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Support vector machine optimal control for mobile wheeled inverted pendulums with unmodelled dynamics ☆

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ABSTRACT

The dynamic balance and motion control based on LS-SVM (least squares support vector machine) are considered for mobile wheeled inverted pendulums (WIP), in the presence of parametric and functional dynamics uncertainties. Based on Lyapunov synthesis, the proposed control mechanisms use the advantage of LS-SVM combined with on-line parameters estimation strategy in order to have an efficient approximation. Under the controller designed, we can ensure that the outputs of the system track the given bounded reference signals within a small neighborhood of zero, and guarantee semi-global uniform boundedness of all the closed loop signals. Simulation results are presented to verify the effectiveness of the proposed control.

1. Introduction

Wheeled inverted pendulums shown in Fig. 1 are not planar and the motors driving the wheels are directly mounted on the pendulum body [4], which are different from cart and pendulums [7–9]. Recently, more researches have been done to investigate the dynamics and control of wheeled inverted pendulums [1–5]. Wheeled inverted pendulums belong to under-actuated dynamic system, where the number of control inputs is less than the number of degrees of freedom to be stabilized [6]. It is difficult to apply the conventional control approach for under-actuated dynamic systems. Therefore, some control designs have been proposed to guarantee stability and robustness for mobile wheeled inverted pendulums. Moreover, because of intrinsic under-actuated system, the dynamics of wheeled inverted pendulums systems is nonlinear and coupled, which can be described by coupled nonlinear differential equations. However, it is often possible to obtain an approximated linearized model around an operating point, where the signals involved are small enough. Several design of controllers and analysis techniques for linear systems were proposed, i.e., motion control using linear state-space model was proposed in [12], and the control was designed based on the dynamic equations linearized around an operating point [1]. In [10], dynamics involving the pitch of the inverted pendulum was studied and the rotation angles of the two wheels as the variables of interest were presented, and in [11], a linear controller was designed. In [13], only a planar model without yaw was considered, a linear stabilizing controller was derived based on this mode. In [14], although the exact dynamics of two-wheeled inverted pendulum was investigated, only the linear feedback control was developed on the dynamic model. In [4], based on partial feedback linearization, a two-level velocity controller and a stabilizing position controller were proposed.

Feedback linearization is an approach to nonlinear control design which has attracted a great deal of research interest in recent years. The central idea of the approach is to algebraically transform a nonlinear system dynamics into a (fully or partly) linear one, so that linear control techniques can be applied. In the transformation, the exact dynamics of system must be known beforehand, the model-based control can provide an effective solution to the problem. However, wheeled inverted pendulum is characterized by unstable balance and unmodelled nonlinear dynamics, and there are time varying external disturbances, in the form of parametric and functional uncertainties, which is not possible to model accurately. Modeling errors might undermine the control approach based on linearized model [1,12] and the control proposed on the velocity level [4]. Therefore, model-based control may not be the ideal choice since the dynamics is not usually available. The presence of uncertainties and disturbances

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Nomenclature

q_v	the vector of generalized coordinates for the mobile platform with $q_v=[q_1,q_2,q_3]^T=[\theta,x,y]^T\in R^3$
x,y	the position coordinates of the mid-point of the two driving wheels
θ	the heading angle in motion relative to the x-axis of the fixed frame
α	the tilt angle relative to z-axis of the fixed frame
$D_v(q)$	the inertia matrices for the mobile platform with $D_v(q)\in R^{3\times 3}$
D_α	the inertia matrix for the inverted pendulum
$D_{v\alpha},D_{\alpha v}$	the coupling inertia matrices of the mobile platform and the inverted pendulum
C_v,C_α	the centripetal and coriolis torques for the mobile platform and the inverted pendulum with $C_v\in R^{3\times 3}$ and $C_\alpha\in R$
$C_{v\alpha},C_{\alpha v}$	the coupling centripetal and coriolis torques of the mobile platform and the inverted pendulum
G_v,G_α	the gravitational torque vectors for the mobile platform and the inverted pendulum with $G_v\in R^3$ and $G_\alpha\in R$
τ_v	the control input vector for the mobile platform with $\tau_v\in R^2$
$f_v(t),f_\alpha(t)$	the external friction force on the mobile platform and the inverted pendulum with $f_v(t)\in R^3$ and $f_\alpha(t)\in R^1$
$d_v(t),d_\alpha(t)$	the external disturbances on the mobile platform and the inverted pendulum with $d_v(t)\in R^3$ and $d_\alpha(t)\in R^1$
B_v	a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system with $B_v\in R^{3\times 2}$
J_v^T	Jacobian matrix with $J_v^T\in R^3$
λ	Lagrangian multipliers corresponding to the nonholonomic constraints

could disrupt the function of the model-based feedback control, and lead to the unstable balance. How to handle the parametric and functional uncertainties, unmodelled dynamics, and disturbances from the environment is one of the important issues in the control of wheeled inverted pendulum. The wheeled inverted pendulum is definitely different from other nonholonomic systems subject to (i) only kinematic constraints which geometrically restrict the direction of mobility, i.e., wheeled mobile robot [27,28], (ii) only dynamic constraints due to dynamic balance at passive degrees of freedom where no force or torque is applied [33], i.e., the manipulator with passive link [29,30]. It belongs to (iii) not only kinematic constraints but also dynamic constraints. Therefore, the wheeled inverted pendulum is more complex than the former two cases, so the previously proposed control approaches suitable for (i) and (ii) could not be applied directly to wheeled inverted pendulums.

A challenging problem is to control a mobile wheeled inverted pendulum system whose cart is no longer constrained to the guide rail like cart-pendulum systems, but moves in its terrain while balancing the pendulum. Moreover, the control for wheeled inverted pendulums is different and difficult compared with other full-actuated systems because they consist of multiple under-actuated configurations.

Least squares support vector machine (LS-SVM) has been proposed for solving nonlinear function estimation problems [23,22]. LS-SVM takes equality instead of inequality constraints of SVM in the problem formulation such that LS-SVM is easy to train. The previous works about SVM learning approaches have been proposed in [15–18] for modeling nonlinear system and SVM-based nonlinear controls; however, those works lack the definite stability proof of the closed-loop system using SVM approaches. In [19], the control design on the support vector

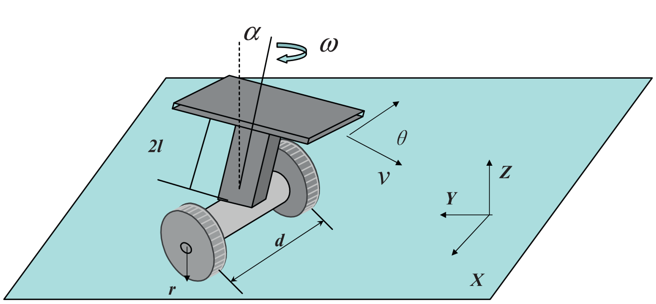

Fig. 1. Mobile wheeled inverted pendulum.

machine (SVM) is developed to achieve accurate haptic display, the approximation model of friction is established off-line through SVM learning for online feed forward friction compensation. However, in practical control applications, it is desirable to have systematic methods to ensure on-line stability, robustness, and performance of the overall system in the unknown environments beforehand. Although the on-line adaptive fuzzy approximation is a well-known technique, for the real-time mechanical system, it is difficult to choose a satisfying fuzzy rule number, the performance would be degraded with less rule number, on the other hand, the "explosion" of rule number would bring great trouble to the limit computation resource. However, support vector machines approach could avoid this problem. While to our best knowledge, there are few works dealing with SVM-based control proposed for the wheeled inverted pendulum up to now.

The previous works, which utilized the SVMs approximation for system modelling, assume beforehand that the state variables of the system are bounded in a compact set since the approximated data can be provided off-line, while for the stability of system by on-line SVM approximation, the system errors cannot be predicted. It is not reasonable to assume that the bounds within the compact set beforehand. The direct SVMs approximation is not valid, while many applications using off-line SVM approximation (all data obtained are bounded) limit the extension of SVMs for the control of nonlinear dynamic systems, for example wheeled inverted pendulums.

In this paper, by discovering and utilizing the unique physical property of the wheeled inverted pendulum, we could decouple the system to simplify the model, such that we can design the control easily. Moreover, we make full use of the physical properties of the wheeled inverted pendulum and then design on-line LS-SVM based control by sliding window to accommodate the presence of parametric and functional uncertainties in the dynamics of wheeled inverted pendulums. The developed SVM-based control combining the physical properties of the system is original.

The main contributions of this paper lie in:

(i) the developed SVM-based control combining the physical properties of the system for achieving better performance and simplifying control design and analysis;
(ii) the use of SVMs in compensating for parametric and functional uncertainties commonly encountered in wheeled inverted pendulums control and the rigorous stability