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New criteria for exponential stability of delayed recurrent neural networks

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ABSTRACT

In this paper, we investigate exponential stability of delayed recurrent neural networks. By using the delay partitioning method, some sufficient conditions are established to guarantee exponential stability of delayed recurrent neural networks under two different conditions with constructing new Lyapunov–Krasvoskii functional. This partitioning approach can reduce the conservatism comparing with some previous results of stability. At last, numerical examples are given out to show the effectiveness and advantage of our results.

1. Introduction

In recent years, recurrent neural networks (RNNs) have been studied extensively because of their many successful applications in many different fields such as signal processing, realizing associative memories, solving certain optimization problems, image processing, and pattern classification [1–9]. It is well known that applications of neural networks rely heavily on the dynamical behaviors of neural networks. For example, when a neural network is designed to function as an associative memory, the designed neural network must have multiple equilibrium points for a particular input vector. However, when problems in the areas of optimization, signal processing neural control systems, the designed neural network is required only one equilibrium point for a particular input vector, and this equilibrium point must be globally asymptotically stable to avoid the risk of having multiple equilibrium points. Hence, the analysis of stability for neural networks especially global exponential stability is an important one of our desired behaviors with designing and applying neural networks. Xia and Wang in [10] have presented a projection neural network to solve a wider kind of variational inequalities and corresponding optimization problems. In [11], Zeng, Wang and Liao have established some sufficient conditions for the global exponential stability of recurrent neural networks with time-varying delays and Lipschitz continuous activation functions. Up to now, the stability analysis for RNNs has been widely researched and a great number of corresponding results have been reported in the published literatures [12–15].

Due to the finite speed of information transmission and the finite switching speed of the amplifiers, time delay is frequently encountered in the practical implement of neural networks, which may lead bad dynamic behaviors such as instability and oscillation. For example, for neural networks with discontinuous activations, Forti, Nistri and Papini in [16] have pointed out that time delay may affect the convergence in finite time of neural networks′ state to the equilibrium point even make the state can not converge to the equilibrium point in finite time. So it is significant to investigate the problem of delayed neural network. So far, the reported results of stability for delayed neural networks have been classified into two types: delay-independent stability and delay dependent stability [17–24]. Since delay-dependent criteria make use of the information on the size of delay, they are generally less conservative than delay-independent ones especially when the delay is small in size, so that there are much more effort devoted to the case of delay-dependent. In [25], authors have established delay-dependent stability criteria for neural networks with time-varying interval delay by means of the free-weighting matrices method.

In addition, according to whether the neuron states (the external states of neurons) or local field states (the internal states of neurons) are chosen as basic variables in the network, neural networks are classified as "static neural networks" or "local field

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neural networks" in [26], respectively. Until now, a great deal of results have been proposed referring to the stability analysis of the local field neural networks and its further extensions such as cellular neural networks, bidirectional associative memory (BAM) neural networks and Cohen–Grossberg neural networks. For example, Faydasicok and Arik in [27] have researched the existence, uniqueness and global asymptotic stability for neural networks with discrete constant time delays and parameter uncertainties. However, referring to "static neural networks", it has attracted very little attention, and very few results corresponding to stability can be found compared with the "local field neural networks". In [28], the problem of robust global exponential stability analysis for RNNs without delay was studied based on the linear matrix inequality (LMI) approach.

Inspired by this case that "static neural networks" have been relatively less studied, we will employ the delay partition technique to discuss the problem of the global exponential asymptotic stability for recurrent neural networks having the form of "static neural networks". The delay partition method's basic idea is to partition time delay (usually constant delay) into several components, and if the time delay is time varying with lower bound, the lower bound is partitioned into several components in this situation. Furthermore, several sufficient delay dependent conditions are presented in the form of linear matrix inequalities by means of Lyapunov–Krasovskii functional. The obtained results are less conservatism than some previous results.

The rest of this paper is organized as follows. In Section 2, corresponding neural networks model and some preliminary lemmas are presented. Sufficient conditions are established to guarantee exponential stability of recurrent neural networks under two different situations in Section 3. In Section 4, two numerical examples are given out to show the effectiveness of our proposed results, and we conclude the paper in Section 5.

2. Problem formulation

Consider a delayed recurrent neural network described as follows:

$$\dot{x}(t) = -Dx(t) + f(Wx(t-\tau(t)) + J) \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \Re^n$ is the neuron state vector associated with the n neurons; $J = [j_1, j_2, \ldots, j_n]^T \in \Re^n$ is a constant input from outside the system; $D = \text{diag}\{d_1, d_2, \ldots, d_n\} > 0$ and $W = [W_1^T, W_2^T, \ldots, W_n^T]^T$ are the connection and the delay connection weight matrix, respectively; $\tau(t)$ is delay of the state; $f(Wx(t)) = [f_1(W_1x(t)), f_2(W_2x(t)), \ldots, f_n(W_nx(t))]^T \in \Re^n$ denotes the neuron activation functions which satisfies the following assumption.

Assumption 1. The neuron activation functions are bounded and satisfy the following condition:

$$\widehat{k}_i \le \frac{f_i(x) - f_i(y)}{x-y} \le \overline{k}_i, \quad \forall x, y \in \Re, \ x \ne y \tag{2}$$

where $\widehat{k}_i, \overline{k}_i, i = 1, 2, \ldots, n$ are known real scalars.

In this paper, time delay $\tau(t)$ is discussed under two different conditions described as follows:

(I) Constant time delay: $0 \le \tau(t) \equiv d$.
(II) Time-varying delay: $0 \le d_1 \le \tau(t) \le d_2$ and $\dot{\tau}(t) \le d_3$.

Suppose that initial conditions of RNNs (1) are given

$$x(t) = \phi(t), \quad -\overline{\tau} \le t \le 0 \tag{3}$$

where $\overline{\tau} = \sup_{t \ge 0}\{\tau(t)\}$.

Remark 1. Suppose that W is invertible and $WD = DW$. Denote $y(t) = Wx(t) + J$. Then, delayed recurrent neural network (1) can be transformed into the local neural network

$$\dot{y}(t) = -Dy(t) + Wf(y(t-\tau(t))) + DJ \tag{4}$$

which has been extensively investigated in many previous literatures. However, most of recurrent neural networks in the form of (1) can not satisfy the above transformation condition. Therefore, it is necessary to study the case of recurrent neural networks described as (1).

Assumption 1 guarantees the existence of one equilibrium point x^* for neural network (1). Now, for simplicity, let $z(t) = x(t) - x^*$. Then, neural network (1) can be written as follows:

$$\begin{cases} \dot{z}(t) = -Dz(t) + g(Wz(t-\tau(t)) \\ z(t) = \psi(t), \ -\overline{\tau} \le t \le 0 \end{cases} \tag{5}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T$ is the state vector of transformed system (5); $\psi(t) = \phi(t) - x^*$ is the initial condition; the transformed neuron activation function is $g(Wx(t)) = [g_1(W_1x(t)), g_2(W_2x(t)), \ldots, g_n(W_nx(t))]^T$ with $g(Wz(t)) = f(W(z(t) + X^*) + J) - f(Wx^* + J)$. Obviously, the functions $g_i(\cdot)$ satisfy

$$\widehat{k}_i \le \frac{g_i(z_i)}{z_i} \le \overline{k}_i, \quad g_i(0) = 0, \ \ \forall z_i \ne 0, \ i = 1, 2, \ldots, n \tag{6}$$

It is obvious that neural network (5) admits the equilibrium point $z(t) = 0$ corresponding to the initial condition $\psi(t) = 0$ when $-\overline{\tau} \le t \le 0$. Based on the above analysis, we can easily know that the problem of how to analyze the exponential stability of neural network (1) at equilibrium point x^* is equivalent to the problem of how to analyze exponential stability of neural network (5) at $z(t) = 0$.

Definition 1. The equilibrium point $z^* = 0$ of recurrent neural network (5) is said to be exponentially stable if the solution $z(t)$ satisfies

$$\|z(t)\| \le \eta \|z_{t_0}\|_C e^{-\lambda(t-t_0)}, \quad \forall t \ge t_0$$

for constants $\eta \ge 1$ and $\lambda > 0$, and when the delay $\tau(t)$ satisfies (I),

$$\|z_t\|_C = \sup_{-d \le \theta \le 0} \{\|z(t+\theta)\|, \quad \|\dot{z}(t+\theta)\|\};$$

otherwise,

$$\|z_t\|_C = \sup_{-d_2 \le \theta \le 0} \{\|z(t+\theta)\|, \quad \|\dot{z}(t+\theta)\|\}.$$

Lemma 1. *For any positive symmetric constant matrix M which has corresponding dimension, scalars a and b, and a vector function $\varphi(t) : [a, b] \to \Re^n$ which is integrable in $[ab]$, Jensen's inequality holds*

$$\left\{\int_a^b \varphi(s)\,ds\right\}^T M \left\{\int_a^b \varphi(s)\,ds\right\} \le (b-a)\left\{\int_a^b \varphi^T(s) M \varphi(s)\,ds\right\}$$

Notation: The superscript 'T' denotes matrix transposition; $\Re^n$ stands for the n-dimensional Euclidean space; I and 0 represent identity matrix and zero matrix with compatible dimensions, respectively; $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix; $A^S = A + A^T$.

3. Exponential stability of delayed recurrent neural networks

In this section, we will employ the delay partition technique to discuss the exponential stability of neural networks under different conditions: constant time-delay and time-varying delay. Firstly, let us consider the constant time-delay case, i.e. (I): $0 \le \tau(t) \equiv d$.