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Robust H_∞ control for uncertain delayed nonlinear systems based on standard neural network models ☆

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Abstract

A neural-network-based robust output feedback H_∞ control design is suggested for control of a class of nonlinear systems both with time delays and with uncertainties. In this paper, a full-order dynamic output feedback controller is designed for the delayed uncertain nonlinear system approximated by the neural network (e.g. multilayer perceptron, recurrent neural network, etc.), of which the activation functions satisfy the sector conditions. The closed-loop neural control system is transformed into a novel neural network model both with uncertainties and with time delays termed standard neural network model (SNNM). Based on the optimal robust H_∞ performance analysis of the SNNM, the parameters of output feedback controllers can be obtained by solving some linear matrix inequalities (LMIs). The optimal H_∞ controller ensures the robust global asymptotic stability of the closed-loop system and eliminates the effect of approximation errors, parametric uncertainties, and external disturbances. Finally, a simple example is presented to illustrate the effectiveness and the applicability of the proposed design approach.

Keywords: Linear matrix inequality (LMI); Nonlinear system; Output feedback; Robust H_∞ control; Standard neural; Network model (SNNM); Time-delay system; Uncertain system

1. Introduction

Neural networks have been successfully employed for the control of nonlinear systems since the 1990s, see for instance [5,15,18] for an overview. In these nonlinear control systems, neural networks have been used either for modeling the system to be controlled, or for designing a controller, or both. In this paper, we focus on control configurations in which an off-line trained neural network serves as a model for the system under control. In this configuration, we must consider two aspects which are important for successful application of the control strategy to real-time systems [20]:

(1) The design of the controller, based on the neural model, should be fairly simple.
(2) Model inaccuracies and external disturbances should be taken into account such that closed-loop robustness (of stability and/or performance) can be guaranteed.

Recently, the robustness issue has been an important focus of research in the neuro-control society, and several robust stability design approaches have been proposed [14,20,22]. However, they only deal with the problems of robust stability analysis and robust stabilization for neural control systems by considering modelling errors resulting from approximation of

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a plant with neural networks. It is well known that the H_∞ performance is closely relative to the capability of disturbance rejection. Regarding H_∞ control by neural networks, to the best of our knowledge, only a few results are published [10,11,18,19]. Refs. [18,19] presented some sufficient conditions for absolute stability and dissipativity of continuous-time recurrent neural networks with two hidden layers. The conditions were employed for nonlinear H_∞ control and imposing closed-loop stability in dynamic backpropagation. Ref. [10] used two multilayer perceptrons to approximate a class of nonlinear systems. The neural networks were piecewisely interpolated to generate a linear differential inclusion model. Based on this model, a state feedback H_∞ control law was designed to eliminate the effect of approximation errors and external disturbances to achieve desired performance. Ref. [11] proposed robust H_∞ controller, which comprises a recurrent neural network (RNN) and a compensating control, and is developed to reduce the influence of parameter variations and external disturbance on a permanent-magnet linear synchronous motor. The RNN is adopted to estimate the dynamics of the lumped plant uncertainty, and the compensating controller is used to eliminate the effect of the higher order terms in Taylor series expansion of the minimum approximation error. Although Refs. [10,11,18,19] provided some novel design approaches of the H_∞ controllers for the neural network systems, they are not applicable to time-delayed systems.

In biological and artificial neural networks, time delays arise in the processing of information storage and transmission. For example, in the electronic implementation of analog neural networks, time delays occur in the communication and response of neurons, owing to the finite switching speed of amplifiers. It is known that they can influence the stability of the entire network by creating oscillatory or unstable phenomena [1]. Liao et al. have summarized the research results about stability analysis of various time-delayed recurrent neural networks in many published literature, and employed Lyapunov–Krasovskii stability theory for functional differential equations and the linear matrix inequality (LMI) approach [2] to investigate the problems about asymptotical stability in [8] and exponential stability in [9] of neural networks with constant or time-varying delays. However, it is well known that the stability of a well-designed neural network may often be destroyed by its unavoidable uncertainty. If deviations and perturbations in parameters are the main sources of uncertainty, and if the deviations and perturbations are all bounded, the neural network is called an interval neural network. Recently, based on the LMI approach, several global and robust stability criteria for interval recurrent neural networks with constant or time-varying delays have been proposed; see, e.g., [7,17]. However, we have noted that, although some robust stability conditions obtained in some literature have explicit expressions and less conservativeness, there does not seem to be much (if any) study on the robust stabilization for neural control systems both with time delays and with uncertainties. The performance of a real-life neural control system is influenced by external disturbances. To eliminate the effect of external disturbances, we must introduce the H_∞ robust technique to design the system. To our knowledge, there is no methodology that can be used in robust H_∞ controller synthesis for uncertain delayed nonlinear systems modeled with neural networks. Furthermore, the problem of robust H_∞ control via output feedback controllers is still open and remains unsolved [23], which motivates the present study.

In this paper, similar to the nominal models in linear robust control theory, we advance a delayed standard neural network model with uncertainties named as standard neural network model (SNNM), which is an extension of the neural network models in [12,16,18]. Firstly, we analyze the robust H_∞ performance of the discrete-time SNNM both with time delays and with uncertainties. The optimal robust H_∞ performance problem is described as an LMI eigenvalue minimization problem. On the other hand, based on the robust H_∞ performance analysis of the interval SNNM, we will develop an output feedback control law for the interval SNNM with inputs and outputs to ensure the robust asymptotic stability of the closed-loop system and optimize the H_∞ performance. The resulting design equations are in the form of a LMI optimization problem which can be solved by various convex optimization algorithms. Most delayed (or non-delayed) uncertain discrete-time nonlinear systems modeled by neural networks can be transformed into SNNMs to be robust H_∞ performance analyzed or robust H_∞ controller synthesized in a unified way. We will provide a detailed design procedure of the control law for the nonlinear system in Section 5.

Notation: Throughout this paper, $\Re^n$ denotes n dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n\times m$ real matrices, I denotes identity matrix of appropriate order, $*$ denotes the symmetric parts. If Λ is a diagonal positive (or semi-positive) definite matrix, $\Lambda^{1/2}$ denotes a diagonal positive (or semi-positive) definite matrix of which the diagonal element is square root of Λ's. The notations $X>Y$ and $X\geqslant Y$, respectively, where X and Y are matrices of same dimensions, mean that the matrix $X-Y$ is positive definite and positive semi-definite, respectively. If $X\in\Re^p$ and $Y\in\Re^q$, $C(X;\,Y)$ denotes the space of all continuous functions mapping $\Re^p\to\Re^q$.

2. Problem formulation

In linear robust control theory, a system with uncertainty can be transformed into a standard form known as linear fractional transformation (LFT) [3]. Similar to the LFT, and referring to [16], we can analyze the performance or synthesize controllers for the nonlinear system composed of neural network by transforming them into SNNMs. The SNNM represents a neural network model as the interconnection of a linear dynamic system and static delayed (or non-delayed) nonlinear operators consisting of bounded activation functions. Here, we discuss only the discrete-time SNNM, though