phi_2(\theta)$, $\phi_3(\theta)$ are continuous functions.

In network (1), $\mu_i > 0\ (i = 1,2,3)$ describe the stability of internal neuron processes on the X-layer and the Y-layer; α_{12}, α_{13}, α_{21} and α_{31} are the interconnection weights through the neurons in two layers: the X-layer and the Y-layer; $f : \mathbb{R} \to \mathbb{R}$ represents the neuron activations; positive constant τ represents the distributed time delay from the X-layer to another Y-layer and from the Y-layer back to the X-layer. There is only one neuron in the X-layer and two neurons in the Y-layer. Here, we suppose $f(0) = 0$ and $f'(0) = 1$.

The object of this paper is to analyze the local asymptotic stability and local Hopf bifurcation at equilibrium point of system (1). Taking the distributed delay τ as parameter, we will show that the equilibrium point losses its stability and Hopf bifurcation occurs when the distributed delay τ passes through a critical value τ_0. The direction of Hopf bifurcation is investigated by using the normal form theory and center manifold theorem as well.

2. The characteristic equation

In this section, we will derive the corresponding characteristic equation of the linearization of (1) at the equilibrium point $(0,0,0)^T$. The linearization of (1) at the equilibrium point is given by

$$\begin{cases} \dfrac{dx(t)}{dt} = -\mu_1 x(t) + \alpha_{21} \int_0^\tau y_1(t-\theta)\, d\theta + \alpha_{31} \int_0^\tau y_2(t-\theta)\, d\theta, \\ \dfrac{dy_1(t)}{dt} = -\mu_2 y_1(t) + \alpha_{12} \int_0^\tau x(t-\theta)\, d\theta, \\ \dfrac{dy_2(t)}{dt} = -\mu_3 y_2(t) + \alpha_{13} \int_0^\tau x(t-\theta)\, d\theta. \end{cases} \tag{2}$$

The associated characteristic equation of (2) is

$$\begin{vmatrix} \lambda + \mu_1 & -\alpha_{21} \int_0^\tau e^{-\lambda\theta}\, d\theta & -\alpha_{31} \int_0^\tau e^{-\lambda\theta}\, d\theta \\ -\alpha_{12} \int_0^\tau e^{-\lambda\theta}\, d\theta & \lambda + \mu_2 & 0 \\ -\alpha_{13} \int_0^\tau e^{-\lambda\theta}\, d\theta & 0 & \lambda + \mu_3 \end{vmatrix} = 0. \tag{3}$$

That is

$$(\lambda+\mu_1)(\lambda+\mu_2)(\lambda+\mu_3) - \alpha_{12}\alpha_{21}(\lambda+\mu_3)\left(\int_0^\tau e^{-\lambda\theta}\, d\theta\right)^2 - \alpha_{13}\alpha_{31}(\lambda+\mu_2)\left(\int_0^\tau e^{-\lambda\theta}\, d\theta\right)^2 = 0. \tag{4}$$

If all roots of (4) have negative real parts, then the equilibrium point $(0,0,0)^T$ is stable. However, if (4) has at least one root with positive real part, then the equilibrium point $(0,0,0)^T$ is unstable, which means that it is necessary to investigate the distribution of roots of (4).

We note that λ is a nonzero root of (4) if and only if λ is a nonzero root of the following equation:

$$\lambda^2(\lambda+\mu_1)(\lambda+\mu_2)(\lambda+\mu_3) - \alpha_{12}\alpha_{21}(\lambda+\mu_3)\left(\lambda\int_0^\tau e^{-\lambda\theta}\, d\theta\right)^2 - \alpha_{13}\alpha_{31}(\lambda+\mu_2)\left(\lambda\int_0^\tau e^{-\lambda\theta}\, d\theta\right)^2 = 0.$$

By integration, we can get

$$\lambda^2(\lambda+\mu_1)(\lambda+\mu_2)(\lambda+\mu_3) - \alpha_{12}\alpha_{21}(\lambda+\mu_3)(1-e^{-\tau\lambda})^2 - \alpha_{13}\alpha_{31}(\lambda+\mu_2)(1-e^{-\tau\lambda})^2 = 0. \tag{5}$$

From (5), by simple calculation, the following fifth-degree exponential polynomial equation is obtained:

$$\lambda^5 + d_0\lambda^4 + d_1\lambda^3 + d_2\lambda^2 - d_3\lambda - d_4 + 2d_3\lambda e^{-\tau\lambda} + 2d_4 e^{-\tau\lambda} - d_3\lambda e^{-2\tau\lambda} - d_4 e^{-2\tau\lambda} = 0, \tag{6}$$

where

$$d_0 = \mu_1 + \mu_2 + \mu_3,$$

$$d_1 = \mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3,$$

$$d_2 = \mu_1\mu_2\mu_3,$$

$$d_3 = \alpha_{12}\alpha_{21} + \alpha_{13}\alpha_{31},$$