Neurocomputing 72 (2009) 3881–3890

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Estimation of epipolar geometry by linear mixed-effect modelling

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A R T I C L E I N F O

Article history:
Received 26 December 2007
Received in revised form
31 December 2008
Accepted 28 April 2009
Communicated by D. Tao
Available online 21 June 2009

Keywords:
Fundamental matrix
Epipolar geometry
Least square
Outlier
Mixed effects

A B S T R A C T

Epipolar geometry relies on the determination of the fundamental matrix. Classical approaches for estimating the fundamental matrix assume that a Gaussian distribution exists in the errors in view of mathematical tractability. However, this assumption will not be justified when the distribution computed is not normally distributed. We propose a new approach that does not make the Gaussian assumption, and so can attain robustness and accuracy in different conditions. The proposed framework, *weighted least squares* (WLS), is the application of linear mixed-effect models considering the correlation between different data subsamples. It provides an unbiased estimation of the fundamental matrix after mitigating the effects of outliers. We test the new model by using synthetic and real images, and comparing it to standard methods.

1. Introduction

Three-dimensional (3-D) reconstruction from a set of images is an important and difficult problem in stereo vision [1,2]. The 3-D reconstruction relies on the consistency between left-based and right-based matching processes, the so-called correspondence problem. To solve the correspondence problem, one of the approaches usually used is to apply the epipolar geometry, which is described by a *fundamental matrix*. The need of estimating the fundamental matrix has been widely recognised since 1980s, and current research focuses on proposing new estimators either to improve the accuracy of the fundamental matrix or to reduce computational cost [3–6].

The estimation of the fundamental matrix is formulated as a constrained least square problem, whose solution is frequently sought by minimising linear or non-linear least square object functions. To find such a solution whilst anticipating the minimum effects of outliers on estimation, a number of techniques have been reported [7,8]. These established algorithms include random sample consensus (RANSAC) [9], least median of squared (LMeS) [10,11] and M-estimator [12]. In spite of their capacities in detection of outliers, these established approaches usually suffer from the non-Gaussian noise in data because the distribution of the relevant residuals computed is not normally distributed [10]. For example, Fig. 1 illustrates the *non-Gaussian* distribution of the residuals computed after the minimum median of squared residuals has been reached.

Therefore, a new approach with no specific assumption on residuals needs to be explored so as to improve robustness and accuracy of the estimation in different conditions. The approach proposed here, *weighted least squares* (WLS), takes into account the covariance between observations in combination of robust outlier-removal, resulting in its consistency, stability and accuracy against other algorithms. The presented algorithm here applies an established linear mixed-effect model with a properly determined threshold to separate outliers from the matched features, which can be considered as an extension of the least median of squares technique reported by Zhang et al. [11].

2. Background feature of the fundamental matrix

Consider a *perspective* or *pinhole* camera model. The fundamental matrix F defines a transformation between corresponding points in the two images, $\mathbf{p}_2^T F \mathbf{p}_1 = 0$, where the positions of points $\mathbf{p}_1$ and $\mathbf{p}_2$ are defined in image coordinates. This equation is equivalent to

$$\mathbf{u}^T \mathbf{f} = 0, \tag{1}$$

where $\mathbf{f}$ is a 9-vector including the entries of the fundamental matrix F and $\mathbf{u}^T$ is the parameter matrix.

To solve Eq. (1) in physical environments that normally lead to an F matrix of non-zero determinant, a least square framework is

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doi:10.1016/j.neucom.2009.04.018

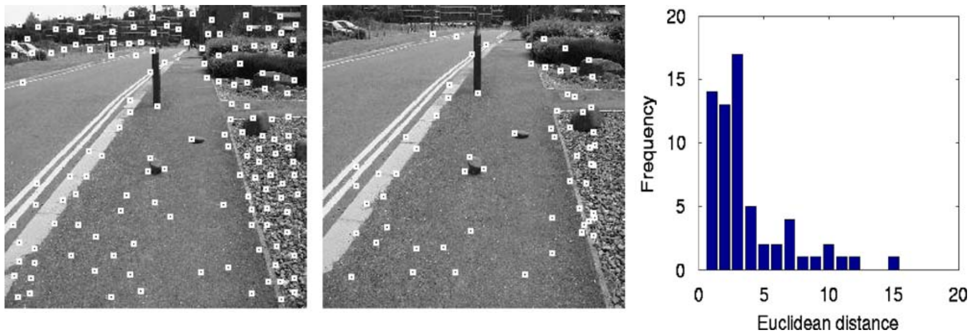

Fig. 1. An example demonstrating the distribution of the residuals computed after the minimum median of squared residuals has been obtained: (a) and (b) a pair of real images with the corner features superimposed and (c) the frequency (point numbers) versus Euclidean distances (pixels), where the width of each bin is 1 pixel.

used as ($\min_F \sum_i (\mathbf{p_2}_i^T F \mathbf{p_1}_i)^2$). One can apply the 2-rank constraint of the F matrix being singular in order to reduce its degrees of freedom, and to utilise the zero determinant. To avoid any ill-posed formulation, the sum of the squared residuals of Euclidean distance from image points to their corresponding *epipolar lines* is minimised [11], which is explained as follows: images 1 and 2 are to be corresponded. When a point in image 2 can be precisely matched to its location in image 1, the Euclidean distance $d(\mathbf{p_2}, F\mathbf{p_1})$ of $\mathbf{p}_2$ to its eipolar line $F\mathbf{p_1}$ should be equal to zero. In a real situation, this may not happen, and so we aim to minimise

$$\sum_i \left(\frac{1}{(F\mathbf{p_1}_i)_1^2 + (F\mathbf{p_1}_i)_2^2} + \frac{1}{(F^T\mathbf{p_2}_i)_1^2 + (F^T\mathbf{p_2}_i)_2^2} \right) (\mathbf{p_2}_i^T F \mathbf{p_1}_i)^2, \quad (2)$$

where $(F\mathbf{p_1})_i$ is the ith component of vector $F\mathbf{p_1}$. We recognise the existence of outliers in correspondences. So, it is assumed that the entire data set for the correspondences has a fraction v of outliers. Then, the number of the subsamples can be found, if at least one of the subsamples generates a good result, whose probability is defined by

$$P_r = 1 - (1 - (1 - v)^\eta)^j, \quad (3)$$

where $\eta \geq 8$. This allows a unique fundamental matrix to be determined by the 8-*point algorithm*, and j is the number of the subsamples. Due to extra sensitivity to noise in the correspondences, the 8-*point algorithm* has been further developed by creating homogeneity in the image coordinates (also called normalisation) [13].

Sampson's method is one of the well studied schemes [14]. It was discovered that if image points are perturbed by Gaussian noise, minimisation of the algebraic distance using the eigenvector of the moment matrix is sub-optimal. For example, Hu et al. [15] presented a novel approach based on the use of evolutionary agents for epipolar geometry estimation by efficiently minimising the Sampson distance optimally. Weng et al. [16] applied a variant of Sampson's method to compute the fundamental matrix, where

$$\mathbf{f} = \min_{\mathbf{f}} \sum_{i=1}^{n} (w_{s_i} \mathbf{u}_i^T \mathbf{f})^2, \quad (4)$$

where i is the index of image points and $w_{s_i} = 1/\nabla r_i$ is the optimal weight defined as the variance of the residual. We also have

$$\nabla r_i = (r_{x_i}^2 + r_{y_i}^2 + r_{x_i'}^2 + r_{y_i'}^2)^{1/2}, \quad (5)$$

where r_{x_i}, r_{y_i}, $r_{x_i'}$ and $r_{y_i'}$ are the partial derivatives.

To our knowledge, algebraic solutions to the nonlinear least square problem represented by Eq. (2) have very good performance in accuracy but still cannot fully cope with potential outliers of anonymous distributions due to the Gaussian assumption in the strategy [17,18]. Our contribution in this paper is to exploit Eq. (2), and incorporate with an *unbiased weighted least square* strategy, in order to seek estimation of the F matrix in different circumstances.

3. Weighted least square strategy

3.1. Preview

The *weighted least square* strategy is the extended version of linear mixed-effect models that have been used for the analysis of balanced or unbalanced grouped data, such as longitudinal data, repeated measures and multilevel data [19]. Conventional modelling techniques do not take account of the correlation between data groups but elicit the agreed criteria out of a number of intermediate outputs for removing outliers. Consequently, a globally optimal solution that fits a linear model like Eq. (1) cannot be directly obtained due to the *biased* property of ordinary least square techniques. The linear mixed-effect model has been proved to provide an unbiased solution to the linear system, as it considers the covariance between observations [19]. One typical example of using the mixed-effect linear model for linear regression is illustrated in Fig. 2.

3.2. Two-stage estimator

The commonly used linear mixed-effect model is expressed in two stages, proposed by Laird and Ware [19]. This model is built up in a hierarchical fashion. The two-stage mixed-effect model is the preferred framework when interest focuses on *inter-individual variation* in the samples. In this paper, the two-stage mixed-effect model is used due to the presence of the outliers or errors of anonymous distributions in the correspondences. This established model will be first summarised before any variations are employed.

The first stage considers a well-known Gauss–Markov linear model. This stage is intended to state the general form of the model. Let $\mathbf{Y}_i$ be a n_i vector of serial measurements on random variable i with the structure

$$\mathbf{Y}_i = \mathbf{X}_i \mathbf{A}_i + \boldsymbol{\varepsilon}_i, \quad i = 1, 2, \ldots, N, \quad (6)$$

where $\mathbf{X}_i$ is an $n_i \times m$ random variable design matrix (full rank), $\mathbf{A}_i$ is an m vector of random coefficients and $\boldsymbol{\varepsilon}_i$ is an independent error with

$$E(\boldsymbol{\varepsilon}_i) = \mathbf{0}, \quad cov(\boldsymbol{\varepsilon}_i) = \sigma^2 \mathbf{I}, \quad (7)$$

where σ is variance, cov is covariance computation and $\mathbf{I}$ is identity matrix. Assume that $\text{rank}(\mathbf{X}_i) = m < n_i$. Indeed, Eq. (6) is the marginal form of Eq. (1).