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Stability analysis of mixed recurrent neural networks with time delay in the leakage term under impulsive perturbations ☆

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ABSTRACT

This paper investigates a class of mixed recurrent neural networks with time delay in the leakage term under impulsive perturbations. The mixed time-delays consist of both discrete and distributed delays. By using the Lyapunov functional method, linear matrix inequality approach and general convex combination technique, two novel sufficient conditions are derived to ensure the global asymptotic stability of the equilibrium point of the networks. The proposed results, which do not require the boundedness, differentiability and monotonicity of the activation functions, can be easily checked via Matlab software. Moreover, they indicate that the stability behavior of neural networks is very sensitive to the time delay in the leakage term. Finally, numerical examples are given to demonstrate the effectiveness of our theoretical results.

1. Introduction

During the past decades, great attention has been paid in the literature to delayed neural networks, because of their applications in many areas such as signal processing, associative memory, pattern recognition, parallel computation and optimization. It is well known that in implementation of neural networks, time delays are unavoidably encountered due to the finite switching speed of neurons and amplifiers. Furthermore, it has been pointed out that time delays may lead to oscillation and instability of a neural network. Therefore, it is of prime importance to consider the delay effects on the dynamical behavior of systems. Recently, neural networks with various types of delay have been widely investigated by many authors, see [1–8] and references therein. However, so far, there has been very little existing work on neural networks with time delay in the leakage (or "forgetting") term [9–15]. This is due to some theoretical and technical difficulties [9]. In fact, time delay in the leakage term also has great impact on the dynamics of neural networks. As pointed out by Gopalsamy [10], time delay in the stabilizing negative feedback term has a tendency to destabilize a system. This observation can be illustrated by Example 1.1 in [12].

On the other hand, besides delays effects, impulsive effects are also likely to exist in the neural network system. Generally speaking, the states of real neural networks are often subject to instantaneous perturbations and experience abrupt change at certain moments of time, which can be caused by switching phenomenon, frequency change, or other sudden noise, i.e., impulsive effects. Moreover, neural networks are subject to impulsive perturbations which in turn affect the dynamical behaviors of the system. Therefore, impulsive perturbations should be taken into account when studying the stability of neural networks. It is inspiring that in recent years considerable attention has been paid to investigating the stability analysis of impulsive neural networks. The reason is twofold: one is that impulsive perturbations occur in many important fields such as medicine, biology, economics, mechanics, electronics, and telecommunications, the other is that an impulsive neural network model belongs to a new category of dynamical systems, which is neither purely continuous-time nor purely discrete-time one. Since this kind of model displays a combination of the characteristics of both continuous-time and discrete-time systems, it is difficult and challenging to discuss the stability analysis of an impulsive neural network [16–20].

Motivated by aforementioned discussion, this paper considers a class of mixed recurrent neural networks with time delay in the leakage term under impulsive perturbations in which the encountered instantaneous perturbations depend on not only the current state of neurons at impulse times but also the state of neurons in recent history. By using the Lyapunov functional method, linear

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matrix inequality approach and general convex combination technique, two novel sufficient conditions are derived to ensure the global asymptotic stability of the equilibrium point of the networks. The proposed results, which do not require the boundedness, differentiability and monotonicity of the activation functions, can be easily checked via Matlab software. Moreover, they indicate that the stability behavior of neural networks is very sensitive to the time delay in the leakage term. In the absence of leakage delay, the obtained results are also new ones. Finally, numerical examples are given to demonstrate the effectiveness and less conservativeness of our theoretical results.

Notations: Throughout this paper, let $\mathbb{Z}_+$ denote the set of positive integers, $\|y\|$ denote the Euclidean norm of a vector $y \in \mathbb{R}^n$, $W^T, W^{-1}, \lambda_M(W), \lambda_m(W)$ and $\|W\| = \sqrt{\lambda_M(W^TW)}$ denote the transpose, the inverse, the largest eigenvalue, the smallest eigenvalue, and the spectral norm of a square matrix W respectively. Let $W>0(\le 0)$ denote a positive (semi-negative) definite symmetric matrix, I denote the identity matrix with appropriate dimension. $\mathbb{N}=\{1,2,\dots,n\}$ and $*$ represents the element below the main diagonal of a symmetric block matrix. The shorthand $\text{col}\{M_1, M_2, \dots, M_k\}$ denotes a column matrix with the matrices $M_1, M_2, \dots, M_k$.

2. Problem description and preliminaries

Consider the following mixed recurrent neural network model with leakage delay and impulsive perturbations

$$\begin{cases} \dot{x}(t) = -Cx(t-h) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} f(x(s))\,ds + U, & t>0, t\ne t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = J_k(x(t_k^-)), & k\in\mathbb{Z}_+, \\ x(s) = \varphi(s), & s\in[-\rho, 0], \end{cases} \tag{2.1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ is the neural state vectors, C is a positive diagonal matrix, A, B, D are the connection weight matrix, the discretely delayed connection weight matrix, and the distributively delayed connection weight matrix, respectively; $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T$ denotes the activation function, $h>0, 0\le\tau(t)\le\overline{\tau}, 0\le\sigma(t)\le\overline{\sigma}$ are bounded and unknown delays, $\overline{\tau}, \overline{\sigma}$ are positive scalars, $U = [u_1, u_2, \dots, u_n]^T$ is the constant external input vector, function $\varphi_i(t) (i\in\mathbb{N})$ is continuous on $[-\rho, 0], \rho = \max\{h, \overline{\tau}, \overline{\sigma}\}$, the norm is defined by

$$\|\varphi\|_\rho = \max\left\{ \sup_{-\rho\le s\le 0} \|\varphi(s)\|, \sup_{-h\le s\le 0}\|\dot{\varphi}(s)\|, \sup_{-\overline{\tau}\le s\le 0}\|\dot{\varphi}(s)\| \right\}.$$

Firstly, we make the following assumptions:

(H1) The delay satisfies that $\dot{\tau}(t)\le\eta<1$.

(H2) There exist constants l_j^-, l_j^+ such that $l_j^- < l_j^+$ and

$$l_j^- \le \frac{f_j(u)-f_j(v)}{u-v} \le l_j^+ \quad \forall u,\ v\in\mathbb{R},\ u\ne v,\ j\in\mathbb{N}.$$

For notational simplicity, we denote

$$\overline{\Sigma} = \text{diag}\{l_1^+, \dots l_n^+\}, \Sigma = \text{diag}\{l_1^-, \dots l_n^-\},$$

$$\Sigma_1 = \text{diag}\{l_1^- l_1^+, \dots, l_n^- l_n^+\}, \Sigma_2 = \tfrac{1}{2}\text{diag}\{l_1^- + l_1^+, \dots, l_n^- + l_n^+\}.$$

Remark 1. As pointed out by Liu et al. [21], the constants l_j^-, l_j^+ are allowed to be positive, negative, or zero. Hence, the resulting activation functions may be non-monotonic, and more general than the usual sigmoid functions in [3,4,6,7].

(H3) Every function $J_k(x):\mathbb{R}^n\to\mathbb{R}^n$ is continuous for any $x\in\mathbb{R}^n$, $k\in\mathbb{Z}_+$.

(H4) The impulsive time instant t_k satisfy $0=t_0<t_1<\dots<t_k\to\infty$ and $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\}>0$.

In order to obtain the results, we need the following lemmas.

Lemma 1 (*see Zhang and Quan [22] and Poznyak and Sanchez [23]*). *Let X, Y and P be real matrices of appropriate dimensions with $P>0$. Then for any positive scalar ε the following matrix inequality holds*:

$$X^TY + Y^TX \le \varepsilon^{-1}X^TP^{-1}X + \varepsilon Y^TPY.$$

Lemma 2 (*Jensen integral inequality, see Gu [24]*). *For any constant matrix $M>0$, any scalars a and b with $a<b$, and a vector function $\chi(t):[a,b]\to\mathbb{R}$ such that the integrals concerned are well defined, then the following inequality holds*:

$$\left(\int_a^b \chi(s)\,ds\right)^T M \left(\int_a^b \chi(s)\,ds\right) \le (b-a)\int_a^b \chi(s)^T M\chi(s)\,ds.$$

Lemma 3 (Yue et al. [25]). *Suppose that $\mho, \mho_{ij}\ge 0 (i,j=1,2)$ are symmetric matrices of appropriate dimensions, $\alpha\in[0,1], \beta\in[0,1]$, then $\mho + [(1-\alpha)\mho_{11} + \alpha\mho_{12}] + [(1-\beta)\mho_{21} + \beta\mho_{22}] < 0$ holds if the following four inequalities $\mho + \mho_{11} + \mho_{21} < 0$, $\mho + \mho_{11} + \mho_{22} < 0$, $\mho + \mho_{12} + \mho_{21} < 0$ and $\mho + \mho_{12} + \mho_{22} < 0$ hold simultaneously.*

Lemma 4. *Assume that $\nu, \mu, \kappa, \zeta, \underline{\vartheta}, \overline{\vartheta}, \underline{\theta}, \overline{\theta}$ are real scalars such that $\nu\le 1, \kappa\le 1, \nu+\mu\le 4, \kappa+\zeta\le 4$, and $\underline{\vartheta}<\overline{\vartheta}, \underline{\theta}<\overline{\theta}$. Let $\vartheta:\mathbb{R}\to(\underline{\vartheta},\overline{\vartheta}), \theta:\mathbb{R}\to(\underline{\theta},\overline{\theta})$ be real functions. Then for any non-negative scalars $\alpha, \beta, \iota, \epsilon$ the following inequality holds*

$$-\frac{\alpha}{\vartheta(t)-\underline{\vartheta}} - \frac{\beta}{\overline{\vartheta}-\vartheta(t)} - \frac{\iota}{\theta(t)-\underline{\theta}} - \frac{\epsilon}{\overline{\theta}-\theta(t)}$$
$$\le \max\left\{ -\frac{\nu\alpha+\mu\beta}{\overline{\vartheta}-\underline{\vartheta}} - \frac{\kappa\iota+\zeta\epsilon}{\overline{\theta}-\underline{\theta}}, -\frac{\nu\alpha+\mu\beta}{\overline{\vartheta}-\underline{\vartheta}} - \frac{\zeta\iota+\kappa\epsilon}{\overline{\theta}-\underline{\theta}}, -\frac{\mu\alpha+\nu\beta}{\overline{\vartheta}-\underline{\vartheta}} - \frac{\kappa\iota+\zeta\epsilon}{\overline{\theta}-\underline{\theta}}, -\frac{\mu\alpha+\nu\beta}{\overline{\vartheta}-\underline{\vartheta}} - \frac{\zeta\iota+\kappa\epsilon}{\overline{\theta}-\underline{\theta}} \right\}.$$

Proof. It is easy to see that we need only verify the following inequality:

$$-\frac{\alpha}{\vartheta(t)-\underline{\vartheta}} - \frac{\beta}{\overline{\vartheta}-\vartheta(t)} \le \frac{1}{\overline{\vartheta}-\underline{\vartheta}}\max\{-\nu\alpha-\mu\beta, -\mu\alpha-\nu\beta\}. \tag{2.2}$$

Without loss of generality, we assume that $\nu\le\mu$. First consider the case that $\alpha\le\beta$. It is easy to see that $\max\{-\nu\alpha-\mu\beta, -\mu\alpha-\nu\beta\} = -\mu\alpha-\nu\beta$. Therefore, we have

$$\begin{aligned} &(\vartheta(t)-\underline{\vartheta})(\overline{\vartheta}-\vartheta(t))(-\mu\alpha-\nu\beta) + (\overline{\vartheta}-\underline{\vartheta})[(\overline{\vartheta}-\vartheta(t))\alpha + (\vartheta(t)-\underline{\vartheta})\beta] \\ &\quad = (\overline{\vartheta}-\vartheta(t))[\overline{\vartheta} + (\mu-1)\underline{\vartheta} - \mu\vartheta(t)]\alpha + (\vartheta(t)-\underline{\vartheta})[(1-\nu)(\overline{\vartheta}-\vartheta(t)) \\ &\qquad + (\vartheta(t)-\underline{\vartheta})]\beta \ge \{(\overline{\vartheta}-\vartheta(t))[\overline{\vartheta} + (\mu-1)\underline{\vartheta} - \mu\vartheta(t)] \\ &\qquad + (\vartheta(t)-\underline{\vartheta})[(1-\nu)(\overline{\vartheta}-\vartheta(t)) + (\vartheta(t)-\underline{\vartheta})]\}\alpha \\ &\quad = \frac{\alpha}{4}[(\nu+\mu)(2\vartheta(t)-\underline{\vartheta}-\overline{\vartheta})^2 + (4-\nu-\mu)(\overline{\vartheta}-\underline{\vartheta})^2] \\ &\quad \ge 0. \end{aligned}$$

That is

$$\frac{1}{\overline{\vartheta}-\underline{\vartheta}}\max\{-\nu\alpha-\mu\beta, -\mu\alpha-\nu\beta\} = \frac{1}{\overline{\vartheta}-\underline{\vartheta}}(-\mu\alpha-\nu\beta) \ge -\frac{\alpha}{\vartheta(t)-\underline{\vartheta}} - \frac{\beta}{\overline{\vartheta}-\vartheta(t)}.$$

Similarly, we can also conclude that inequality (2.2) holds for $\alpha>\beta$. Now, proof of Lemma 4 is completed. □

3. Main result

As usual, a vector $x^* = [x_1^*, x_2^*, \dots, x_n^*]^T$ is said to be an equilibrium point of system (2.1) if it satisfies $Cx^* = [A + B + \sigma(t)D]f(x^*) + U$. In this paper, we assume that some conditions are satisfied such that