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Synchronization of linearly coupled neural networks with reaction–diffusion terms and unbounded time delays ☆

Xiwei Liu *

Key Laboratory of Nonlinear Science of Chinese Ministry of Education, School of Mathematics, Fudan University, Shanghai 200433, PR China

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ABSTRACT

In this paper, the author investigates the global synchronization problem for linearly coupled neural networks with reaction–diffusion terms and unbounded time delays. The main difference of this paper from previous works in literature is that the time delay can be unbounded and non-differential. Moreover, the pinning control problem of such neural networks is also investigated. Some sufficient criteria for synchronization are given by means of the linear matrix inequality (LMI). Finally, numerical simulations are also given to show the validity of the obtained criteria.

1. Introduction

In the past few decades, various neural networks, such as Hopfield neural networks, cellular neural networks, bidirectional associative neural networks, and Cohen–Grossberg neural networks, etc., have been widely investigated and successfully applied in many areas, such as combinatorial optimization, signal or image processing, pattern recognition and associative memory design, see [1,2] and references therein.

Generally, the neural network model is described by ordinary differential equations; but in the real world, diffusion effect cannot be avoided in the neural network model when electrons are moving in asymmetric electromagnetic field, so we must consider the space varying with the time, and in this case the model should be expressed by partial differential equations. For example, employing the properties of diffusion operator, Liang and Cao [3] investigated the existence, uniqueness and global exponential stability of the equilibrium point of delayed reaction–diffusion recurrent neural networks by applying general Halanay inequality; the global asymptotic stability of bi-directional associative memory neural networks with distributed delays and reaction–diffusion terms were studied by [4]; Qui [5] and Li and Song [6] investigated the dynamical behaviors of impulsive neural networks; Yang and Xu [7] had estimated the existence range of the attracting sets and the periodic attractors for non-autonomous reaction–diffusion neural networks with time-varying delays; Lu [8,9] and Wang and Lu [10] gave a better result on stability analysis by analyzing the role of reaction–diffusion terms; and it was also common to consider the diffusion effect in biological systems, such as immigration, see [11].

Moreover, in neural processing and signal transmission, axonal signal transmission delays often occur; moreover, in electronic implementation of analog neural networks, time delay is usually time varying due to the finite switching speed of amplifiers. It is known that time delays may cause undesirable dynamical network behaviors such as oscillation and instability. Therefore, it is of great importance to study the global stability of neural networks with delays. And in this case, the neural network depends on not only the time but also the time delay, thus the model is a functional differential equation [12]. Until now, a large amount of results have been reported in the literature, and the delay type can be constant [13], time varying [14], distributed [15], unbounded [16–18], etc.

Therefore, the neural networks with reaction–diffusion effect and time delays are more applicable in the real world, and many such neural networks can result in a complex network by mutually coupling. Recently, investigation of dynamical behaviors in complex networks, especially the synchronization problem [19], has attracted numerous scientists from diverse fields including physics, biology, neuroscience, mathematics, chemistry and ecology, etc. For example, in [20,21], the left eigenvector corresponding to the zero eigenvalue of the diffusive coupling

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* Tel.: +86 21 65109710.
E-mail address: xwliu.sh@gmail.com

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matrix was utilized to investigate the global synchronization for linearly coupled networks; in [22,23], the synchronization of nonlinearly coupled networks was investigated; in [24], the adaptive synchronization algorithm was proposed and proved strictly; and in [25], the pinning control problem for complex networks was studied by adding a single controller. More concretely, as for linearly coupled neural networks with reaction–diffusion terms and time delays, asymptotic and exponential synchronization problems were investigated by [26–28], respectively.

To the best of our knowledge, few authors have considered global synchronization problem of reaction–diffusion neural networks with the Dirichlet boundary conditions and unbounded and non-differential time varying delays, which is challenging and important both in theories and applications. Motivated by above discussions, in this paper, we will investigate the global synchronization and pinning control problems for a class of linearly coupled reaction–diffusion neural networks with the Dirichlet boundary conditions and unbounded and non-differential time delays.

The paper is organized as follows. In Section 2, a model of linearly coupled neural networks with reaction–diffusion terms and unbounded time-varying delays is proposed at first, and a new synchronization definition, called the μ- synchronization, is also defined. In Section 3, some necessary definitions, lemmas and hypotheses are given. In Section 4, some criteria for the global synchronization and pinning control of such linearly coupled neural networks are derived. In Section 5, numerical examples are presented to show the validity of the theoretical results. We conclude this paper in Section 6.

Notations: Throughout this paper, we denote the vector $(1,\dots,1)^T$ by $\mathbf{1}$. The identity matrix is denoted by I, the transpose (or inverse) of any square matrix A is expressed as A^T (or A^{-1}). $A>0$ ($A<0, A\geq 0, A\leq 0$) is used to denote a positive- (negative-, semi-positive-, semi-negative-) definite matrix A. The dimension of these vectors and matrices will be clear in the context. If all eigenvalues of a matrix $A\in R^{N\times N}$ are real, then we sort them as $\lambda_1(A)\geq\lambda_2(A)\geq\cdots\geq\lambda_N(A)$. mes Ω denotes the measure of the region Ω. The Kronecker product of a n by m matrix $A=(a_{ij})$ and a p by q matrix B is the np by mq matrix $A\otimes B$, defined as

$$A\otimes B=\begin{pmatrix} a_{11}B & \cdots & a_{1m}B \\ \vdots & \ddots & \\ a_{n1}B & & a_{nm}B \end{pmatrix}$$

2. Model description

A single reaction–diffusion neural network with time-delays and Dirichlet boundary conditions can be described by the following differential equations:

$$\begin{cases} \dfrac{\partial u_j(t,x)}{\partial t}=\displaystyle\sum_{r=1}^{m}\frac{\partial}{\partial x_r}\left(D_{jr}\frac{\partial u_j(t,x)}{\partial x_r}\right)-w_j^0u_j(t,x)+\sum_{k=1}^{n}w_{jk}^1g_k(u_k(t,x)) \\ \qquad+\displaystyle\sum_{k=1}^{n}w_{jk}^2h_k(u_k(t-\tau(t),x))+I_j, \quad x\in\Omega \\ \dfrac{\partial u_j(t,x)}{\partial \underline{n}}:=\left(\dfrac{\partial u_j(t,x)}{\partial x_1},\dfrac{\partial u_j(t,x)}{\partial x_2},\dots,\dfrac{\partial u_j(t,x)}{\partial x_m}\right)^T=0, \quad x\in\partial\Omega \\ u_j(s,x)=\phi_j(s,x), \quad (s,x)\in(-\infty,0]\times\Omega \end{cases} \tag{1}$$

where $j=1,\dots,n$; $\Omega=\{x=(x_1,x_2,\dots,x_m)^T\}$ is a compact set with smooth boundary and mes $\Omega>0$ in space R^m; $u_j(t,x)$ is the state of jth neuron at time t and in space x; g_k,h_k denote the activation functions of the jth neuron in space x; $w_j^0>0$ represents the rate with which the jth neuron will reset its potential to the resting state; $w_{jk}^{1,2}$ are the connection weights; $D_{jr}\geq 0$ means the transmission diffusion coefficient along the jth neuron; I_j denotes the external bias on the jth neuron; $\tau(t)$ is the unbounded and non-differential time delays; $\phi_j(s,x)$ are bounded and continuous initial functions.

The discussion of existence, uniqueness and global convergence of the equilibrium of neural network (1) can be found in [3] and references therein. Without loss of generality, we let $I_j=0$, $j=1,\dots,n$. Denote

$$W^0=\mathrm{diag}(w_1^0,\dots,w_n^0)\in R^{n\times n}, \quad W^1=(w_{jk}^1)\in R^{n\times n}, \quad W^2=(w_{jk}^2)\in R^{n\times n}$$

$$u(t,x)=(u_1(t,x),\dots,u_n(t,x))^T\in R^n, \quad g(u)=(g_1(u_1),\dots,g_n(u_n))^T\in R^n$$

$$h(u(t-\tau(t),x))=(h_1(u_1(t-\tau(t),x)),\dots,h_n(u_n(t-\tau(t),x)))^T\in R^n$$

$$f^{rd}(u(t,x))=\left(\sum_{r=1}^{m}\frac{\partial}{\partial x_r}\left(D_{1r}\frac{\partial u_1(t,x)}{\partial x_r}\right),\dots,\sum_{r=1}^{m}\frac{\partial}{\partial x_r}\left(D_{nr}\frac{\partial u_n(t,x)}{\partial x_r}\right)\right)\in R^n$$

$$f(u(t,x);u(t-\tau(t),x))=-W^0u(t,x)+W^1g(u(t,x))+W_2h(u(t-\tau(t),x))$$

Thus, the above model (1) can be represented in the compact form as

$$\begin{aligned}\frac{\partial u(t,x)}{\partial t}&=f^{rd}(u(t,x))-W^0u(t,x)+W^1g(u(t,x))+W_2h(u(t-\tau(t),x)) \\ &=f^{rd}(u(t,x))+f(u(t,x);u(t-\tau(t),x))\end{aligned} \tag{2}$$

where $f^{rd}(u(t,x))$ is the reaction–diffusion terms, and $f(u(t,x);u(t-\tau(t),x))$ is the canonical Hopfield neural network term.

Assumption 1. The neuron activation functions $g(\cdot),h(\cdot)$ satisfy the following Lipschitz condition:

$$|g(u(t,x))-g(u'(t,x))|\leq|G(u(t,x)-u'(t,x))| \quad \text{for any } u,u'\in R^n$$

$$|h(u(t,x))-h(u'(t,x))|\leq|H(u(t,x)-u'(t,x))| \quad \text{for any } u,u'\in R^n$$

where $G,H\in R^{n\times n}$ are known constant matrices.

Therefore, N such neural networks (2) can be linearly coupled into a complex network, which is described as

$$\frac{\partial u^i(t,x)}{\partial t}=f^{rd}(u^i(t,x))+f(u^i(t,x);u^i(t-\tau(t),x))+\alpha\sum_{l\neq i}c_{il}(u^l(t,x)-u^i(t,x)) \tag{3}$$

where the initial functions and boundary values are both defined in (1), $\alpha>0$ denotes the coupling strength, $c_{il}\geq 0$ is the coupling strength from u^l to u^i.

Definition 1. This network (3) is said to be globally μ-synchronized if there exists a constant $M>0$, $T>0$, and a differential function $\mu(t)\to+\infty$ as $t\to+\infty$, such that for any initial values $\phi_j(s,x)$, $j=1,\dots,n$, and $t>T$,

$$\|u^i(t,x)-u^j(t,x)\|\leq M\frac{1}{\mu(t)}, \quad i,j=1,\dots,N \tag{4}$$

where the norm $\|\cdot\|$ is defined as: for $u(t,x)=(u_1(t,x),\dots,u_n(t,x))^T\in R^n$,

$$\|u(t,x)\|=\left[\int_\Omega u(t,x)^Tu(t,x)\,dx\right]^{1/2}$$

On the other hand, for the complex network model (3), if it can be pinned to a specified trajectory $\overline{u}(t,x)$ by adding a single controller, then the pinning control problem can be described by