Neurocomputing 73 (2010) 2754–2764

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Impulsive effect on the delayed Cohen–Grossberg-type BAM neural networks ☆

Yonghui Xia

Department of Mathematics, Zhejiang Normal University, Jinhua, Zhejiang 321004, China

ARTICLE INFO

Article history:
Received 3 November 2009
Received in revised form
28 March 2010
Accepted 16 April 2010
Communicated by J. Liang
Available online 4 May 2010

Keywords:
Cohen–Grossberg network
Equilibrium
Distributed delay
Exponential stability

ABSTRACT

In the present paper, an impulsive Cohen–Grossberg-type bi-directional associative memory (BAM) neural network with distributed delays is studied. A set of new sufficient conditions are established for the existence and global exponential stability of a unique equilibrium without strict conditions imposed on self-regulation functions. Applying the results to some special cases, the obtained results generalize some previously known results. A variety of methods are employed to investigate the issue. The approaches are based on Banach fixed point theory, Brower fixed point theory, Laypunov–Kravsovskii functional, homeomorphism theory and the matrix spectral theory. It is believed that these results are helpful for the design and applications of the impulsive Cohen–Grossberg BAM type artificial neural networks.

1. Introduction

In 1983, Cohen–Grossberg [1] proposed an artificial feedback neural network which is called Cohen–Grossberg neural network (CGNN). It can be described as follows:

$$\dot{u}_i(t) = -a_i(u_i(t))\left[b_i(u_i(t)) - \sum_{j=1}^{n} a_{ij} g_j(u_j(t)) + I_i\right], \quad i = 1,2,\ldots,n.$$

These Cohen–Grossberg neural networks were designed to include Hopfield-type neural networks, shunting neural networks and some ecological systems. Cohen–Grossberg networks have their promising potential for the tasks of classification, associative memory, parallel computation and have great ability to solve difficult optimization problems. Thus they have been received great attentions and they have been extensively studied in the literature (see e.g. [1,3,6–18,21–28,37–44,58] and the references cited therein).

On the other hand, based on the framework of Hopfield neural networks, Kosko [2] has generalized the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched hetero-associative circuit, and proposed a class of the bi-directional associative memory model (BAM) which has been extensively studied with or without impulses (e.g. see [4,5,19,20,32–37,59–61]). On basis of the bi-directional associative memory neural networks and Cohen–Grossberg neural networks model, in [29], the authors proposed the following Cohen–Grossberg-type BAM neural network model:

$$\begin{cases} \dot{x}_i(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{m} h_{ij} f_j(\lambda_j y_j(t-\tau_{ij})) - r_i\right], & i = 1,2,\ldots, \quad n, \\ \dot{y}_j(t) = -c_j(y_j(t))\left[d_j(y_j(t)) - \sum_{i=1}^{n} w_{ji} g_i(\mu_i x_i(t-\sigma_{ji})) - s_j\right], & j = 1,2,\ldots, \quad m. \end{cases}$$

In this paper, the asymptotic stability was investigated for Cohen–Grossberg-type BAM neural networks. As pointed out in [11], it is preferable and desirable that the neural network not only converges to an equilibrium point but also has a convergence rate which is as fast as possible. It is noted that the exponential stability gives a fast convergence rate to the equilibrium point. So it is important to determine the exponential stability and to estimate the exponential convergence rate. For this reason, the authors studied the exponential stability of Cohen–Grossberg-type BAM neural networks in [11,12], respectively. Then, a few works on the stability of continuous Cohen–Grossberg-type BAM neural networks have been reported in [51–55].

In reality, however, many physical systems undergo abrupt changes at certain moments due to instantaneous perturbations, which lead to impulsive effects. Since the existence of delays and impulses is frequently a source of instability, bifurcation and chaos for dynamical systems, it is important to study the delay and impulsive effects on the stability of dynamical systems. Though many known results are done for the BAM neural networks and Cohen–Grossberg neural networks (e.g. see [6,7,14,15,32–34,36,41,43,51,52]), there are few works considering

☆ This work was supported by the National Natural Science Foundation of China under Grant (No.10901140), Natural Science Foundation of Zhejiang Province under Grant (no. Y6100029) and Zhejiang Innovation Project under Grant (No. T200905).

E-mail addresses: yhxia@zjnu.cn, xiadoc@163.com.

0925-2312/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.neucom.2010.04.011

the impulsive effect on the Cohen–Grossberg-type BAM neural networks. Recently, the authors [30,31,56,57] attempted to study the stability of the impulsive Cohen–Grossberg-type BAM neural networks. But the conditions are quite strict and they can be reduced to be less conservative. Motivated by the aforementioned discussion, in the present paper, we consider the stability characteristics of impulsive Cohen–Grossberg-type BAM neural networks with distributed delays which modelled as follows:

$$\begin{cases} \dot{x}_i(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{m} h_{ij}^0 f_j(\lambda_j y_j(t-\tau_{ij})) - \sum_{j=1}^{m} h_{ij}^1 \int_0^\infty H_{ij}(s) f_j(\lambda_j y_j(t-s))\,ds - r_i\right], \quad t \neq t_k,\ t \geq 0, \\ \Delta x_i(t_k) = I_i(x_i(t_k)), \quad i = 1,2,\ldots,n,\ k = 1,2,\ldots, \\ \dot{y}_j(t) = -c_j(y_j(t))\left[d_j(y_j(t)) - \sum_{i=1}^{n} w_{ji}^0 g_i(\mu_i x_i(t-\sigma_{ji})) - \sum_{j=1}^{m} w_{ji}^1 \int_0^\infty K_{ji}(s) g_i(\mu_i x_i(t-s))\,ds - s_j\right], \quad t \neq t_k,\ t \geq 0, \\ \Delta y_j(t_k) = J_j(y_j(t_k)), \quad j = 1,2,\ldots,m,\ k = 1,2,\ldots, \end{cases} \tag{1}$$

where $x_i(t)$ and $y_j(t)$ are the state of the i-th neuron from the neural field F_U and the j-th neuron from the neural field F_V at time t , respectively; f_j, g_i denote the activation functions of the j-th neuron from F_V and the i-th neuron from F_U, respectively; r_i and s_j are constants, which denote the external inputs on the i-th neuron from F_U and the j-th neuron from F_V, respectively; τ_{ij} and σ_{ji} correspond to the transmission delays; (τ_{ij} and σ_{ji} are positive constants); $a_i(x_i(t))$ and $c_j(y_j(t))$ represent amplification functions; $b_i(x_i(t))$ and $d_j(y_j(t))$ are appropriately behaved functions such that the solutions of model (1) remain bounded; $h_{ij}^0, h_{ij}^1, w_{ji}^0$ and w_{ji}^1 denote the connection strengths and λ_j and μ_i are positive constants, which correspond to the neuronal gains associated with the neuronal activations. Here $\Delta x_i(t_k) = x_i(t_k+0) - x_i(t_k-0)$ and $\Delta y_j(t_k) = y_j(t_k+0) - y_j(t_k-0)$ are the impulsive jumps at moment t_k and $0 < t_1 < t_2 < \cdots$ is an increasing sequence. As usual in the theory of impulsive differential equations, by a solution of (1) we mean $z(t) = (x_1(t),\ldots,x_n(t),y_1(t),\ldots,y_m(t))^T \in R^{n+m}$ in which $x_i(\cdot), y_j(\cdot)$ is piecewise continuous on $(0,\beta)$ for some $\beta > 0$ such that $z(t_k^+)$ and $z(t_k^-)$ exist and $z(\cdot)$ is differentiable on intervals of the form $(t_{k-1},t_k) \subset (0,\beta)$ and satisfies (1); we assume that $z(t)$ is left continuous with $z(t_k-0) = z(t_k)$ the functions $I_k, J_k : (\cdot) : R \to R$ are assumed to be continuous. And system (1) is supplemented with initial values given by

$$x_i(s) = \varphi_{x_i}(s), \quad s \in (-\infty,0],\ i = 1,2,\ldots n,$$

$$y_j(s) = \varphi_{y_j}(s), \quad s \in (-\infty,0],\ j = 1,2,\ldots m,$$

where $\varphi_{x_i}(s)$ and $\varphi_{y_j}(s)$ denote the real-valued continuous functions defined on $(-\infty,0]$. Obviously, the systems considered in [30,31] are special cases of (1).

- Throughout this paper, we always use $i = 1,\ldots,n$; $j = 1,\ldots,m$, unless otherwise stated.

For convenience, we introduce the following assumptions:

(H_1) $h_{ij}^0, w_{ji}^0, h_{ij}^1, w_{ji}^1, r_i, s_j \in R$. The amplification functions $a_i(\cdot)$ and $c_j(\cdot)$ are bounded, locally Lipschitzian and $0 < \underline{a}_i \leq a_i(x) \leq \overline{a}_i$, $x \in R$, $0 < \underline{c}_j \leq c_j(y) \leq \overline{c}_j$, $y \in R$.

(H_2) For activation functions, there exists positive numbers L_j^f and L_i^g such that

$$|f_j(x) - f_j(y)| \leq L_j^f |x-y|, \quad |g_i(x) - g_i(y)| \leq L_i^g |x-y| \ \text{ for all } x,y \in R.$$

(H_3) Assume that the kernels $H_{ij}(\cdot)$ and $K_{ji}(\cdot)$ are nonnegative continuous functions defined on $[0,\infty)$ and there exist positive numbers λ and μ such that

$$\int_0^\infty H_{ij}(s)\,ds = 1, \quad \int_0^\infty H_{ij}(s) e^{\lambda s}\,ds < +\infty, \quad \int_0^\infty H_{ij}(s) s e^{\lambda s}\,ds < +\infty,$$

$$\int_0^\infty K_{ji}(s)\,ds = 1, \quad \int_0^\infty K_{ji}(s) e^{\mu s}\,ds < +\infty, \quad \int_0^\infty K_{ji}(s) s e^{\mu s}\,ds < +\infty.$$

The rest of this paper is organized as follows. In next section, by using different methods, a variety of interesting sufficient conditions are established for the existence and uniqueness of equilibrium of (1). In Section 3, we devote ourself to studying exponential stability of the unique equilibrium of impulsive Cohen–Grossberg-type BAM networks with distributed delays. Meanwhile, we shall apply our results to the famous BAM networks with or without impulses. As you will see, our results generalize and improve the preciously known results in the literature. Finally, an example is presented to show the effectiveness and feasibility of our results.

2. Existence of a unique equilibrium

In this section, some easily verifiable and new sufficient conditions are established for the existence of a unique equilibrium of (1). An equilibrium solution of (1) is a constant vector $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T \in R^{n+m}$ which satisfies the following algebraic equation:

$$\begin{cases} a_i(x_i^*)\left[b_i(x_i^*) - \sum_{j=1}^{m}(h_{ij}^0 + h_{ij}^1) f_j(\lambda_j y_j^*) - r_i\right] = 0, \\ c_j(y_j^*)\left[d_j(y_j^*) - \sum_{i=1}^{n}(w_{ji}^0 + w_{ji}^1) g_i(\mu_i x_i^*) - s_j\right] = 0, \end{cases}$$

when the impulsive jumps are assumed to satisfy $I_i(x_i^*) = 0, J_j(y_j^*) = 0$. From the assumption ($H_1$), it follows that

$$\begin{cases} b_i(x_i^*) = \sum_{j=1}^{m}(h_{ij}^0 + h_{ij}^1) f_j(\lambda_j y_j^*) + r_i, \\ d_j(y_j^*) = \sum_{i=1}^{n}(w_{ji}^0 + w_{ji}^1) g_i(\mu_i x_i^*) + s_j. \end{cases} \tag{2}$$

For convenience, we introduce some notations. We will use $z = (x_1, x_2, \ldots, x_n, y_1, \ldots, y_m)^T \in R^{n+m}$ to denote a column vector, in which the symbol $(^T)$ denotes the transpose of a vector. For matrix $\mathcal{D} = (d_{ij})_{n\times n}$, $\mathcal{D}^T$ denotes the transpose of $\mathcal{D}$, and E_n denotes the identity matrix of size n. diag$(\cdot)$ represents a diagonal matrix with specified diagonal entries. A matrix or vector $\mathcal{A} \geq 0$ means that all entries of $\mathcal{A}$ are greater than or equal to zero. $\mathcal{A} > 0$ can be defined similarly. For matrices or vectors A and B, $\mathcal{A} \geq \mathcal{B}$ (resp. $\mathcal{A} > \mathcal{B}$) means that $\mathcal{A} - \mathcal{B} \geq 0$ (resp. $\mathcal{A} - \mathcal{B} > 0$). We denote the spectral radius of the matrix $\mathcal{A}$ by $\rho(\mathcal{A})$.

Definition 2.1. (*see Berman and Plemmons [47], Lasalle [48], Horn and Johnson [49]*). A real $n \times n$ matrix $\mathcal{A} = (a_{ij})$ is said to be an M-matrix if $a_{ij} \leq 0$, $i,j = 1,2,\ldots,n$, $i \neq j$, and $\mathcal{A}^{-1} \geq 0$.

Lemma 2.1. (Xia [19]). *Let N be a positive integer and B be a Banach space. If the mapping $\Phi^N : B \to B$ is a contraction mapping, then $\Phi : B \to B$ has exactly one fixed point in B, where $\Phi^N = \Phi(\Phi^{N-1})$.*