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Letters

Mean square exponential stability of stochastic fuzzy Hopfield neural networks with discrete and distributed time-varying delays ☆

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ABSTRACT

It is well known that a complex nonlinear system can be represented as a Takagi–Sugeno (T–S) fuzzy model that consists of a set of linear sub-models. This paper is concerned with the problem of mean square exponential stability for a class of stochastic fuzzy Hopfield neural networks with discrete and distributed time-varying delays. By using the stochastic analysis approach and Itô differential formula, delay-dependent conditions ensuring the stability of the considered neural networks are obtained. The conditions are expressed in terms of linear matrix inequalities (LMIs) and can be easily checked by standard software. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Over the past decades, Hopfield neural networks [13] have been extensively studied because of their important applications in various fields such as combinatorial optimization, signal processing, image processing and pattern recognition problems [8]. Both in biological and artificial neural networks, the interactions between neurons are generally asynchronous, which inevitably results in time delays. It is known that time delays are often a source of instability of neural networks [2]. Therefore, considerable attention has been paid to the problem of stability analysis of neural networks with delays, and a lot of research results have been reported for the neural networks with various types of time delays, see for example, [3,4,9,19,22,23,32–34], and the references therein. In real nervous systems, stochastic disturbances are nearly inevitable and affect the stability of neural networks. The results in [21] showed one neural network could be stabilized or destabilized by certain stochastic inputs. It is shown that the stability analysis of stochastic neural networks has primary importance in the design and applications of neural networks. Recently, stability analysis of stochastic neural networks with time-delays has received much attention; see, for example, [6,14,16,18,25–27,30]. The problem of stability analysis for stochastic neural networks with discrete and distributed delays was investigated in [16,25,26].

Fuzzy logic theory has shown to be an appealing and efficient approach to dealing with the analysis and synthesis problems for complex nonlinear systems. The well-known Takagi–Sugeno (T–S) fuzzy model [24] is a popular and convenient tool in functional approximations. During the last decades, the problems of stability analysis and control synthesis for systems in T–S fuzzy model with time-delay have been studied extensively, and a lot of research results have been reported in the literature [1,5,7,17,29]. Recently, the T–S fuzzy control approach has been extended to the study of nonlinear stochastic time-delay systems. For example, some delay-independent stability criteria for a class of T–S fuzzy stochastic systems with constant delays have been given in [28], while the delay-dependent stabilization problem has been investigated in [31]. In [12], the authors have dealt with fuzzy sliding-mode control problem for uncertain nonlinear stochastic time-delay systems by means of T–S fuzzy modeling approach, and a sufficient condition for the exponential stability in the mean square of the sliding motion has been proposed.

Quite recently, more attention has been paid to apply T–S fuzzy models to describe the delayed Hopfield neural networks. The problem of exponential stability for T–S fuzzy model in which the consequent parts are composed of a set of stochastic Hopfield neural networks with time-varying delays has been considered in [15]. The overall fuzzy model can be achieved by fuzzy blending of these nonlinear neural networks [15]. In [20], the global

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asymptotic stability problem of fuzzy bi-directional associative memories neural networks with time-varying delays and parameter uncertainties has been investigated by means of T–S fuzzy modeling approach. However, in [15] and [20], the results on stability analysis for fuzzy delayed neural networks are delay-independent. To the best of our knowledge, the mean square exponential stability problem for stochastic fuzzy Hopfield neural networks with discrete and distributed time-varying delays has not been fully investigated, which is very challenging and remains as an open issue.

Motivated by the ideas in Refs. [15,20], in this paper, we further extend the ordinary T–S fuzzy models to describe the stochastic Hopfield neural networks with discrete and distributed time-varying delays. By using the stochastic analysis approach and free-weighting matrices method [10,11], mean square exponential stability criteria for the stochastic fuzzy delayed Hopfield neural networks are established in the form of linear matrix inequalities (LMIs), which can be readily verified by using standard numerical software. A numerical example is provided to illustrate the usefulness and less conservativeness of the developed techniques.

Notation. Through this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. I is the identity matrix with appropriate dimensions; "T" stands for the transpose of a matrix; For symmetric matrices X and Y, the notation $X>Y$ respectively $X\geqslant Y$ means that the $X-Y$ is positive definite (respectively, positive semi-definite); $|\cdot|$ refers the Euclidean vector norm; $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geqslant 0},P)$ is a probability space with a filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e. the filtration contains all P-null sets and is right continuous); Denoted by $L^2_{\mathscr{F}_0}([-2\tau,0];\mathbb{R}^n)$ the family of all $\mathscr{F}_0$-measurable $C([-2\tau,0];\mathbb{R}^n)$-valued random variable $\xi=\{\xi(\theta):-2\tau\leqslant\theta\leqslant 0\}$ such that $\sup_{-2\tau\leqslant\theta\leqslant 0}\mathbb{E}|\xi(\theta)|<\infty$, where $\mathbb{E}(\cdot)$ stands for the mathematical expectation; The symmetric terms in a symmetric matrix are denoted by "$*$"; Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following neural networks with discrete and distributed time-varying delays:

$$\dot{y}_i(t) = -a_i y_i(t) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} c_{ij} g_j(y_j(t-d(t))) + \sum_{j=1}^{n} d_{ij}\int_{t-h(t)}^{t} g_j(y_j(s))\,ds + I_i,\quad i=1,2,\ldots,n, \tag{1}$$

or equivalently the vector form

$$\dot{y}(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-d(t))) + D\int_{t-h(t)}^{t} g(y(s))\,ds + I, \tag{2}$$

where $y(t)=[y_1(t),y_2(t),\ldots,y_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$ denotes the state vector associated with n neurons. The matrix $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ is a diagonal matrix with positive entries $a_i>0$. $B=(b_{ij})_{n\times n}$, $C=(c_{ij})_{n\times n}$ and $D=(d_{ij})_{n\times n}$ are connection weight matrices representing the weighting coefficients of the neurons. $g(x)=[g_1(x_1),g_2(x_2),\ldots,g_n(x_n)]^{\mathrm{T}}\in\mathbb{R}^n$ is the activation function with $g(0)=0$. $I=[I_1,I_2,\ldots,I_n]^{\mathrm{T}}$ is a constant vector. The bounded function $d(t)$ and $h(t)$ represents unknown discrete and distributed time-varying delays, and satisfy $0<d(t)\leqslant\bar{d}$, $\dot{d}(t)\leqslant\mu$ and $0<h(t)\leqslant\bar{h}$, respectively.

(H1) The activation function g is bounded and satisfies Lipschitz condition

$$|g(x_1)-g(x_2)|\leqslant K|x_1-x_2|,\quad \forall x_1,x_2\in\mathbb{R},$$

where $K=\mathrm{diag}(k_1,k_2,\ldots,k_n)>0$ is a positive diagonal matrix. Then, by (H1) we can have

$$|g(x)|\leqslant K|x|,\quad \forall x\in\mathbb{R}. \tag{3}$$

As discussed in [15], it is reasonable to assume that the neural network (2) has only one equilibrium point $y^*=[y_1^*,y_2^*,\ldots,y_n^*]^{\mathrm{T}}$. Then, we will shift the equilibrium point y^* to the origin. The transformation $x(\cdot)=y(\cdot)-y^*$ puts system (2) into the following form:

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t-d(t))) + D\int_{t-h(t)}^{t} f(x(s))\,ds, \tag{4}$$

where $x(t)$ is the state vector of the transformation system, $f_j(x_j(t))=g_j(y_j(t)+y_j^*)-g_j(y_j^*)$ with $f_j(x_j(0))=0$ for $j=1,2,\ldots,n$. Then, from (3), we have

$$f^{\mathrm{T}}(x)f(x)\leqslant x^{\mathrm{T}}(t)K^{\mathrm{T}}Kx(t). \tag{5}$$

In the following section, we will consider the following stochastic fuzzy Hopfield neural network with discrete and distributed time-varying delays, which is represented by a T–S fuzzy model as [15]. The ith rule of this T–S fuzzy model is of the following form:

Plant Rule i: IF $\theta_1(t)$ is N_{i1} and $\cdots$ $\theta_p(t)$ is N_{ip} THEN

$$\begin{aligned} dx(t) &= \left[-A_i x(t) + B_i f(x(t)) + C_i f(x(t-d(t))) + D_i\int_{t-h(t)}^{t} f(x(s))\,ds\right]dt \\ &\quad + \sigma_i\left(t,x(t),x(t-d(t)),\int_{t-h(t)}^{t} f(x(s))\,ds\right)d\omega(t), \end{aligned}\tag{6}$$

$$x(t)=\phi(t),\quad \forall t\in[-2\tau,0],\quad \tau=\max\{\bar{d},\bar{h}\}, \tag{7}$$

where N_{ij} is the fuzzy set, $\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_m(t)]^{\mathrm{T}}$ is a m-dimensional Brownian motion defined on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geqslant 0},P)$. As discussed in Ref. [15], we assume that σ_i: $\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times m}$ is locally Lipschitz continuous and satisfies the linear growth condition. Moreover, σ_i satisfies

$$\begin{aligned} &\mathrm{trace}\left[\sigma_i\left(t,x(t),x(t-d(t)),\int_{t-h(t)}^{t} f(x(s))\,ds\right)^{\mathrm{T}} \right.\\ &\quad\left.\times\sigma_j\left(t,x(t),x(t-d(t)),\int_{t-h(t)}^{t} f(x(s))\,ds\right)\right] \\ &\quad\leqslant |F_1x(t)|^2 + |F_2x(t-d(t))|^2 + \left|F_3\int_{t-h(t)}^{t} f(x(s))\,ds\right|^2. \end{aligned}\tag{8}$$

Scalar k is the number of IF–Then rules. $\theta_1(t)$, $\theta_2(t),\ldots,\theta_p(t)$ are the premise variables. It is assumed that the premise variables do not depend on the noise-input variables $\omega(t)$ explicitly. The defuzzified output of system (6) is inferred as follows:

$$\begin{aligned} dx(t) &= \sum_{i=1}^{k} h_i(\theta(t))\Big\{-A_i x(t) + B_i f(x(t)) + C_i f(x(t-d(t))) \\ &\quad + D_i\int_{t-h(t)}^{t} f(x(s))\,ds\Big\}dt + \sum_{i=1}^{k} h_i(\theta(t)) \\ &\quad \times\left\{\sigma_i\left(t,x(t),x(t-d(t)),\int_{t-h(t)}^{t} f(x(s))\,ds\right)\right\}d\omega(t), \end{aligned}\tag{9}$$

where $h_i(\theta(t))=\mu_i(\theta(t))\big/\sum_{i=1}^{k}\mu_i(\theta(t))$, $\mu_i(\theta(t))=\prod_{j=1}^{p}N_{ij}(\theta_j(t))$ and $N_{ij}(\theta_j(t))$ is the degree of the membership of $\theta_j(t)$ in fuzzy set N_{ij}. In this paper, we assume that $\mu_i(\theta(t))\geqslant 0$ for $i=1,2,\ldots,k$ and $\sum_{i=1}^{k}\mu_i(\theta(t))>0$ for all t. Therefore, $h_i(\theta(t))\geqslant 0$ (for $i=1,2,\ldots,k$) and $\sum_{i=1}^{k}h_i(\theta(t))=1$.

Throughout this paper, we assume that all membership functions are continuous and piecewise continuously differentiable and the defuzzified model is also continuous. Clearly, based on the above discussion, (9) has a unique global solution on $t\geqslant 0$ through the initial value $x(\vartheta)=\phi(\vartheta)$ on $-2\tau\leqslant\vartheta\leqslant 0$ in