Neurocomputing 72 (2009) 3751–3759

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Almost periodic solution to Cohen–Grossberg-type BAM networks with distributed delays

Hongjun Xiang [a,*], Jinhua Wang [a], Jinde Cao [b]

[a] Department of Mathematics, Xiangnan University, Chenzhou 423000, China
[b] Department of Mathematics, Southeast University, Nanjing 210096, China

ARTICLE INFO

Article history:
Received 9 November 2008
Received in revised form
16 March 2009
Accepted 27 May 2009
Communicated by T. Heskes
Available online 21 June 2009

Keywords:
Cohen–Grossberg-type
BAM neural networks
Almost periodic solution
Exponential stability

ABSTRACT

In this paper, a class of Cohen–Grossberg-type bi-directional associative memory (BAM) neural networks with distributed delays is discussed. Based on inequality analysis method and combining the exponential dichotomy with fixed point theorem, some novel sufficient conditions are obtained to ensure the existence and globally exponential stability of almost periodic solution to this system. Moreover, an example is given to demonstrate the feasibility of our results.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

Recently, bi-directional associative memory (BAM) neural network has been intensively studied due to its potential applications in many fields such as pattern recognition and automatic control, image and signal processing since it was presented by Kosko [1] in 1988. In the design and applications of networks, the stability of the designed neural network is one of the most important issues. There were many important results concerning mainly on the existence and stability of equilibrium of BAM neural networks in Refs. [1–5] and references therein.

As we know, Cohen–Grossberg neural network (CGNN) [6] is a kind of important neural network described as follows:

$$\frac{dx_i(t)}{dt} = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} c_{ij} g_j(x_j(t)) - I_i\right],$$

where $i = 1, 2, \ldots, n$; $n \geq 2$ is the number of neurons in the network; $x_i(t)$ denotes the state variable of the ith neuron at time t; $g_j(x_j(t))$ denotes the activation function of the jth neuron at time t; the feedback matrix $C = (c_{ij})_{n\times n}$ indicates the strength of the neuron interconnections within the network; $a_i(x_i(t))$ represents an amplification function; $b_i(x_i(t))$ is an appropriately behaved function such that the solutions to the model remain bounded; I_i represents external input at the time t. Researches have shown their potential applications in classification, parallel computation, associative memory, especially their ability to solve some optimization problems, see for example [6–17,19–22].

In [15], by constructing a suitable Lyapunov functional, the asymptotic stability was investigated for Cohen–Grossberg-type BAM neural networks with both constant delays and time-varying delays described as

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(x_i(t))\left[b_i(x_i(t)) - \displaystyle\sum_{j=1}^{m} p_{ij} f_j(\lambda_j y_j(t-\tau_j))\right], \\ \quad i = 1, 2, \ldots, n, \\ \dfrac{dy_j(t)}{dt} = -c_j(y_j(t))\left[d_j(y_j(t)) - \displaystyle\sum_{i=1}^{n} q_{ji} g_i(\mu_i x_i(t-\sigma_i))\right], \\ \quad j = 1, 2, \ldots, m. \end{cases}$$

In [16], the authors discussed the Cohen–Grossberg-type bi-directional associative memory (CGBAM) with time-varying delays

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(x_i(t))\left[b_i(x_i(t)) - \displaystyle\sum_{j=1}^{m} p_{ij} f_j(\lambda_j y_j(t-\tau_{ij}(t))) - I_i\right], \\ \quad i = 1, 2, \ldots, n, \\ \dfrac{dy_j(t)}{dt} = -c_j(y_j(t))\left[d_j(y_j(t)) - \displaystyle\sum_{i=1}^{n} q_{ji} g_i(\mu_i x_i(t-\sigma_{ji}(t))) - J_j\right], \\ \quad j = 1, 2, \ldots, m. \end{cases}$$

They gave out several sufficient conditions to guarantee the existence, uniqueness and global exponential stability of the

* Corresponding author.
E-mail addresses: hunxhjxhj67@126.com (H. Xiang). jdcao@seu.edu.cn (J. Cao).

0925-2312/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.neucom.2009.05.014

equilibrium point and made out the estimation of the exponential convergence rate for the model. In [17], Yang et al. discussed global stability of impulsive BAM type Cohen–Grossberg neural networks with delays.

In addition, experiments show that time delays can affect the stability of neural networks and lead to some other dynamical behaviors, for example periodic or almost periodic oscillation, bifurcation, chaos, and so on. As we know, the non-autonomous phenomenon often occurs in many realistic systems. For example, when we consider a long-time dynamical behavior of a system, the parameters of the system usually bring change along with time. Moreover, the property of periodic oscillatory solutions to neural networks also is very interest in many applications. Many scholars have studied the periodicity of neural networks and derived some sufficient conditions for checking the existence and stability of periodic solutions to delayed BAM neural networks, refer to [18–22] and references therein. In [24], the authors studied the exponential stability of periodic solution to Cohen–Grossberg-type BAM networks with time-varying delays.

In [21], the authors proposed a class of bi-directional Cohen–Grossberg neural networks with distributed delays as follows:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(x_i(t)) \\ \qquad \left[b_i(t, x_i(t)) - \displaystyle\sum_{j=1}^{m} p_{ij}(t) \int_0^{\infty} K_{ji}(u) f_j(t, \lambda_j y_j(t-u))\, du - I_i(t) \right], \\ \quad i = 1, 2, \ldots, n, \\ \dfrac{dy_j(t)}{dt} = -c_j(y_j(t)) \\ \qquad \left[d_j(t, y_j(t)) - \displaystyle\sum_{i=1}^{n} q_{ji}(t) \int_0^{\infty} L_{ij}(u) g_i(t, \mu_i x_i(t-u))\, du - J_j(t) \right], \\ \quad j = 1, 2, \ldots, m. \end{cases}$$

By using the Lyapunov functional method and some analytical techniques, some sufficient conditions are obtained for the globally exponential stability of periodic solutions to this networks.

However, to the best of our knowledge, the almost periodic oscillation for the bi-directional Cohen–Grossberg neural networks is hardly considered and investigated. Motivated by the above arguments, in this paper, we will consider the following Cohen–Grossberg-type BAM neural network with distributed delays:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(x_i(t)) \\ \qquad \left[b_i(x_i(t)) - \displaystyle\sum_{j=1}^{m} p_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s) f_j(y_j(s))\, ds - I_i(t) \right], \\ \dfrac{dy_j(t)}{dt} = -c_j(y_j(t)) \\ \qquad \left[d_j(y_j(t)) - \displaystyle\sum_{i=1}^{n} q_{ji}(t) \int_{-\infty}^{t} L_{ji}(t-s) g_i(x_i(s))\, ds - J_j(t) \right], \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; $n, m \geq 2$ are the number of neurons in the networks; $x_i(t)$ and $y_j(t)$ are the states of the ith neuron from the neural field F_X and the jth neuron from the neural field F_Y at the time t, respectively; f_j, g_i denote the activation functions of jth neuron from F_Y and the ith neuron from F_X, respectively; $p_{ij}(t)$ weights the strength of the ith neuron on the jth neuron at the time t; $q_{ji}(t)$ weights the strength of the jth neuron on the ith neuron at the time t; The kernels $K_{ij} : [0, +\infty) \to [0, +\infty)$ and $L_{ji} : [0, +\infty) \to [0, +\infty)$ $(i, j = 1, 2, \ldots, n)$ are continuous functions. $I_i(t), J_j(t)$ denote the external inputs on the ith neuron from F_X and the jth neuron from F_Y, respectively; the functions $p_{ij}(t)$, $q_{ji}(t)$ and $I_i(t)$, $J_j(t)$ are continuous almost periodic functions; $a_i(x_i(t))$ and $c_j(y_j(t))$ represent amplification functions; $b_i(x_i(t))$ and $d_j(y_j(t))$ are appropriately behaved functions such that the solution of model (1) remain bounded.

System (1) includes a number of models from neurobiology and population biology, such as the Lotka–Votterra system, and BAM neural networks as a special case. The main objective of this paper is to give some sufficient conditions to ensure the existence and globally exponential stability of the almost periodic oscillation solution to the model (1).

The remaining part of the paper is organized as follows. In Section 2, we shall introduce some notations, definitions and preliminaries which will be used later. In Section 3, sufficient conditions are obtained to ensure the existence, globally exponential stability of the almost periodic solution. In Section 4, an example is given to show the effectiveness of the obtained results. Finally, the conclusion is drawn in Section 5.

2. Preliminaries

There are several known equivalent definitions of almost periodic functions and here we use the Bohr definition. Combining Definitions 1.10 and 1.11 with Definition 1.12 in [23], we can have the following definition.

Definition 1 (*Fink [23]*). Let $x(t) : \mathbb{R} \to \mathbb{R}^n$ be continuous in t. $x(t)$ is said to be almost periodic (Bohr) on $\mathbb{R}$, if for any $\varepsilon > 0$, the set $T(x, \varepsilon) = \{\omega \mid |x(t+\omega) - x(t)| < \varepsilon,\ \forall t \in \mathbb{R}\}$ is relatively dense, i.e. $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, for any interval with length $l(\varepsilon)$, there exists a number $\omega = \omega(\varepsilon)$ in this interval such that $|x(t+\omega) - x(t)| < \varepsilon$, for $\forall t \in \mathbb{R}$.

Definition 2 (*Fink [23]*). System

$$\dot{x}(t) = A(t)x(t) \tag{2}$$

is said to admit an exponential dichotomy if there exists a projection P and positive constants α, β so that the fundamental solution matrix $X(t)$ satisfy

$$|X(t)PX^{-1}(s)| \leq \beta e^{-\alpha(t-s)}, \quad t \geq s,$$

$$|X(t)(I-P)X^{-1}(s)| \leq \beta e^{-\alpha(s-t)}, \quad t \leq s.$$

Lemma 1 (*Fink [23]*). *If the linear system* (2) *admits an exponential dichotomy, then the almost periodic system*

$$\dot{x}(t) = A(t)x(t) + f(t),$$

in which $A(t)$ is an almost periodic matrix and $f(t)$ is an almost periodic vector, has a unique almost periodic solution $\varphi(t)$, and

$$\varphi(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)f(s)\, ds - \int_{t}^{+\infty} X(t)(I-P)X^{-1}(s)f(s)\, ds.$$

Lemma 2 (*Fink [23]*). *Let $a_i(t)$ be an almost periodic function on $\mathbb{R}$ and*

$$M[a_i] = \lim_{T \to +\infty} \frac{1}{T} \int_{t}^{t+T} a_i(s)\, ds > 0, \quad i = 1, 2, \ldots, n.$$

Then the linear system

$$\dot{x}(t) = \text{diag}(-a_1(t) \quad ,- \quad a_2(t), \ldots \quad ,- \quad a_n(t))x(t),$$

admits exponential dichotomy on $\mathbb{R}$.

Download English Version:

https://daneshyari.com/en/article/410465

Download Persian Version:

https://daneshyari.com/article/410465

Daneshyari.com