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The optimal Mexican hat wavelet filter de-noising method based on cross-validation method

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A R T I C L E I N F O

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A B S T R A C T

A new de-noising method based on parameter optimized Mexican hat wavelet was put forward in this paper. For the similar shape to the mechanical shock vibration signal, the Mexican hat wavelet is chosen as the mother wavelet and improved by the shape parameters optimization. The noise jamming in the raw vibration signals can be filtered by the continue wavelet transform (CWT) using the improved Mexican hat wavelet as the mother wavelet. The shape parameters of the Mexican hat wavelet are optimized by the cross-validation method (CVM). In the CWT process, the optimal scale factor is also obtained by the circle CVM. The useful components can be extracted by the CWT with the optimal shape parameters and scale factor. The experimental result shows that the proposed method can not only de-noise the useless noise effectively but also extract the fault feature availably.

1. Introduction

In mechanical condition monitoring and fault diagnosis, transient signals always contain important information of the monitored objects. Vibration analysis is the best-known technology applied in mechanical condition monitoring, especially for rotating equipments. Signals under considerations are known to be non-stationary, for which the signal parameters are time-varying [1,2,3]. For spectral analysis of such type signals, the time–frequency analysis techniques have been widely used [4,5]. However, if the signal feature components' energy is low, the amplitude of the interfering noise will be higher than the signal feature components, and at the same time the frequency spectrum of the noise and signal will be mixed together. In this situation, traditional time–frequency analysis methods cannot separate the useful signal features from the noise jamming effectively. Therefore, it is important to pre-process raw signals before analysis since the raw data contain some redundant information.

Wavelet analysis, which is the most popular one for non-stationary signal analysis, overcomes the drawbacks of other techniques by means of analytical functions that are local in both time and frequency. Wavelet transform (WT) method is widely used in mechanical signal de-noising process [6,7,8]. However, traditional wavelet de-noising method has some difficulties in the analysis process, such as the selection of the mother wavelet function, the decomposition level of signal, the order of the mother wavelet function, etc. Some papers used wavelet de-noising methods in mechanical fault de-noising and feature extraction, but had not given some useful theoretical methods to ensure the decomposition level or the order of mother wavelet function [9,10]. Most researchers used comparison methods to choose some optimal mother wavelet functions and decomposition level. In this situation, it is a waste of time and energy to do a lot of contrastive experiments [11]. Wavelet transform can be regarded as the inner product of a time domain signal with the translated wavelet-base function. Therefore, the WT can be regarded as the filter process to the signal, by which the noise in the signal can be de-noised effectively. The WT resulting coefficients reflect the different features of the noise and the useful signal, which can be separated by the filter process. Therefore, by choosing suitable wavelet function and taking the continue wavelet transform (CWT) process, it is feasible to de-noise the interfering noise in the raw signals and keep the effective feature components.

This paper chooses Mexican hat wavelet, which is in shape similar to the mechanical shocking signal, as the mother wavelet function in CWT process. The parameters of the improved Mexican hat wavelet and the wavelet transform scale factor are optimized by the cross-validation method (CVM). The paper is structured as follows. An introduction is given in Section 1. The principle of the Mexican hat wavelet filter de-noising method is discussed in Section 2. The parameters of the improved Mexican hat wavelet are

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optimized in Section 3. The application of the proposed de-noising method in the gear fault experiment is presented in Section 4. And some conclusions are drawn in Section 5.

2. Principle of Mexican hat filter de-noising

The wavelet analysis results are series of wavelet coefficients, which indicate the comparability between the signal and the particular wavelet. In order to extract the fault features of the signal more effectively, an appropriate wavelet base function should be selected [12–14]. The corresponding wavelet family consists of a series of daughter wavelets, which are generated by dilation and translation operations from the mother wavelet and shown as follows:

$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right) \; a,b \in R, \quad a \neq 0 \tag{1}$$

where the parameter a is the scale factor, b denotes the time location, which is used to keep energy preservation.

For any function $f(t) \in L^2(R)$, its WT can be expressed as

$$W_f(a,b) = |a|^{-1/2} \int_{-\infty}^{+\infty} f(t)\psi * \left(\frac{t-b}{a}\right) dt \tag{2}$$

where $*$ is the conjugate of the function.

In particular, the WT can be regarded as the inner product of the signal and mother wavelet function. Therefore, it can be deduced that

$$W(a,b) = \sqrt{a}F^{-1}\{X(f)\psi^*(af)\} \tag{3}$$

where $X(f)$is the Fourier transform of signal $x(t)$, $\psi^*(af)$ is the Fourier transform of the wavelet base $\psi^*(at)$, F^{-1} is the inverse Fourier transform.

The above function can be regarded as a filter process. In other words, the WT of the signal can be regarded as the filter process passing the band-pass filter whose frequency response is $\psi^*(af)$. Therefore, it is feasible to filter de-noising the signal by choosing suitable mother wavelet function to do CWT.

The wavelet transform resulting coefficients reflect the correlation between the signal and the selected wavelet base function. To increase the amplitude of the generated wavelet coefficients related to the fault impulses and enhance the fault detection process, the selected wavelet base function should be similar to the mechanical shock response in characteristics [3–5,11–14]. From the time domain wavelet of the mechanical shock signal, we notice that the Mexican hat wavelet is similar to the mechanical impulse signal [15–19]. Therefore, wavelet de-noising using a Mexican hat wavelet as a base function can be used to extract the impulses for mechanical faults detection.

Mexican hat wavelet can also be called Marr wavelet is the second derivative of the Gaussian function. If we define the Gaussian function as $\theta(x)$, then we can obtain

$$\theta(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x^2/2\sigma)} \tag{4}$$

The wavelet function of the Mexican hat wavelet can be derived as

$$\psi(x) = \frac{d^2\theta(x)}{dx^2} = \frac{2}{\sqrt{3\sqrt{\pi}}}(1-x^2)e^{-(x^2/2)} \tag{5}$$

The time domain wavelet is shown in Fig. 1(a). Set the shift factor $b=0$, by changing the scale factor a, we can obtain the filter characteristic of the Mexican hat wavelet under different factors, as shown in Fig. 1(b).

As shown in Fig. 1, the time domain characteristics of Mexican hat wavelet are exponential decay, incompact support, good time–frequency localization and symmetry at the zero point.

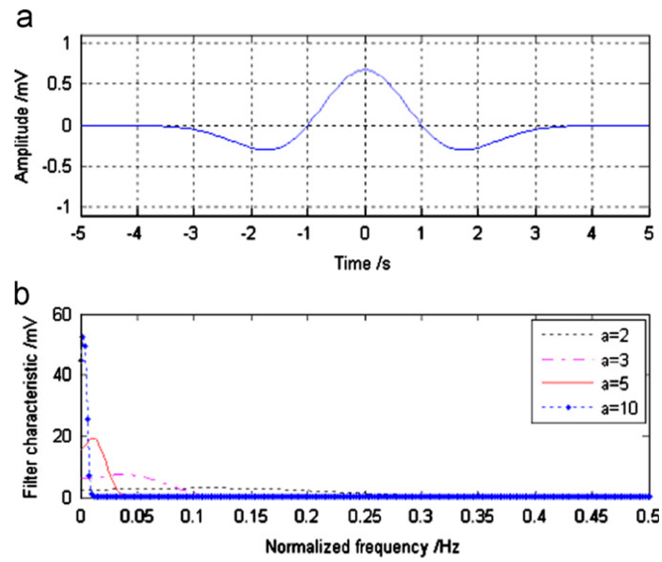

Fig. 1. Mexican hat wavelet: (a) time domain wavelet; (b) filter characteristic in frequency domain.

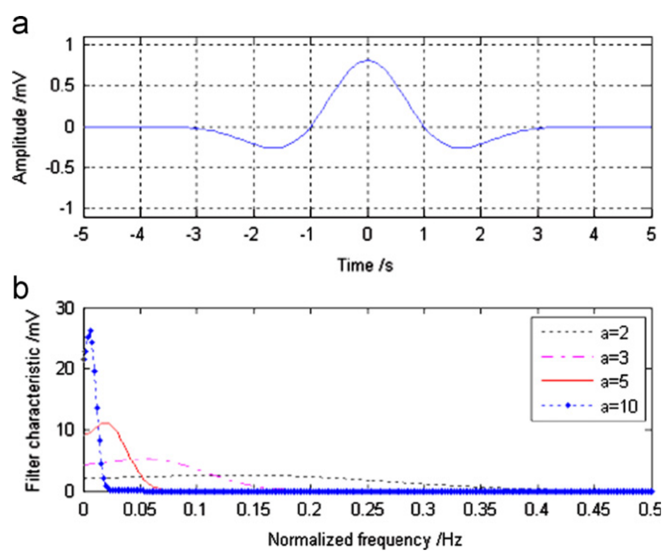

Fig. 2. Mexican hat wavelet when $m=0.8$, $n=0.6$, $b=0$: (a) time domain wavelet; (b) frequency filter characteristic.

Fig. 1(b) shows the obvious band-pass filter characteristic of Mexican hat wavelet. Compared to other low-pass filters, Mexican hat wavelet is much more suitable to extract the middle frequency signal. We can obtain different band-pass filters by changing the scale factor a. However, the wavelet shape in formula (1) is fixed and cannot fit the fast decay signal or big amplitude signal. The analysis affect cannot be ideal in this manner. Some papers adjust the mother wavelet shape by improving some parameters, but only limited to the inner small changing in the exponential function. In order to expand the application of the Mexican hat wavelet, we improve the parameters in the base of formula (1) and increase two parameters m and n, finally obtain the improved Mexican hat wavelet as

$$\psi(x) = m(1-x^2)e^{-nx^2} \tag{6}$$

where m and n are the wavelet shape adjusting parameters.

Select $m=0.8$, $n=0.6$, $b=0$, by changing the scale factor a, we can obtain the wavelet and filter shape of Mexican hat wavelet, as shown in Fig. 2.