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On the exponential synchronization of stochastic jumping chaotic neural networks with mixed delays and sector-bounded non-linearities ☆

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ABSTRACT

This paper is concerned with the problem of exponential synchronization for stochastic jumping chaotic neural networks (SJCNNs) with mixed delays and sector non-linearities. Based on Lyapunov–Krasovskii functional and free-weighting matrix method, a delay-dependent feedback controller with sector non-linearities is proposed to achieve the synchronization in mean square in terms of linear matrix inequalities (LMIs). The activation functions are assumed to be of more general descriptions. Finally, the corresponding simulation results show the effectiveness of the proposed criteria.

1. Introduction

It is widely believed that there exist many benefits of having synchronization or chaos synchronization in many fields, such as secure communication, modeling brain activity and pattern recognition phenomena [3,18,17,32,1,21,9]. Specially, as special complex networks, it has been shown that synchronization of chaotic neural networks (CNNs) has been a hot topic nowadays. With respect to some recent representative works on this topic, we refer the reader to [31,2,11,20,33,19,16,12,23,22] and references therein.

Recently, neural networks with stochastic perturbations or distributed delays have received much attention [18,17,22–27,29,12–15,4,10,6]. In [33], synchronization control of stochastic neural networks with time-varying delays has been considered. In [12], some sufficient criteria for synchronization of CNNs with distributed delay have been proposed by linear matrix inequalities (LMIs).

On the other hand, recently, it has been found that the jumping between different neural networks modes can be governed by a Markovian chain. Therefore, neural networks with Markovian jumping parameters are of great significance in modeling neural networks with finite network modes. Several works have been published concerning the stability analysis of the neural networks with Markovian jumping parameters [15,28]. However, it is worth pointing out that, up to now, the synchronization problem for CNNs with Markovian jumping has arrested little research attention, despite its practical importance.

Furthermore, in many practical situations, the control inputs are frequently subject to non-linearity due to physical limitations. It has been shown that input non-linearity can lead to a serious degradation of the system performance, and in a worst-case scenario, system failure if the controller is not well designed [31,30,5,8]. Therefore, it is obvious that the influence of input non-linearity should be taken into consideration when implementing a synchronization control scheme. In [31], the synchronization of CNNs with sector non-linearities has been considered. However, the methods are not presented in terms of LMIs, which makes their checking by the developed algorithms somewhat difficult and inconvenient. To the best of our knowledge, the problem of delay-dependent exponential synchronization for stochastic jumping chaotic neural networks (SJCNNs) with mixed time delays and sector non-linearities is available in the literature, which remains open and challenging.

Motivated by the above discussion, in this paper, we investigate the problem of exponential synchronization for SJCNNs with mixed time delays and sector non-linearities. Based on free-weighting matrix method and exponential synchronization

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criteria in mean square are presented in terms of LMIs. A numerical example shows that the results developed in this paper are feasible.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all real matrices. The superscript 'T' denotes matrix transposition and the notation $X\geqslant Y$ (respectively, $X>Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). $\lambda_{\max}(\cdot)$ or $\lambda_{\min}(\cdot)$ denotes the largest or smallest eigenvalue of a matrix, respectively. Let $h>0$ and $C([-h,0;\mathbb{R}^n])$ denote the family of continuous functions ϕ from $[-h,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|=\sup_{-h\leqslant\theta\leqslant 0}|\phi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$; $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator. Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geqslant 0},P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L^p_{\mathscr{F}_0}([-h,0];\mathbb{R}^n)$ the family of all $\mathscr{F}_0$-measurable $C([-h,0];\mathbb{R}^n)$-valued random variables $\xi=\{\xi(\theta):-h\leqslant\theta\leqslant 0\}$ such that $\sup_{-h\leqslant\theta\leqslant 0}\mathbb{E}|\xi(\theta)|^p<\infty$. I is an identity matrix; $*$ represents a block that is readily inferred by symmetry.

2. Problem formulation

Let $\{r(t),t\geqslant 0\}$ be a right-continuous Markov chain on the probability space taking values in a finite state space $\mathscr{S}=\{1,2,\ldots,N\}$ with following transition probabilities:

$$P\{r(t+\Delta t)=j:r(t)=i\}=\begin{cases}\gamma_{ij}\Delta t+O(\Delta t) & \text{if } i\neq j,\\ 1+\gamma_{ij}\Delta t+O(\Delta t) & \text{if } i=j,\end{cases}$$

where $\Delta t>0$ and $\lim_{\Delta t\to 0}O(\Delta t)/\Delta t=0$. Here, $\gamma_{ij}\geqslant 0$ is the transition rate from i to j if $i\neq j$, while $\gamma_{ii}=-\sum_{j=1,j\neq i}^{N}\gamma_{ij}$.

Consider, on a probability space $\{\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geqslant 0},P\}$, the following SJCNNs with mixed delays and Markovian switching of the form:

$$\begin{aligned}dx(t)=&\Bigg[-C(r(t))x(t)+A(r(t))\tilde{f}(x(t))+B(r(t))\tilde{g}(x(t-\tau(t)))\\&+D(r(t))\int_{t-\mu(t)}^{t}\tilde{h}(x(s))\,ds+I(t)\Bigg]dt\\&+[E(r(t))x(t)+F(r(t))x(t-\tau(t))]\,d\omega(t),\end{aligned}\tag{1}$$

$$x(t)=\phi(t),t\in[-\bar{\tau},0],\bar{\tau}=\max\{\tau_m,\mu_m\},\tag{2}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ is the state vector associated with the neurons; $C(r(t))$ is a diagonal matrix representing self-feedback term; $A(r(t))$, $B(r(t))$ and $D(r(t))$ denote the connection weight matrix, the discrete time-varying delay connection weight matrix and distributed time-varying connection weight matrix, respectively; $E(r(t))$ and $F(r(t))$ are known constant matrices with appropriate dimensions; $I(t)=[I_1(t),I_2(t),\ldots,I_n(t)]^{\mathrm{T}}$ is an external input vector; for a fixed mode $r(t)\in\mathscr{S}$; $\omega(t)$ is a one-dimensional Brownian motion defined on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geqslant 0},P)$ satisfying $\mathbb{E}\{d\omega(t)\}=0$ and $\mathbb{E}\{[d\omega(t)]^2\}=dt$; $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ are the activation functions of neurons, where

$$\tilde{f}(x(\cdot))=(\tilde{f}_1(x_1(\cdot)),\tilde{f}_2(x_2(\cdot)),\ldots,\tilde{f}_n(x_n(\cdot)))^{\mathrm{T}}\in\mathbb{R}^n,$$

$$\tilde{g}(x(\cdot))=(\tilde{g}_1(x_1(\cdot)),\tilde{g}_2(x_2(\cdot)),\ldots,\tilde{g}_n(x_n(\cdot)))^{\mathrm{T}}\in\mathbb{R}^n,$$

$$\tilde{h}(x(\cdot))=(\tilde{h}_1(x_1(\cdot)),\tilde{h}_2(x_2(\cdot)),\ldots,\tilde{h}_n(x_n(\cdot)))^{\mathrm{T}}\in\mathbb{R}^n.$$

$\tau(t)$ and $\mu(t)$ is the discrete and distributed time-varying delay satisfying

$$0<\tau(t)\leqslant\tau_m,\quad \dot{\tau}(t)\leqslant\upsilon<1,\quad 0<\mu(t)\leqslant\mu_m,\tag{3}$$

where τ_m, υ and μ_m are constants.

The response system can be given as follows:

$$\begin{aligned}dy(t)=&\Bigg[-C(r(t))y(t)+A(r(t))\tilde{f}(y(t))+B(r(t))\tilde{g}(y(t-\tau(t)))\\&+D(r(t))\int_{t-\mu(t)}^{t}\tilde{h}(y(s))\,ds+I(t)+K(r(t))\psi(e(t))\\&+K_d(r(t))\psi(e(t-\tau(t)))\Bigg]dt+[E(r(t))y(t)\\&+F(r(t))y(t-\tau(t))]\,d\omega(t),\end{aligned}\tag{4}$$

$$y(t)=\xi(t),t\in[-\bar{\tau},0],\bar{\tau}=\max\{\tau_m,\mu_m\},\tag{5}$$

where $C(r(t)),A(r(t)),B(r(t)),D(r(t)),E(r(t)),F(r(t))$ are matrices which are the same as (1); $\psi(\cdot)$ represents the sector non-linearity, $K(r(t))$ and $K_d(r(t))$ are the control gain.

Note that the set $\mathscr{S}$ consists of different operation modes of systems (1) and (4) for each possible values of $r(t)=i,i\in\mathscr{S}$. For the sake of simplicity, we denote the matrix associated with the ith mode by

$$\Gamma_i\triangleq\Gamma(r(t)=i),$$

where the matrix Γ could be $C,A,B,D,E,F,P,J,J_d,K,K_d,U,V,W,N,M,\varepsilon_1,\varepsilon_2$.

Let the error states be $e(t)=y(t)-x(t)$. Then, subtracting (1) from (4), yields the error dynamical system as follows:

$$\begin{aligned}de(t)=&\Bigg[-C_ie(t)+A_if(e(t))+B_ig(e(t-\tau(t)))\\&+D_i\int_{t-\mu(t)}^{t}h(e(s))\,ds+K_i\psi(e(t))\\&+K_{di}\psi(e(t-\tau(t)))\Bigg]dt+[E_ie(t)+F_ie(t-\tau(t))]\,d\omega(t),\end{aligned}\tag{6}$$

where

$$f(e(t))=[f_1(e_1(t)),f_2(e_2(t)),\ldots,f_n(e_n(t))]:=\tilde{f}(y(t))-\tilde{f}(x(t)),$$

$$g(e(t))=[g_1(e_1(t)),g_2(e_2(t)),\ldots,g_n(e_n(t))]:=\tilde{g}(y(t))-\tilde{g}(x(t)),$$

$$h(e(t))=[h_1(e_1(t)),h_2(e_2(t)),\ldots,h_n(e_n(t))]:=\tilde{h}(y(t))-\tilde{h}(x(t)),$$

and $\varphi(t)=\xi(t)-\phi(t)$ is the initial condition of (6).

Remark 1. In many real applications, we are interested in designing a memoryless state-feedback controller $U(t)=K_ie(t)$, where $K_i\in\mathbb{R}^n$ is a constant gain matrix. Furthermore, the control inputs are frequently subject to non-linearity due to physical limitations. For a special case where the information on the size of $\tau(t)$ is available, the delayed feedback controller with sector non-linearity of the form $U(t)=K_i\psi(e(t))+K_{di}\psi(e(t-\tau(t)))$ is considered.

In the following, the following assumptions are needed:

(H1 see Refs. [29,13]) For $i\in\{1,2,\ldots,n\}$, the neuron activation functions in (1) and (4) satisfy

$$\begin{aligned}&\sigma_i^-\leqslant\frac{\tilde{f}_i(x)-\tilde{f}_i(y)}{x-y}\leqslant\sigma_i^+,\\&\delta_i^-\leqslant\frac{\tilde{g}_i(x)-\tilde{g}_i(y)}{x-y}\leqslant\delta_i^+,\\&\rho_i^-\leqslant\frac{\tilde{h}_i(x)-\tilde{h}_i(y)}{x-y}\leqslant\rho_i^+,\quad\forall\, x,\ y\in\mathbb{R}^n,\ x\neq y,\ i=1,2,\ldots,n,\end{aligned}\tag{7}$$

where $\sigma_i^-,\sigma_i^+,\delta_i^-,\delta_i^+,\rho_i^-,\rho_i^+$ are constants.

(H2) The non-linear function $\psi(\cdot)$ in stochastic error system (6) represents the sector non-linearities satisfying the following