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Neural network approximation for periodically disturbed functions and applications to control design

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ABSTRACT

This paper addresses the approximation problem of functions affected by unknown periodically time-varying disturbances. By combining Fourier series expansion into multilayer neural network or radial basis function neural network, we successfully construct two kinds of novel approximators, and prove that over a compact set, the new approximators can approximate a continuously and periodically disturbed function to arbitrary accuracy. Then, we apply the proposed approximators to disturbance rejection in the first-order nonlinear control systems with periodically time-varying disturbances, but it is straightforward to extend the proposed design methods to higher-order systems by using adaptive backstepping technique. A simulation example is provided to illustrate the effectiveness of control schemes designed in this paper.

1. Introduction

Recent years have witnessed a rapid development of function approximation theory and their successful applications. As far as the theoretical researches are concerned, lots of approximators have been developed in past years [1–6]. For instance, neural networks (NNs) including radial basis function neural network (RBFNN) [1], multilayer neural network (MNN) [2], and high-order neural network (HONN) [3], etc., have been proven to have good capabilities in function approximation. In other words, NNs can approximate a continuous function arbitrarily closely over a compact set, which is well known as the NN universal approximation property. In parallel with NNs, the universal approximation property of fuzzy logic system (FLS) [4] and wavelet network [5] is also well known. One of the drawbacks of traditional approximators is that their approximation property is only for continuous functions. To encounter this drawback, by combining jump functions into NNs, a new NN approximator is proposed to approximate piecewise continuous functions [6]. From the viewpoint of applications, within the framework of system control, various kinds of existing approximators have been successfully applied to solving the problem of identification and control for uncertain nonlinear systems, see [7–15,28], to name just a few. However, all existing approximators are only used to approximate unknown functions without unknown time-varying disturbances.

It is well known that time-varying disturbances often exist in many practical physical systems. Their existence can lead to some bad effects on system identification and control. For example, disturbances can destroy the universal approximation property of approximators or deteriorate the control system performances. In general, it is extremely difficult to construct a suitable function approximator to model the disturbed functions to any accuracy. A more realistic way is first to classify the time-varying disturbances into subclasses, e.g., periodic vs. non-periodic, and then look for an appropriate suitable approach for each subclass. In fact, many works have been done to control systems with periodically time-varying disturbances (also called time-varying parameters) [16–22]. However, in these works, disturbances are assumed to be added to (or multiplied by) system equalities. To the best of authors' knowledge, up to now no works have been done to investigate the systems in which the periodic disturbances, as partial variables, appear in system functions, especially in unknown system functions.

Motivated by the above discussion, in this paper we will consider the approximation problem of nonlinear functions affected by continuously and periodically time-varying disturbance. I.e., $f(\chi, \theta(t))$, where χ is a measured signal and $\theta(t)$ is an unmeasured disturbance. Main design difficulty is that the unmeasured time-varying disturbance nonlinearly appears in system structure. As far as we know, no works have been reported

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to solve this difficulty in the literature at present stage. Fortunately, it is shown in this paper that the above difficulty can be overcome. The main contributions of this paper include:

(1) From the viewpoint of disturbance rejection, for the first time, we combine Fourier series expansion (FSE) into NNs to construct two kinds of new approximators to reject the unknown periodically time-varying disturbances in unknown functions. One is constructed by combining FSE into MNN, and the other is established by combining FSE into RBFNN. Then, we prove their universal approximation property. Compared with traditional NNs, the main advantage of new approximators is their disturbance rejection capability due to the introduction of FSE.
(2) From the viewpoint of practical applications, we further apply two kinds of approximators to the disturbance rejection in adaptive control systems. Concretely speaking, we employ the proposed approximators to solve the adaptive tracking control problem for uncertain systems with periodically time-varying and unknown disturbances, which further demonstrates the approximation capability and disturbance rejection capability of the proposed approximators.

The rest of this paper is organized as follows. In Section 2, we construct two kinds of approximators for periodically disturbed functions and prove their approximation capability. In Section 3, we apply the new approximators to the disturbance rejection in adaptive control system with periodic disturbances, and give the stability analysis. In Section 4, a simulation example is provided to illustrate the effectiveness of the proposed control schemes. In Section 5, we conclude the work of this paper.

Throughout this paper, $\|\cdot\|$ denote Euclidean norm of a vector or its induced matrix norm; $\mathrm{tr}\{\cdot\}$ represents the trace operator; $\|B\|_F$ denotes the Frobenius norm, i.e., for a given matrix $B=[b_{i,j}]\in R^{m\times n}$, $\|B\|_F=\mathrm{tr}\{B^{\mathrm T}B\}$; $|A|_1=\sum_{i=1}^m |a_i|$ with $A=[a_1,a_2,\ldots,a_m]^{\mathrm T}\in R^m$, and $\lambda_{\max}(C)$ and $\lambda_{\min}(C)$ denote the largest and smallest eigenvalues of a square matrix C, respectively.

2. New approximators for periodically disturbed functions

In this section, we will consider the approximation problem of an unknown function affected by periodically time-varying disturbance

$$y=h(\chi,\theta(t)), \tag{1}$$

where $h: R^l\times R^m\to R$ is an unknown and continuous function; $\chi=[\chi_1,\ldots,\chi_l]^{\mathrm T}\in\Omega\subset R^l$ denotes the measured input vector with Ω being a compact set, and $y\in R$ denotes the output variable; $\theta(t)=[\theta_1(t),\theta_2(t),\ldots,\theta_m(t)]^{\mathrm T}\in R^m$ is a continuously and periodically time-varying disturbance with known period T, i.e., $\theta(t+T)=\theta(t)$, which is assumed to be unmeasured. The design objective is to construct novel function approximators to model the unknown function $h(\chi,\theta(t))$.

In fact, the main design obstacle is that the time-varying disturbance vector $\theta(t)$ is unmeasured. To overcome this obstacle, considering the period property of $\theta(t)$, each component of $\theta(t)$, i.e., $\theta_i(t)$, $1\le i\le m$, can be also modelled by a linearly parameterized FSE as follows [23]:

$$\theta_i(t)=\Psi_i^{\mathrm T}\Phi(t)+\varsigma_{\theta_i}(t),\quad |\varsigma_{\theta_i}(t)|\le\bar\xi_{\theta_i}, \tag{2}$$

where $\Psi_i=[\psi_{i,1},\ldots,\psi_{i,r}]^{\mathrm T}\in R^r$ is a vector including the first r coefficients of the FSE of $\theta_i(t)$ (r is an odd integer), $\varsigma_{\theta_i}(t)$ is the truncation error with the minimum upper bound $\bar\xi_{\theta_i}>0$, which can be arbitrarily decreased by increasing r, and $\Phi(t)=[\phi_1(t),\ldots,\phi_r(t)]^{\mathrm T}$ with $\phi_1(t)=1$, $\phi_{2j}(t)=\sqrt{2}\sin(2\pi jt/T)$ and $\phi_{2j+1}(t)=\sqrt{2}\cos(2\pi jt/T)$, $j=1,\ldots,(r-1)/2$. Then, we have

$$\theta(t)=\begin{bmatrix}\theta_1(t)\\ \vdots\\ \theta_m(t)\end{bmatrix}=\begin{bmatrix}\Psi_1^{\mathrm T}\\ \vdots\\ \Psi_m^{\mathrm T}\end{bmatrix}\Phi(t)+\begin{bmatrix}\varsigma_{\theta_1}(t)\\ \vdots\\ \varsigma_{\theta_m}(t)\end{bmatrix}=:\Psi^{\mathrm T}\Phi(t)+\varsigma_\theta(t), \tag{3}$$

where $\|\varsigma_\theta(t)\|\le\bar\xi_\theta$ with $\bar\xi_\theta=\sqrt{\bar\xi_{\theta_1}^2+\cdots+\bar\xi_{\theta_m}^2}$. It is obvious that $\bar\xi_\theta$ is also arbitrarily decreased by increasing r.

2.1. Construction of FSE-MNN-based approximator

In this subsection, by combining FSE into MNN, we will construct an FSE-MNN-based approximator to solve the approximation problem of unknown function (1).

On the one hand, when the disturbance $\theta(t)$ is known, the three-layer NN [2], as a representative of MNN, can be used to approximate the function (1) as follows:

$$h(\chi,\theta(t))=W^{\mathrm T}S(V^{\mathrm T}Z)+\delta_h(\chi,\theta(t)),\quad |\delta_h(\chi,\theta(t))|\le\bar\delta_h, \tag{4}$$

where $Z=[\chi^{\mathrm T},\theta^{\mathrm T}(t),1]^{\mathrm T}\in R^{l+m+1}$ is the NN input vector; $W=[w_1,w_2,\ldots,w_p]^{\mathrm T}\in R^p$ is the first-to-second layer weight with the NN node number $p>1$;

$$V=[V_\chi^{\mathrm T},V_\theta^{\mathrm T},V_0]^{\mathrm T}\in R^{(l+m+1)\times(p-1)}$$

is the second-to-third layer weight with

$$V_\chi=[v_{\chi,1},v_{\chi,2},\ldots,v_{\chi,p-1}]\in R^{l\times(p-1)},$$
$$V_\theta=[v_{\theta,1},v_{\theta,2},\ldots,v_{\theta,p-1}]\in R^{m\times(p-1)},$$
$$V_0=[v_{0,1},v_{0,2},\ldots,v_{0,p-1}]^{\mathrm T}\in R^{p-1},$$
$$S(V^{\mathrm T}Z)=[s(v_1^{\mathrm T}Z),s(v_2^{\mathrm T}Z),\ldots,s(v_{p-1}^{\mathrm T}Z),1]^{\mathrm T},$$

where $s(\star)=1/(1+e^{-\gamma\star})$, $\gamma>0$ and $v_i=[v_{\chi,i}^{\mathrm T},v_{\theta,i}^{\mathrm T},v_{0,i}]^{\mathrm T}$ $(1\le i\le p-1)$; $\delta_h(\chi,\theta(t))$ is the inherent NN approximation error with the minimum upper bound $\bar\delta_h\ge 0$, which can be arbitrarily decreased by increasing the NN node number p.

Now, we begin to construct a novel approximator for the periodically disturbed function $h(\chi,\theta(t))$. Note $V^{\mathrm T}Z=V_\chi^{\mathrm T}\chi+V_\theta^{\mathrm T}\theta(t)+V_0$, in which by replacing the unknown periodically time-varying vector $\theta(t)$ with (3), we get

$$V^{\mathrm T}Z=V_\chi^{\mathrm T}\chi+V_\theta^{\mathrm T}\Psi^{\mathrm T}\Phi(t)+V_0+V_\theta^{\mathrm T}\varsigma_\theta(t)=U^{\mathrm T}\bar Z(\chi,t)+V_\theta^{\mathrm T}\varsigma_\theta(t), \tag{5}$$

where $U=[U_1^{\mathrm T},U_2^{\mathrm T},\ldots,U_{p-1}^{\mathrm T}]^{\mathrm T}=[V_\chi^{\mathrm T},V_\theta^{\mathrm T}\Psi^{\mathrm T},V_0]^{\mathrm T}$ and $\bar Z(\chi,t)=[\chi^{\mathrm T},\Phi^{\mathrm T}(t),1]^{\mathrm T}$. Substituting (5) into (4) yields

$$h(\chi,\theta(t))=W^{\mathrm T}S[U^{\mathrm T}\bar Z(\chi,t)+V_\theta^{\mathrm T}\varsigma_\theta(t)]+\delta_h(\chi,\theta(t)). \tag{6}$$

Remark 1. In (5), it seems that the state variable χ can be separated from the unknown periodic disturbance $\theta(t)$. This is because χ and $\theta(t)$ are two independent input variables of $f(\chi,\theta(t))$, so the structure property of MNN allows Eq. (5) holds (see [2,8]), but this does not mean that $\theta(t)$ can be separated from the unknown function $f(\chi,\theta(t))$. In fact, $\theta(t)$ still nonlinearly appears in MNN, which can be easily seen from (6), where $S(\cdot)$ is still a nonlinear function.