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# Image deblurring with filters learned by extreme learning machine

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ABSTRACT

Image deblurring is a basic and important task of image processing. Traditional filtering based image deblurring methods, e.g. enhancement filters, partial differential equation (PDE) and etc., are limited by the hypothesis that natural images and noise are with low and high frequency terms, respectively. Noise removal and edge protection are always the dilemma for traditional models.

In this paper, we study image deblurring problem from a brand new perspective—classification. And we also generalize the traditional PDE model to a more general case, using the theories of calculus of variations. Furthermore, inspired by the theories of approximation of functions, we transform the operator-learning problem into a coefficient-learning problem by means of selecting a group of basis, and build a filter-learning model. Based on extreme learning machine (ELM) [1–4], an algorithm is designed and a group of filters are learned effectively. Then a generalized image deblurring model, learned filtering PDE (LF-PDE), is built.

The experiments verify the effectiveness of our models and the corresponding learned filters. It is shown that our model can overcome many drawbacks of the traditional models and achieve much better results.

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## 1. Introduction

Image deblurring, or restoration, is a classical and important task of image processing. Nowadays there are large numbers of theories and algorithms for image deblurring. As a type of famous and effective image deblurring methods, partial differential equation (PDE) models, e.g. total variation (TV) model [7], Mumford–Shah model [8], Perona–Malik (P–M) PDE [9] and etc., improve the traditional linear filtering methods, e.g. constrained least square model [5], Weiner filter [6], and play a very important role in image processing. See [6,10] for a comprehensive and detailed introduction.

However, the limitations of traditional PDE based methods are also obvious: frequency characteristic is not a good way to distinguish the features in the natural images from noise. So noise removal and edge protection are always the dilemma for traditional filtering based methods. Essentially, this is caused by the lack of statistic information of image category, i.e. the image priors. Statistic learning methods have proposed an effective way to achieve the image priors from the samples of image category. Many existing models and algorithms, e.g. [13–17], can be used to learn good image priors.

Some studies have also been made on using a neural classifier to learn an image deblurring model. Basu and Su proposed the projection pursuit learning network (PPLN) based method [11], and a 3-stage hybrid learning system [12] for blind deconvolution. Different from the existing methods, our model, which is derived form PDE models, focuses on the continuous case. It is an extension of traditional learning models and closely connected with the theories of PDE and inverse problems.

In this paper, we revisit traditional PDE models from a classification point of view, and build a theoretical framework unifying both PDE models and learning methods. Based on the theories of approximation of functions [18,19], we skillfully transform the operator (or functions) learning problem into a coefficient-learning problem with a group of basis.

Then, we design a learning algorithm based on extreme learning machine (ELM) [1–4]. And a group of filters are achieved for the effective classification of noise and natural images. Furthermore, we propose a new and effective image deblurring model: learned filtering PDE (LF-PDE) model.

Experimental results show that our model can overcome many drawbacks of the traditional PDE models, e.g. spots caused by isolated noise points, "piecewise-constant" characters and etc., and achieve much better results.

## 2. Image restoration

Image deblurring, or restoration, is a typical inverse problem in image processing area. Due to the ill-posed character, regularization is necessary for a stable inverse process. We first propose a

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mathematic description of image restoration, then build an optimization model – energy functional model, and then derive the algorithm model – PDE model. Finally, we point out the substantial limitations of traditional models.

### 2.1. Image blurring and restoration

For linear and shift invariant (LSI) system, the blurred image $v(\boldsymbol{x})$ is modeled as the original image $u_0(\boldsymbol{x})$ convoluted by a point spread function (PSF) $G(\boldsymbol{x})$, and added by the additional noise $\varepsilon$. Mathematically, it is written as

$$v(\boldsymbol{x}) = (Au_0)(\boldsymbol{x}) + \varepsilon = G(\boldsymbol{x}) \circledast u_0(\boldsymbol{x}) + \varepsilon, \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2) \in \Omega \subset \mathbb{R}^2$ denotes a two-dimensional (2D) variable, $A$ denotes a convolution operator, $\circledast$ denotes the 2D convolution, and $G(\boldsymbol{x})$ is also called the kernel of $A$. $\varepsilon$ is supposed to be the independent and identically distributed (i.i.d.) Gaussian noise in this paper.

Image deblurring, or deconvolution, is the problem of restoring the original sharp image $u_0(\boldsymbol{x})$ from the blurred image $v(\boldsymbol{x})$. It is an inverse problem of image blurring, and can be modeled as an optimization problem:

$$\inf_{u \in \boldsymbol{I}} \|(Au)(\boldsymbol{x}) - v(\boldsymbol{x})\|^2, \tag{2}$$

where $\boldsymbol{I}$ denotes the functional space, e.g. $C[\Omega]$, $L^1[\Omega]$, $L^2[\Omega]$ and etc., $\|Au - v\| := (\int_\Omega (Au - v)^2 \, d\boldsymbol{x})^{1/2}$. Due to the ill-posed character of the inverse problem, the solution of Eq. (2) may not be unique; even it is unique, it may not depend on the blurred image in a continuous way. Therefore, some regularization methods should be used to stabilize the inverse process, i.e. restricting $\boldsymbol{I}$ by additional constrains or priors of $u$[1]:

$$\|R(u)\|_\Psi := \int_\Omega \Psi(R(u)) \, d\boldsymbol{x} \le C_0, \tag{3}$$

where $R : \boldsymbol{I} \to \boldsymbol{I}_S$ denotes an operator, $\boldsymbol{I}_S$ denotes the feature space, i.e. the range of $R$, and $\|\bullet\|_\Psi$ denotes a special energy form defined by $\Psi$. For instance, let $\Psi(s(\boldsymbol{x})) := s^2(\boldsymbol{x})$, then $\|\bullet\|_\Psi = \|\bullet\|_2^2$ denotes the traditional energy form defined by the square of $l_2$-*norm*; let $\Psi(s(\boldsymbol{x})) := |s(\boldsymbol{x})|$, then $\|\bullet\|_\Psi = \|\bullet\|_1$ denotes the energy defined by $l_1$-*norm*.

### 2.2. Energy functional model

Image restoration can be considered as a constrained optimization problem, i.e. Eqs. (2) and (3). By Lagrange multiplier method, it can be modeled as an energy functional optimization problem as follows:

$$\inf_{u \in \boldsymbol{I}} \int_\Omega [|v - Au|^2 + \lambda \cdot \Psi(R(u))] \, d\boldsymbol{x}, \tag{4}$$

where $A$ is typically modeled as a low-pass filtering process, and its kernel $G(\boldsymbol{x})$ varies according to the blurring process, e.g. out of focus, motion blur, Gaussian blur and etc. $\lambda$ is called the regularization parameter. The restored image $u_{opt}(\boldsymbol{x})$ is achieved by solving Eq. (4).

Eq. (4) is the generalized form of the energy functional model for image deblurring. Traditional linear or nonlinear models, e.g. Tikhonov method ($R = \nabla$, $\Psi(\bullet) = \|\bullet\|_2^2$) [6], TV model ($R = \nabla$, $\Psi(\bullet) = \|\bullet\|_1$) [7] and etc., are the specialized cases of Eq. (4).

$R$ and $\Psi$ decide the existence and uniqueness of $u_{opt}$ (i.e. the solution of Eq. (4)), and play a very important role in image restoration.

[1] In the view of statistic, $\|R(u)\|_\Psi$ is assumed to obey the exponential family of distributions, and the constrain Eq. (3) is equivalent to a prior term of $u$.

### 2.3. PDE model

Energy functional model Eq. (4) is the mathematical description of the task of image restoration, yet, for the implementation of the task, the corresponding PDE model should be derived. The numerical solution method of the PDE model is the image restoration algorithm.

By calculus of variations theory, if $\exists\, u_{opt}(\boldsymbol{x}) \in \boldsymbol{I}$ such that $u_{opt}$ is the solution of Eq. (4), then $u_{opt}$ must satisfy the equation:

$$\frac{\delta}{\delta u}\left(\int_\Omega |v - Au|^2 + \lambda \cdot \Psi(R(u)) \, d\boldsymbol{x}\right) = 0. \tag{5}$$

Let $s(\boldsymbol{x}) = (R(u))(\boldsymbol{x})$. From Eq. (5), one can derive the general Euler equation:

$$A^*Au - A^*v + \lambda \cdot (R_u)^* \left(\frac{\partial}{\partial s}\Psi(s)\bigg|_{s = R(u)}\right) = 0, \tag{6}$$

where $R_u$ denotes the *Frechet* derivative. $A^*$ and $(R_u)^*$ denote the adjoint operator of $A$ and $R_u$, respectively. In general, Eq. (6) is hard to solve. Solving the variational gradient flow (VGF) PDE:

$$\frac{\partial u}{\partial t} = -\left[A^*Au - A^*v + \lambda \cdot (R_u)^* \left(\frac{\partial}{\partial s}\Psi(s)\bigg|_{s = R(u)}\right)\right] \tag{7}$$

is a way to approach $u_{opt}$. VGF PDE is appropriate for deriving iterative algorithms. There are many numerical methods for solving VGF PDE, e.g. finite difference, finite element and etc.

Let $R = \nabla$, then the VGF PDE Eq. (7) can be specified as the traditional linear (with $\Psi(\bullet) = \|\bullet\|_2^2$) heat diffusion PDE:

$$\frac{\partial u}{\partial t} = \lambda \triangle u + f(u) \tag{8}$$

and the nonlinear (with $\Psi(\bullet) = \|\bullet\|_1$) heat diffusion PDE:

$$\frac{\partial u}{\partial t} = \lambda \cdot div\left(\frac{1}{|\nabla u|}\nabla u\right) + f(u), \tag{9}$$

where $f(u) = A^*v - A^*Au$, and $\triangle := div(\nabla) = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$.

### 2.4. Limitations of traditional models

Now let us reconsider the effect of $R$ and $\Psi$ more deeply. Suppose that $u_0 \in \boldsymbol{I}$, such that any function $u \in \boldsymbol{I}$ can be decomposed as $u = u_0 + u_\delta$, where $u_\delta$ denotes the disparity between $u_0$ and $u$. If $u_\delta \ne 0$ but $u_\delta \in null(A)$, then $Au = Au_0$, thus $u$ and $u_0$ are inseparable by Eq. (2). In order to separate $u$ and $u_0$, additional criterion, i.e. Eq. (3), is necessary. The validity of the regularization method relies on the choice of $R$ and $\Psi$ such that for any $u_0, u \in \boldsymbol{I}$, if $u_\delta \in null(A)$ but $u_\delta \ne 0$, then $\|R(u)\|_\Psi > \|R(u_0)\|_\Psi$.

Let $\boldsymbol{I}_0 \subset \boldsymbol{I}$ denotes the subspace consisting of all feasible solutions $u_0$. Let $(\boldsymbol{I}_0)^c$ denotes the complement of $\boldsymbol{I}_0$, consisting of all infeasible images. Then the regularization is essentially the separation of $\boldsymbol{I}_0$ and $(\boldsymbol{I}_0)^c$, and the ideal result of $R$ is to map $\boldsymbol{I}_0$ and $(\boldsymbol{I}_0)^c$ into different parts (denoted by $\boldsymbol{I}_S^0$ and $(\boldsymbol{I}_S^0)^c$, respectively) in the feature space $\boldsymbol{I}_S$. $\boldsymbol{I}_S^0$ and $(\boldsymbol{I}_S^0)^c$ are separated by the range of $\|\bullet\|_\Psi$. If $u \in \boldsymbol{I}_0$, $\|R(u)\|_\Psi$ will be small, otherwise $\|R(u)\|_\Psi$ will be large.

Then let us review the existing PDE models. The basic hypothesis of Tikhonov method is that $\boldsymbol{I}_0 \subset C_\Omega$ such that $\boldsymbol{I}_S^0 \subset L_\Omega^2$ for all feasible $u \in \boldsymbol{I}_0$. But as is known to all, many natural images contain jumps and edges, i.e. $\boldsymbol{I}_0 \not\subseteq C_\Omega$, thus, many $u \in \boldsymbol{I}_0$ are excluded by Tikhonov method.

As an improvement, TV model breaks the hypothesis of Tikhonov methods by releasing the constrains from $\boldsymbol{I}_S^0 \subset L_\Omega^2$ to $\boldsymbol{I}_S^0 \subset L_\Omega^1$. $\boldsymbol{I}^0$ is extended to contain some $u(\boldsymbol{x})$ with finite jumps and edges. For example, if $u(\boldsymbol{x})$ contains jumps and edges, then $|\nabla u|$ will contain the Dirac function $\delta(\boldsymbol{x})$. $\|\delta(\boldsymbol{x})\|_2^2 = \infty$, but $\|\delta(\boldsymbol{x})\|_1 = 1$.

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