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Global exponential stability of cellular neural networks with time-varying discrete and distributed delays

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A B S T R A C T

This paper is concerned with the problem of global exponential stability analysis for a class of cellular neural networks with time-varying discrete and distributed delays (DDCNNs). A new delay-dependent sufficient condition is derived for the global exponential stability of the DDCNNs by using the integral inequality method and the newly proposed Lyapunov–Krasovskii functional. The obtained stability condition is less conservative than some of the existing results in the literature. Numerical examples are given to demonstrate the effectiveness and superiority of the proposed results.

1. Introduction

An artificial neural network (ANN) is a network of interconnected elements which behave like biological neurons. A neuron can be mathematically described by a system of differential equations or difference equations. For each single neuron, the structure may be simple, but once some of them are connected together to form a network, the whole network could have very rich dynamics and thus can admit many applications [15]. These ANNs can either be realized via electronic circuits or be simulated by computer program. During the past decade, considerable attention was devoted to the study of ANNs, and numerous useful results have been presented in the literature. The applications of ANNs can nowadays been found in a variety of areas, such as electron science, computer science, control engineering, mathematics, biology, etc. One of the commonly used ANNs is the cellular neural network (CNN), which has been successfully applied in many areas, such as signal processing, image processing and solving nonlinear algebraic equations.

The practical applicability of CNN depends on the existence and stability of the equilibrium point of the CNN. Moreover, time delay occurs due to the finite switching speed of electronic networks. During the implementation on very large scale integrated chips, parameter perturbations and transmitting delays will degrade the system performance and even cause the instability of the neural networks. Therefore, the stability study of delayed cellular neural networks (DCNNs) is of great theoretical and practical significance, and it has become an important topic of theoretical studies. Existing criteria for the stability of DCNNs can be classified into two categories, namely, delay-independent criteria [1,3,4,10,11,16,17] and delay-dependent criteria [2,5,7,8,13,18]. The former does not contain any information on the size of delay while the latter employs such information. It is generally recognized that the delay-dependent criteria are less conservative than the delay-independent ones, especially when the delays are small. Therefore, more attention has been paid to the delay-dependent stability analysis. DCNNs with distributed time delays (DDCNNs) is another important type of DCNNs, and a lot of asymptotic stability results for such DDCNNs have been presented in the literature [9,14]. Except for stability, fast convergence is also essential for real-time computations in NNs. It is commonly known that exponential stability does not only guarantee the asymptotic stability but also certain convergence rate for the NNs. The exponential stability property is particularly important when the exponential-convergence rate is used to determine the speed of neural computations. Therefore, it is not only theoretically interesting but also practically important to determine the exponential stability and to estimate the exponential-convergence rate for the dynamical NNs. However, only a few results in the existing literature are concerned with the exponential stability of the DDCNNs, and among these results only delay-independent stability conditions were presented in [12] for the global exponential stability of the DDCNNs. To the best of the authors' knowledge, the delay-dependent exponential stability of

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the DDCNNs has not yet been well investigated. This motivates the present research.

In this paper, the problem of delay-dependent global exponential stability is studied for a class of DDCNNs. Delay-dependent sufficient conditions are derived for such NNs by using a newly proposed Lyapunov–Krasovskii functional and the integral inequality method. The conditions are presented in terms of LMIs, and thus can be solved by many efficient convex optimization algorithms. Moreover, the obtained stability conditions are less conservative than some of the existing results in the literature. Numerical examples are finally given to demonstrate the effectiveness of the proposed results.

2. Preliminary

Consider the following CNN with time-varying discrete and distributed delays

$$\dot{x}(t) = -Ax(t) + W_0 f(x(t)) + W_1 f(x(t-h(t))) + W_2\int_{t-\tau(t)}^{t} f(x(s))\mathrm{d}s + J, \tag{1}$$

where $x(t)=[x_1(t),\ldots,x_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$ corresponds to the state vector at time, $J=[J_1,\ldots,J_n]^{\mathrm{T}}\in\mathbb{R}^n$ is the external bias vector, $f(x(\cdot))=[f_1(x_1(\cdot)),\ldots,f_n(x_n(\cdot))]^{\mathrm{T}}\in\mathbb{R}^n$ denotes the activation function of the neurons with $f_i(0)=0$, $h(t)$ and $\tau(t)$ represent time-varying discrete and distributed delays of systems with $0\leqslant h(t)\leqslant h_M$, $\dot{h}(t)\leqslant h_D$, $0\leqslant\tau(t)\leqslant\tau_M$. The matrices $A=\mathrm{diag}[a_i]$, $a_i>0$, $W_j\in\mathbb{R}^{n\times n}$, $j=0,1,2$ are some constant matrices. The activation functions $f_i(x_i(\cdot))$, $i=1,2,\ldots,n$, are bounded monotonically non-decreasing and satisfy

$$0\leqslant\frac{f_i(\xi_1)-f_i(\xi_2)}{\xi_1-\xi_2}\leqslant\eta_i, \tag{2}$$

where $\xi_2,\xi_2\in\mathbb{R}$, $\xi_1\neq\xi_2$, $\eta_i>0$, $i=1,2,\ldots,n$ are given constants. Denote $\Gamma=\mathrm{diag}\{\eta_1,\ldots,\eta_n\}\in\mathbb{R}^{n\times n}$.

Assume that $\tilde{x}=[\tilde{x}_1,\ldots,\tilde{x}_n]^{\mathrm{T}}\in\mathbb{R}^n$ is an equilibrium point of system (1), one can derive from (1) that the transformation $z(t)=x(t)-\tilde{x}(t)$ transforms system (1) into the following system:

$$\dot{z}(t) = -Az(t) + W_0 g(z(t)) + W_1 g(z(t-h(t))) + W_2\int_{t-\tau(t)}^{t} g(z(s))\mathrm{d}s, \tag{3}$$

where

$$\begin{aligned} z(t) &= [z_1(t)\, z_2(t)\ldots z_n(t)]^T\in\mathbb{R}^n,\\ g(z(\cdot)) &= [g_1(z_1(\cdot))\, g_2(z_2(\cdot))\ldots g_n(z_n(\cdot))]^T\in\mathbb{R}^n,\\ g_i(z_i(\cdot)) &= f_i(x_i(\cdot))-f_i(\tilde{x}_i)=f_i(z_i(\cdot)+\tilde{x}_i)-f_i(\tilde{x}_i),\\ g_i(0) &= 0,\quad i=1,2,\ldots,n. \end{aligned} \tag{4}$$

The initial condition is $z(s)=\phi(s)$, $s\in[-2\tau^*,0]$, $\tau^*\in\max\{h_M,\tau_M\}$.

It follows from (2) and (4) that each $g_i(\cdot)$ satisfies

$$\begin{aligned} &0\leqslant\frac{g_i(\zeta_1)-g_i(\zeta_2)}{\zeta_1-\zeta_2}=\frac{f_i(\zeta_1+\tilde{x}_i)-f_i(\tilde{x}_i)-[f_i(\zeta_2+\tilde{x}_i)-f_i(\tilde{x}_i)]}{(\zeta_1+\tilde{x}_i)-(\zeta_2+\tilde{x}_i)}\leqslant\eta_i,\\ &\zeta_1\neq\zeta_2,\ |g_i z_i(\cdot))|=|f_i(z_i(\cdot)+\tilde{x}_i)-f_i(\tilde{x}_i)|\\ &\quad\leqslant\eta_i|z_i(\cdot)|,\quad i=1,2,\ldots,n. \end{aligned} \tag{5}$$

In the derivation of the main results, we need the following definitions and lemmas:

Definition 1. The equilibrium point $\tilde{x}=[\tilde{x}_1,\ldots,\tilde{x}_n]^{\mathrm{T}}\in\mathbb{R}^n$ of system (1) is said to be global exponentially stable, if there exist some constants $k>0$ and $m\geqslant 1$ such that

$$\|x(t)-\tilde{x}\|\leqslant m\phi\mathrm{e}^{-kt},\quad\forall t\geqslant 0,$$

where $\phi=\sup_{-\tilde{\tau}\leqslant\theta\leqslant 0}\|x(\theta)-\tilde{x}\|$ and $\|x\|$ is the Euclidean norm of x, and k is called the exponential-convergence rate.

Lemma 1 (*Zhang et al. [19]*). *Let $z(t)\in\mathbb{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for matrix $R_1=R_1T>0$ and any matrices M_1, M_2, and a scalar function $h_M:=h_M(t)\geqslant 0$*

$$-\int_{t-h_M}^{\tau}\dot{z}^{\mathrm{T}}(s)R_1\dot{z}(s)\mathrm{d}s\leqslant\zeta^{\mathrm{T}}(t)\begin{bmatrix} M_1^{\mathrm{T}}+M_1 & -M_1^{\mathrm{T}}+M_2\\ * & -M_2^{\mathrm{T}}-M_2\end{bmatrix}\zeta(t) + h_M\zeta^{\mathrm{T}}(t)Y^{\mathrm{T}}R_1^{-1}Y\zeta(t), \tag{6}$$

where $Y=[M_1\ M_2]\in\mathbb{R}^{n\times 2n}$ and $\zeta(t)=[z(t)\ z(t{-}h_m)]^{\mathrm{T}}$.

Lemma 2 (*Gu [6]*). *For any constant matrix $\Sigma\in\mathbb{R}^{n\times n}$, $\Sigma=\Sigma^{\mathrm{T}}>0$, scalar $\gamma>0$, vector function $\omega:[0,\gamma]\to\mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^{\gamma}\omega(s)\mathrm{d}s\right)^{\mathrm{T}}\sum\left(\int_0^{\gamma}\omega(s)\mathrm{d}s\right)\leqslant\gamma\int_0^{\gamma}\omega^{\mathrm{T}}(s)\sum\omega(s)\mathrm{d}s. \tag{7}$$

3. Main results

In this section, a new Lyapunov–Krasovskii functional is used to analyze the exponential stability of the equilibrium point of system (1), and the main result is given in the following theorem.

Theorem 1. *The equilibrium point $\tilde{x}$ of system (1) with $h_D<1$ is globally exponentially stable with convergence rate $k>0$, if there exist symmetric positive definite matrices P_0, P_1, P_2, R_1 and R_2, positive diagonal matrices D, Q and S, and any matrices M_1 and M_2, such that the following LMI*

$$\Theta=\begin{bmatrix}\sum & h_M\mathrm{e}^{-kh_M}M^{\mathrm{T}}\\ * & -R_1\end{bmatrix}<0, \tag{8}$$

is satisfied, where

$$M=\begin{bmatrix}M_1 & M_2 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Sigma=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & 0\\ * & \Sigma_{22} & 0 & \Sigma_{24} & 0 & 0\\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & 0\\ * & * & * & \Sigma_{44} & \Sigma_{45} & 0\\ * & * & * & * & \Sigma_{55} & 0\\ * & * & * & * & * & \Sigma_{66}\end{bmatrix},$$

$$\begin{aligned}
\Sigma_{11} &= 2kP_0+P_1+h_M\mathrm{e}^{-2kh_M}(M_1^{\mathrm{T}}+M_1)-(P_0A+A^{\mathrm{T}}P_0)+h_M^2A^{\mathrm{T}}R_1A,\\
\Sigma_{12} &= h_M\mathrm{e}^{-2kh_M}(-M_1^{\mathrm{T}}+M_2),\\
\Sigma_{13} &= P_0W_0+2kD+\Gamma S-AD-h_M^2A^{\mathrm{T}}R_1W_0,\\
\Sigma_{14} &= P_0W_1-h_M^2A^{\mathrm{T}}R_1W_1,\\
\Sigma_{15} &= P_0W_2-h_M^2A^{\mathrm{T}}R_1W_2,\\
\Sigma_{22} &= -(1-h_D)\mathrm{e}^{-2kh_M}P_1+h_M\mathrm{e}^{-2kh_M}(-M_2^{\mathrm{T}}-M_2),\\
\Sigma_{24} &= \Gamma Q,\\
\Sigma_{33} &= h_MP_2+DW_0+W_0^{\mathrm{T}}D+h_M^2W_0^{\mathrm{T}}R_1W_0+\tau_M^2R_2-2S,\\
\Sigma_{34} &= DW_1+h_M^2W_0^{\mathrm{T}}R_1W_1,\\
\Sigma_{35} &= DW_2+h_M^2W_0^{\mathrm{T}}R_1W_2,\\
\Sigma_{44} &= h_M^2W_1^{\mathrm{T}}R_1W_1-2Q,\\
\Sigma_{45} &= h_M^2W_1^{\mathrm{T}}R_1W_2,\\
\Sigma_{55} &= h_M^2W_2^{\mathrm{T}}R_1W_2-\mathrm{e}^{-2k\tau_M}R_2,\\
\Sigma_{66} &= -h_M^{-1}\mathrm{e}^{-2kh_M}P_2.
\end{aligned}$$