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Delay-distribution-dependent stability of stochastic discrete-time neural networks with randomly mixed time-varying delays ☆

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ABSTRACT

In this paper, the stability analysis problem for a new class of discrete-time neural networks with randomly discrete and distributed time-varying delays has been investigated. Compared with the previous work, the distributed delay is assumed to be time-varying. Moreover, the effects of both variation range and probability distribution of mixed time-delays are taken into consideration in the proposed approach. The distributed time-varying delays and coupling term in complex networks are considered by introducing two Bernoulli stochastic variables. By using some novel analysis techniques and Lyapunov–Krasovskii function, some delay-distribution-dependent conditions are derived to ensure that the discrete-time complex network with randomly coupling term and distributed time-varying delay is synchronized in mean square. A numerical example is provided to demonstrate the effectiveness and the applicability of the proposed method.

1. Introduction

In the past decade, neural networks (NNs) have attracted considerable attention because of their potential applications in engineering applications, such as associative memory, pattern recognition, optimization and signal processing [1–3]. As is known to all, stability is one of the preconditions in the design. For example, if a neural network is employed to solve some optimization problems, it is highly desirable for the NNs to have a unique globally stable equilibrium. Therefore, stability analysis of NNs is a very important issue and has been well studied in [4–20]. In particular, the stability of discrete-time neural networks (DNNs) have been studied in [12–20], since DNNs play a more important role than their continuous-time counterparts in today's digital life [21–23]. On the other hand, it has now been well recognized that stochastic disturbances are mostly inevitable owing to thermal noise in electronic implementations. It has also been revealed that certain stochastic inputs could make a neural network unstable. Therefore, the stability problem of stochastic DNNs becomes more significant from the practical point of view, see, e.g. [24–26].

Recently, the dynamics analysis problem for DNNs with or without time delays has received much research interest, see, e.g. [13,16,18,19] and references therein. On the other hand, due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, a neural network usually has a spatial nature. Therefore, it is necessary to take continuously distributed delays into account for modeling a realistic neural network such that the distant past has less influence compared with the recent behavior of the state. Note that the dynamics analysis problem of distributed delay in continuous-time neural networks has been well studied in [8,24–26]. Very recently, Liu and Wang introduced the infinite distributed delay and distributed delay in the form of constant delay into the DNNs [17,18]. However, in practice, time-varying delay in DNNs occurs commonly in most designs. Therefore, the study of DNNs with distributed time-varying delay is more important than those with constant delays.

In the existing references for DNNs, the deterministic time-delay case was well studied, see, e.g. [16,18,27]. Actually, the time delay in some NNs exist in a stochastic fashion [28–30]. For example, to control and propagate the stochastic signals, a probabilistic universal learning networks (PULN) was proposed in [30]. In [30], if some values of the time delay are very large but the probabilities of the delay taking such large values are very small, then it may result in a more conservative result if only the information of variation range of the time delay is considered. For this case, if we derive the criteria by only using the variation range

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of time delay, the results could lead to more conservative. In [19,20], the stochastic discrete time-delay was introduced into DNNs and some so-called delay-distribution-dependent criteria have been derived. However, to the best of authors' knowledge, so far, the stability analysis of DNNs with distributed time-varying delay has received little attention in the literature, not to mention that stochastic distributed delay is also involved.

Motivated by the above discussions, the aim of this paper is to investigate the stability of DNNs with randomly mixed time-varying delays in mean square. By using two stochastic variables which satisfy Bernoulli random binary distribution, we propose a new model of DNNs composed of stochastic mixed time-delays. The effects of both variation range and probability distribution of the discrete and distributed time-varying delays is considered to derive the stability criteria. The proposed results will result in less conservativeness and takes some well-studied models as special cases. The stochastic disturbances are described in terms of a Brownian motion. Via a Lyapunov–Krasovskii functional and some new analysis techniques, some sufficient conditions for global stability in mean square are established for the addressed stochastic DNNs with randomly mixed time-varying delays. An illustrative example is given to show the effectiveness of the proposed results.

2. Model formulation and preliminaries

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all real matrices. The superscript 'T' denotes matrix transposition and the notation $X\geq Y$(respectively, $X>Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. $|\cdot|$ stands for the Euclidean vector norm in $\mathbb{R}^n$. $\mathbb{Z}_{\geq 0}$denotes the the set including zero and positive integers. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ denote the expectation of x and the expectation of x conditional on y. $[a:b]$ denotes a set involving all integers between a and b. $(\Omega,\mathscr{F},\mathscr{P})$ is a probability space, where Ω is the sample space, $\mathscr{F}$ is the σ-algebra of subsets of the sample space and $\mathscr{P}$ is the probability measure on $\mathscr{F}$.

Consider the following n-neuron DNNs with mixed time-varying delays:

$$u(k+1)=Cu(k)+A\tilde{f}(u(k))+B\tilde{g}(u(k-\tau(k)))+D\sum_{i=-d(k)}^{-1}\tilde{h}(u(k+i))+\tilde{J},\tag{1}$$

where $u(k)=(u_1(k),u_2(k),\ldots,u_n(k))^T$ is the neural state vector, $C=\mathrm{diag}\{c_1,c_2,\ldots,c_n\}$ is the state feedback coefficient matrix; the $n\times n$ matrices $A=[a_{ij}]_{n\times n}$, $B=[b_{ij}]_{n\times n}$ and $D=[d_{ij}]_{n\times n}$ are, respectively, the connection weight matrix, the discretely delayed connection weight matrix and distributively delayed connection weight matrix; $\tilde{J}=[\tilde{J}_1,\tilde{J}_2,\ldots,\tilde{J}_n]^T$ is the exogenous input; $\tau(k)$ and $d(k)$ denote the discrete and distributed time-varying delays, respectively. $\tilde{f}(u(k))=[\tilde{f}_1(u_1(k)),\tilde{f}_2(u_2(k)),\ldots,\tilde{f}_n(u_n(k))]^T$, $\tilde{g}(u(k))=[\tilde{g}_1(u_1(k)),\tilde{g}_2(u_2(k)),\ldots,\tilde{g}_n(u_n(k))]^T$and $\tilde{h}(u(k))=[\tilde{h}_1(u_1(k)),\tilde{h}_2(u_2(k)),\ldots,\tilde{h}_n(u_n(k))]^T$ denote the activation functions.

Remark 1. Note that the distributed delay in continuous time systems has received much attention in [8,24,26]. Very recently, Liu and Wang introduced the distributed delay in discrete-time systems [17,18,27]. However, the distributed delay is considered in the case of constant delay or infinite distributed delay. Due to the time-varying delay takes the usual time delay as special cases, in this paper, we aim to investigate stability analysis of neural networks with the time-varying distributed delay. In the following, we use some novel techniques to deal with the stochastic interval time-varying distributed delay in discrete-time systems.

Assumption 1. For $i\in\{1,2,\ldots,n\}$, the neuron activation functions in (1) satisfy

$$\begin{aligned}&H_1\leq\frac{\tilde{f}_i(x)-\tilde{f}_i(y)}{x-y}\leq H_2,\\&L_1\leq\frac{\tilde{g}_i(x)-\tilde{g}_i(y)}{x-y}\leq L_2,\\&M_1\leq\frac{\tilde{h}_i(x)-\tilde{h}_i(y)}{x-y}\leq M_2,\quad\forall x,y\in\mathbb{R}^n,\ x\neq y,\\&i=1,2,\ldots,n,\end{aligned}\tag{2}$$

where H_1,H_2,L_1,L_2,M_1,M_2 are constants.

Remark 2. This assumption was first introduced in Refs. [8,11] and has been subsequently studied in many recent NN papers (see, e.g. [16,19,25,27]). Obviously, the conditions in Assumption 1 are more general than the usual sigmoid functions and the recently commonly used Lipschitz conditions, see, e.g. [5,14,26]. Such a description is very precise in quantifying the lower and upper bounds of the activation functions, therefore very helpful for employing LMI-based method to reduce the possible conservatism.

Under Assumption 1, let u_* be the equilibrium point of (1). We shift the intended equilibrium u_* to the origin by letting $x(k)=u(k)-u_*$. Then, system (1) with stochastic disturbances can be written as

$$\begin{aligned}x(k+1)=&\,Cx(k)+Af(x(k))+Bg(x(k-\tau(k)))+D\sum_{i=-d(k)}^{-1}h(x(k+i))\\&+\sigma(k,x(k))\omega(k),\end{aligned}\tag{3}$$

where $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T$ is the state vector of the transformed system, and the transformed neuron activation functions are $f(x(k))=\tilde{f}(u(k))-\tilde{f}(u_*)$, $g(x(k))=\tilde{g}(u(k))-\tilde{g}(u_*)$ and $h(x(k))=\tilde{h}(u(k))-\tilde{h}(u_*)$. $\sigma(\cdot,\cdot):\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ is the noise intensity function vector; $\omega(k)$ is a scalar Wiener process on a probability space $(\Omega,\mathscr{F},\mathscr{P})$ with

$$\mathbb{E}\{\omega(k)\}=0,\quad\mathbb{E}\{\omega^2(k)\}=1,\quad\mathbb{E}\{\omega(i)\omega(j)\}=0\quad(i\neq j).\tag{4}$$

It can be verified from (2) and (3) that

$$\begin{aligned}&H_1\leq\frac{f_i(x)-f_i(y)}{x-y}\leq H_2,\\&L_1\leq\frac{g_i(x)-g_i(y)}{x-y}\leq L_2,\\&M_1\leq\frac{h_i(x)-h_i(y)}{x-y}\leq M_2,\quad\forall x,y\in\mathbb{R}^n,\ x\neq y,\\&f_i(0)=g_i(0)=h_i(0)=0,\quad i=1,2,\ldots,n,\end{aligned}\tag{5}$$

where H_1,H_2,L_1,L_2,M_1,M_2 are constants.

Assumption 2. The noise intensity function vector $\sigma(\cdot,\cdot):\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ satisfies the Lipschitz condition, i.e. there exist a constant γ such that the following inequality:

$$\sigma(k,x(k))^T\sigma(k,x(k))\leq\gamma x^T(k)x(k).\tag{6}$$

Assumption 3. The discrete time-varying delay $\tau(k)$ and distributed time-varying delay $d(k)$ are bounded, namely $0<\tau_m\leq\tau(k)\leq\tau_M$, $0<d_m\leq d(k)\leq d_M$, and its probability distribution can be observed, i.e. assume that $\tau(k)$ takes values in $[\tau_m:\tau_0]$ or $(\tau_0:\tau_M]$ and $\mathrm{Prob}\{\tau(k)\in[\tau_m:\tau_0]\}=\rho_0$, where τ_0,τ_m,τ_M are integers satisfying $\tau_m\leq\tau_0<\tau_M$, and $0\leq\rho_0\leq 1$. Similarly, $d(k)$ takes values in $[d_m:d_0]$ or $(d_0:d_M]$ and $\mathrm{Prob}\{d(k)\in[d_m:d_0]\}=\xi_0$, where d_0,d_m,d_M are integers also satisfying $d_m\leq d_0<d_M$, and $0\leq\xi_0\leq 1$.