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Convergence behavior of Cohen–Grossberg neural networks with time-varying delays in the leakage terms ☆

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ABSTRACT

In this paper, we study the convergence behavior of Cohen–Grossberg neural networks with time-varying delays in the leakage terms. Under proper conditions, we establish some criteria to guarantee that the solutions of the networks converge locally exponentially to zero point, which are new and complement of previously known results. Moreover, we give an example to illustrate the effectiveness of our main results.

1. Introduction

Since the model of Cohen–Grossberg neural networks (CGNNs) was first introduced by Cohen and Grossberg in [1], the dynamical characteristics (including stable, unstable and periodic oscillatory) of CGNNs have been widely investigated for the sake of theoretical interest as well as application considerations. Many good results on the problem of the existence and stability of the equilibriums for CGNNs are given out in many literatures. We refer the reader to [2–8] and the references cited therein. Recently, very little attention has been paid to neural networks with time delay in the leakage (or "forgetting") term (see [9–15]). As pointed out in [15], the time delays in the leakage terms are difficult to handle. Hence, to the best of our knowledge, few authors have investigated the convergence behavior for CGNNs with delays in the leakage terms. Motivated by this, in this present paper, we shall consider the convergence behavior of CGNNs with time-varying delays in the leakage terms as follows:

$$x'_i(t) = -a_i(t, x_i(t))\left[b_i(t, x_i(t-\delta_i(t))) - \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) - \sum_{j=1}^{n} d_{ij}(t)\int_0^{\infty} K_{ij}(u) g_j(x_j(t-u))\,du + I_i(t)\right], \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where a_i and b_i are continuous functions on R^2, δ_i, τ_{ij}, f_j, g_j, c_{ij}, d_{ij} and I_i are continuous functions on R; n corresponds to the number of units in a neural network; $x_i(t)$ denotes the potential (or voltage) of cell i at time t; a_i represents an amplification function; b_i is an appropriately behaved function; $c_{ij}(t)$ and $d_{ij}(t)$ denote the strengths of connectivity between cells i and j at time t respectively; the activation functions $f_i(\cdot)$ and $g_i(\cdot)$ show how the ith neuron reacts to the input, $\delta_i(t) \geq 0$ and $\tau_{ij}(t) \geq 0$ denote the leakage delay and transmission delay respectively, and $I_i(t)$ denotes the ith component of an external input source introduced from outside the network to cell i at time t for $i, j = 1, 2, \ldots, n$. As pointed out in Kosko [16], the first term $b_i(t, x_i)$ in each of the right side of (1.1) corresponds to a stabilizing negative feedback of the system which acts instantaneously without time delay $\delta_i(t)$; these terms are variously known as "forgettin" or leakage terms (see for instance Haykin [17], Gopalsamy [18]). In real world, the transmission delays often appear in these terms, which are called as leakage delays by Gopalsamy [11,16].

Throughout this paper, for $i, j = 1, 2, \ldots, n$, it will be assumed that $t-\delta_i(t) \geq 0$ for all $t \geq 0$, and there exist constants τ_{ij}^+, δ_i^+, $\overline{c_{ij}}$, $\overline{d_{ij}}$ and $\overline{I_i}$ such that

$$\begin{cases} \tau_{ij}^+ = \sup_{t\in R} \tau_{ij}(t), \quad \delta_i^+ = \sup_{t\in R} \delta_i(t), \quad \overline{c_{ij}} = \sup_{t\in R} |c_{ij}(t)|, \\ \overline{d_{ij}} = \sup_{t\in R} |d_{ij}(t)|, \quad \overline{I_i} = \sup_{t\in R} |I_i(t)|. \end{cases} \tag{1.2}$$

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We also assume that the following conditions (H_1)–(H_6) hold.

(H_1) For each $j\in\{1,2,\ldots,n\}$, there exist nonnegative constants m, m^*, $\tilde{L}_j$ and L_j such that

$$m\geq 1,\ m^*\geq 1,\ |f_j(u)|\leq \tilde{L}_j|u|^m,\ |g_j(u)|\leq L_j|u|^{m^*}\quad \text{for all } u\in R. \tag{1.3}$$

(H_2) For $i=1,2,\ldots,n$, there exist positive constants such that $\underline{a_i}$ and $\overline{a_i}$ such that

$\underline{a}_i\leq a_i(t,u)\leq\overline{a_i}\quad$ for all $t>0, u\in R$.

(H_3) For $i=1,2,\ldots,n$, $b_i(t,0)\equiv 0$, and there exist positive constants $\underline{b_i}$ and $\overline{b_i}$ such that

$$\underline{b}_i|u-v|\leq \operatorname{sgn}(u-v)(b_i(t,u)-b_i(t,v))$$
$$\leq \overline{b_i}|u-v|\quad \text{for all } t>0,\ u,v\in R.$$

(H_4) There exists a constant $\eta>0$ such that for all $t>0$, there holds

$$-\eta>-[\underline{a}_i\underline{b}_i-\delta_i(t)(\overline{a_i}\overline{b_i})^2]+\overline{a_i}\left[\sum_{j=1}^n(|c_{ij}(t)|+\delta_i(t)\overline{a_i}\overline{b_i}\overline{c_{ij}})\tilde{L}_j\right.$$
$$\left.+\sum_{j=1}^n(|d_{ij}(t)|+\delta_i(t)\overline{a_i}\overline{b_i}\overline{d_{ij}})\int_0^\infty|K_{ij}(u)|\,duL_j\right],\quad i=1,2,\ldots,n.$$

(H_5) There exists a constant $\tilde{\lambda}>0$ such that $I_i(t)=O(e^{-\tilde{\lambda}t})$, $i=1,2,\ldots,n$.

(H_6) For $i,j\in\{1,2,\ldots,n\}$, the delay kernels $K_{ij}:[0,\infty)\to R$ are continuous, and $|K_{ij}(t)|e^{\kappa t}$ are integrable on $[0,+\infty)$ for a certain positive constant κ.

The initial conditions associated with system (1.1) are of the form

$$x_i(s)=\varphi_i(s),\quad s\in(-\infty,0], i=1,2,\ldots,n, \tag{1.4}$$

where $\varphi_i(\cdot)$ denotes real-valued bounded continuous function defined on $(-\infty,0]$, and

$$\sup_{-\infty\leq s\leq 0}\max_{1\leq j\leq n}|\varphi_j(s)|<\frac{\max_{1\leq i\leq n}\{\overline{a_i}\overline{b_i}\delta_i^+\overline{a_i}\overline{I_i}+\overline{a_i}\overline{I_i}\}}{\eta}. \tag{1.5}$$

The remaining part of this paper is organized as follows. In Section 2, we present some new sufficient conditions to ensure that all solutions of CGNNs (1.1) with initial conditions (1.4) and (1.5) converge exponentially to the zero point. In Section 3, we shall give some examples and remarks to illustrate our results obtained in the previous sections.

2. Main results

Lemma 2.1. *Let*

$$\frac{\max_{1\leq i\leq n}\{\overline{a_i}\overline{b_i}\delta_i^+\overline{a_i}\overline{I_i}+\overline{a_i}\overline{I_i}\}}{\eta}:=Q\leq 1. \tag{2.1}$$

Suppose that the conditions (H_1)–(H_4) hold. Then, for every solution $Z(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T$ of CGNNs (1.1) with initial conditions (1.4) and (1.5), there holds

$$|x_i(t)|<Q\quad \text{for all } t>0,\ i=1,2,\ldots,n. \tag{2.2}$$

Proof. Assume, by way of contradiction, that (2.2) does not hold. Then, one of the following two cases must occur.

(1) There exist $i\in\{1,2,\ldots,n\}$ and $T_1>0$ such that
$$x_i(T_1)=Q,\quad |x_j(t)|<Q\ \text{ for all } t<T_1,\ j=1,2,\ldots,n. \tag{2.3}$$

(2) There exist $i\in\{1,2,\ldots,n\}$ and $T_2>0$ such that
$$x_i(T_2)=-Q,\quad |x_j(t)|<Q\ \text{ for all } t<T_2,\ j=1,2,\ldots,n. \tag{2.4}$$

Now, we consider two cases.

Case (i): If (2.3) holds. Then, from (2.1), (2.3) and (H_1)–(H)

$$=-a_i(T_1,x_i(T_1))\left[b_i(T_1,x_i(T_1-\delta_i(T_1)))-\sum_{j=1}^n c_{ij}(T_1)f_j(x_j(T_1-\tau_{ij}(T_1)))\right.$$
$$\left.-\sum_{j=1}^n d_{ij}(T_1)\int_0^\infty K_{ij}(u)g_j(x_j(T_1-u))\,du+I_i(T_1)\right]$$
$$=a_i(T_1,x_i(T_1))\big[-b_i(T_1,x_i(T_1))+(b_i(T_1,x_i(T_1))-b_i(T_1,x_i(T_1-\delta_i(T_1))))$$
$$+\sum_{j=1}^n c_{ij}(T_1)f_j(x_j(T_1-\tau_{ij}(T_1)))$$
$$\left.+\sum_{j=1}^n d_{ij}(T_1)\int_0^\infty K_{ij}(u)g_j(x_j(T_1-u))\,du-I_i(T_1)\right]$$
$$\leq-\underline{a}_i\underline{b}_ix_i(T_1)+\overline{a_i}\overline{b_i}(x_i(T_1)-x_i(T_1-\delta_i(T_1)))$$
$$+\sum_{j=1}^n|c_{ij}(T_1)|\overline{a_i}\tilde{L}_j|x_j(T_1-\tau_{ij}(T_1))|^m$$
$$+\sum_{j=1}^n|d_{ij}(T_1)|\overline{a_i}L_j\int_0^\infty|K_{ij}(u)||x_j(T_1-u)|^{m^*}\,du+\sup_{t\geq 0}|\overline{a_i}I_i(t)|$$
$$=-\underline{a}_i\underline{b}_ix_i(T_1)+\overline{a_i}\overline{b_i}\int_{T_1-\delta_i(T_1)}^{T_1}x'_i(s)\,ds$$
$$+\sum_{j=1}^n|c_{ij}(T_1)|\overline{a_i}\tilde{L}_j|x_j(T_1-\tau_{ij}(T_1))|^m$$
$$+\sum_{j=1}^n|d_{ij}(T_1)|\overline{a_i}L_j\int_0^\infty|K_{ij}(u)||x_j(T_1-u)|^{m^*}\,du+\sup_{t\geq 0}|\overline{a_i}I_i(t)|$$
$$\leq-\underline{a}_i\underline{b}_ix_i(T_1)+\overline{a_i}\overline{b_i}\int_{T_1-\delta_i(T_1)}^{T_1}\Big|a_i(s,x_i(s))\big[b_i(s,x_i(s-\delta_i(s)))$$
$$-\sum_{j=1}^n c_{ij}(s)f_j(x_j(s-\tau_{ij}(s)))-\sum_{j=1}^n d_{ij}(s)\int_0^\infty K_{ij}(u)g_j(x_j(s-u))\,du$$
$$+I_i(s)\big]\Big|\,ds+\sum_{j=1}^n|c_{ij}(T_1)|\overline{a_i}\tilde{L}_j|x_j(T_1-\tau_{ij}(T_1))|^m$$
$$+\sum_{j=1}^n|d_{ij}(T_1)|\overline{a_i}L_j\int_0^\infty|K_{ij}(u)||x_j(T_1-u)|^{m^*}du+\sup_{t\geq 0}|\overline{a_i}I_i(t)|$$
$$\leq-\underline{a}_i\underline{b}_iQ+\overline{a_i}\overline{b_i}\delta_i(T_1)\left[\overline{a_i}\overline{b_i}Q+\overline{a_i}\sum_{j=1}^n\overline{c_{ij}}\tilde{L}_jQ^m\right.$$
$$\left.+\overline{a_i}\sum_{j=1}^n\overline{d_{ij}}L_jQ^{m^*}\int_0^\infty|K_{ij}(u)|\,du+\overline{a_i}\overline{I_i}\right]$$
$$+\sum_{j=1}^n|c_{ij}(T_1)|\overline{a_i}\tilde{L}_jQ^m+\sum_{j=1}^n|d_{ij}(T_1)|\overline{a_i}L_jQ^{m^*}\int_0^\infty|K_{ij}(u)|\,du$$
$$+\sup_{t\geq 0}|\overline{a_i}I_i(t)|$$
$$=-[\underline{a}_i\underline{b}_i-\delta_i(T_1)(\overline{a_i}\overline{b_i})^2]Q+\overline{a_i}\left[\sum_{j=1}^n(|c_{ij}(T_1)|+\delta_i(T_1)\overline{a_i}\overline{b_i}\overline{c_{ij}})\tilde{L}_jQ^m\right.$$
$$\left.+\sum_{j=1}^n(|d_{ij}(T_1)|+\delta_i(T_1)\overline{a_i}\overline{b_i}\overline{d_{ij}})\int_0^\infty|K_{ij}(u)|\,duL_jQ^{m^*}\right]$$
$$+\left(\overline{a_i}\overline{b_i}\delta_i(T_1)\overline{a_i}\overline{I_i}+\sup_{t\geq 0}|\overline{a_i}I_i(t)|\right)$$