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Dynamics of an adaptive higher-order Cohen–Grossberg model

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ABSTRACT

In this paper, we study the dynamical behavior of an adaptive higher-order Cohen–Grossberg model and choose a biologically plausible rule specifying how the connection weights will vary in time, i.e., we incorporate an unsupervised Hebbian-type learning rule with a higher-order Cohen–Grossberg model. By constructing several Lyapunov functions, some sufficient conditions for the asymptotic and exponential stability of the equilibrium are derived. Furthermore, we also study how a temporally varying, in particular, a periodic environment, can influence on the dynamics of this model, i.e., the neuronal parameters, synaptic weights, and gains can either be temporally uniform or be periodic with same period as that of the stimulus. Sufficient condition for the existence of a globally attractive periodic solution associated with a given periodic external stimulus is also derived. Some numerical examples are employed to illustrate our theoretical results.

1. Introduction

Generally, a neural network can be thought of as a network of interconnected neurons in which activity levels at each neuron are inhibited or exited by the activity of the other neurons. This can also be seen as logically equivalent to an interacting set of neurons with the activation of each neuron affected negatively or positively by its competition or cooperation with other neurons. Recently, research on artificial neural networks has become an exciting new branch of artificial intelligence. It is a fascinating field both from a fundamental point of view (nonlinear dynamical systems) as well as for application prospects (non-programmed adaptive information processing). A large variety of models for artificial neural networks(ANNs) have been proposed in the past with strongly increased research activities over the past few years [1–30].

One of best-analyzed mathematical models for neural activity is the Cohen–Grossberg model [3,4,14–16,20,22,23,25]. With constant values of the synaptic connection weights, however, this system does not have capability for modeling adaptation, which is a vital component in any neural network or learning model. Adaptation in neural networks is most often incorporated by allowing the weights of the synaptic connections between neurons to vary in time. Literature dealing with time-varying connection weights or network parameters seems to be scarce; such studies are, however, important to understand the dynamical characteristics of neuronal behavior in time-varying connection weight's environments. Furthermore, in the Cohen–Grossberg model, this may be accomplished by allowing the interaction coefficients to vary in time [19,23,24].

In the previous papers [17,23,24], the authors analyzed a neural network model with nonconstant interaction coefficients, and showed that their asymptotic behavior was distinctly different from that of the corresponding constant-coefficient systems. In [19,24], the authors have extended that analysis of systems of connection weight depending on neuron states, and showed that for certain choices of the parameters, the resulting dynamical systems have an asymptotically stable equilibrium point and can therefore be embedded in a simple type of classification network. Usually, a neural network should be dynamically stable, i.e., it should not exhibit chaotic behavior but go into stationary states for large times. And at the same time, the dynamical behavior of the activation states and connection strengths of an ANN should, at least on a rudimentary level, be consistent with that of the corresponding neurons and synapses in the brain. The thrust of this paper is to choose a biologically plausible rule specifying how the weights will vary and then determine analytically which type of stability results.

The importance of dynamical stability has been particularly emphasized in the adaptive bidirectional associative memory models [5–7]. However, a symmetric connection matrix and a global time scale are employed therein, which does not apply with biological plausibility. The main point of the present work is to preserve stability while replacing the symmetric weights used in stable neural systems by asymmetric connections of a specific kind, as well as to study its consequences.

Recently, limit cycles, strange attractors and other dynamical phenomena have been found and used by many authors to represent encoded temporal patterns as associative memories

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[17,26,27]. Most of the existing literature on theoretical studies of artificial neural network is predominantly concerned with autonomous systems containing temporally uniform network parameters and external input stimuli. In [28,29], the authors have found that assembles of cells in the visual cortex oscillate synchronously in response to external stimuli. Such a synchrony is a manifestation of the encoding process of temporally varying external stimuli. Hence, we will study how a temporally varying, in particular, a periodic environment, can influence the dynamics of a higher-order Cohen–Grossberg model, in addition to the temporal variant of the input, we also incorporate an unsupervised learning algorithm which is similar to Hebbian-type learning rule in the processing part of the neuronal architecture [12,13].

The organization of this paper is as follows. In Section 2, by combining an unsupervised Hebbian-type learning rule with higher-order Cohen–Grossberg model, we obtain an adaptive higher-order Cohen–Grossberg model. Our learning rule which is only similar to an unsupervised Hebbian-type learning rule is conjectural since it is not validated by neurophysiological evidence or experiments of Amari and Gopalsamy [2,18]. In artificial neural networks, it is shown that such higher-oder synaptic connections can improve the storage capacity [30]. convergence rate and solve more general optimization problems [9–11,21]. In Section 3, by using Brouwer's fixed point theorem, we can derive sufficient conditions for the existence of the equilibrium of our proposed model. It will be interesting to obtain sufficient conditions for the existence of an equilibrium which will also guarantee some sort of the stability of the equilibrium also. In Section 4, we derive sufficient conditions for the asymptotic and exponential stability of the equilibrium by constructing several Lyapunov functions. In Section 5, we obtain sufficient conditions for the existence (or encoding) of a globally attractive (heteroassociative recall) periodic solution(or a pattern) associated with a given periodic external stimulus. The neuronal parameters, synaptic connection weights and gains can either be temporally uniform or be periodic with the same period as that of the stimulus. In Section 6, several examples are employed to illustrate the correctness of our obtained results. Some conclusions and further works are also given in Section 7.

2. Model formulations and preliminaries

In this section, by combining an unsupervised Hebbian-type learning rule with Cohen–Grossberg model, we derive the following a class of higher-order Cohen–Grossberg models with learning rule:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum\limits_{j=1}^{n} t_{ij}s_j(x_j(t)) - \sum\limits_{j=1}^{n}\sum\limits_{k=1}^{n} T_{ijk}s_j(x_j(t))s_k(x_k(t)) \right. \\ \qquad \left. +B_i \sum\limits_{j=1}^{n} w_{ij}(t)p_j + I_i \right], \\ \dfrac{dw_{ij}(t)}{dt} = -\alpha_i w_{ij}(t) + \beta_i s_i(x_i(t))p_j, \quad i = 1,2,\cdots,n, \end{cases} \tag{1}$$

where $t>0$; $x_i(t),\ i=1,2,\cdots,n$, denote the state variable of the ith neuron, $s_j(\cdot),\ j=1,2,\cdots,n$, denote the activation functions of the jth neuron at time t; $a_i(\cdot),\ i=1,2,\cdots,n$, represent amplification functions; $b_j(\cdot),\ i=1,2,\cdots,n$,are appropriately behavioral functions; $t_{ij}, T_{ijk}, i,j,k=1,2,\cdots,n$, are the first-order and second-order connection weights of neural network at time t, respectively; $w_{ij}(t),\ i,j=1,2,\ldots,n$, denote the synaptic vector of the neuron i undergoing the temporal process of learning the signal vector $P=(p_1,p_{2,}\cdots,p_n)^T$; also $w_{ij}(t)$ models the efficiency with which the ith neuron's jth synapse can transform presynaptic potential to post-synaptic potential; $B_i,\ i=1,2,\cdots,n$,denote the uptake of the input signal by the ith neuron; α_i and β_i denote disposable scaling constants with $\alpha_i>0$ and the vector $I=(I_1,I_{2,}\cdots,I_n)^T$ denotes the external input signal vector simulating the networks.

Throughout this section, we assume that

(H$_1$). The amplification functions $a_i(\cdot)\ i=1,2,\cdots,n$ are continuous and there exist constants $\underline{a}_i,\overline{a}_i$ such that $0<\underline{a}_i \le a_i(x_i) \le \overline{a}_i$, for all $x_i \in R, i=1,2,\cdots,n$;

(H$_2$). The behavioral functions $b_i(\cdot)$ and it's inverse $b_i^{-1}(\cdot)$, $i=1,2,\cdots,n$, are locally Lipschitz continuous and there exist $\gamma_i>0, i=1,2,\cdots,n$, such that $\frac{b_i(x)-b_i(y)}{x-y} \ge \gamma_i > 0$, for all $x \ne y$, $\quad i=1,2,\cdots,n$;

(H$_3$). For activation functions $s_j(\cdot)$, there exist positive constants L_j and M_j such that

$$L_j = \sup_{x\ne y}\left|\frac{s_j(y)-s_j(x)}{y-x}\right| \text{ and } \quad |s_j(\cdot)| \le M_j.$$

The initial conditions of system (1) are given by

$$x_i(0)=\varphi_i(0), \quad w_{ij}(0)=\Phi_{ij}(0), \quad i,j=1,2,\cdots,n.$$

We note that the second equation of system (1) corresponds to an unsupervised Hebbian-type learning algorithm with a feedback term or forgetting terms which is introduced in order to control the synaptic vector from becoming excessively large; such forgetting term was not present in the original version of the Hebbian learning algorithms [2]. There is a wide variety of learning algorithms used in the literature of neural network. The learning dynamics incorporated in system (1) are based on a Hebbian postulate along with a forgetting factor. For more details of several learning algorithms, the readers may refer to [1,2,5,7].

For convenience, we introduce another auxiliary state variable $y_i(t), i=1,2,\cdots,n$, which are defined by

$$y_i(t) = \sum_{j=1}^{n} w_{ij}(t)p_j, \quad i=1,2,\cdots,n, \quad t>0, \tag{2}$$

so that the second equation of system (1) becomes

$$\frac{dy_i(t)}{dt} = -\alpha_i y_i(t) + \beta_i s_i(x_i)\sum_{j=1}^{n} p_j^2.$$

Let $\sum_{j=1}^{n} p_j^2 = c$ for some nonnegative c and simplify system (1) to the form

$$\begin{cases} \frac{dx_i(t)}{dt} = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum\limits_{j=1}^{n} t_{ij}s_j(x_j(t)) - \sum\limits_{j=1}\sum\limits_{k=1}^{n} T_{ijk}s_j(x_j(t))s_k(x_k(t)) \right. \\ \qquad \left. + B_i y_i(t) + I_i\right] \\ \frac{dy_i(t)}{dt} = -\alpha_i y_i(t) + c\beta_i s_i(x_i) \end{cases} \tag{3}$$

We note that that if we set $c=\sum_{j=1}^{n} p_j^2 = 0$ in the above equation, then system (1) will become uncoupled and will contain the higher-order type Cohen–Grossberg networks. The dynamics of such networks without learning components have been considered recently by some authors [3,4–16,20,22,23,25].

Thus, system (3) and its analysis should contain some of the results known for higher-order networks with no learning components in it.

3. Existence of the equilibrium

In this section, by using the contraction mapping principle, we can derive sufficient conditions for the existence of the