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Letters

Exponential state estimation for recurrent neural networks with distributed delays

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Abstract

In this paper, the delay-dependent state estimation problem for recurrent neural networks with both time-varying and distributed time-varying delays is investigated. Through available output measurements, a delay-dependent criterion is established to estimate the neuron states such that the dynamics of the estimation error is globally exponentially stable. The derivative of a time-varying delay satisfies $\dot{\tau}(t) \leqslant \mu$ and the activation functions are assumed to be neither monotonic nor differentiable, and more general than the recently commonly used Lipschitz conditions. Finally, two illustrative examples are given to demonstrate the usefulness of the obtained condition.

Keywords: State estimator; Recurrent neural networks; Exponential stability; Distributed delay; Linear matrix inequality

1. Introduction

In the past decade, neural networks have been studied intensively. They have witnessed a large amount of successful applications in many fields such as signal processing, pattern recognition, and static image processing and these applications depend on the dynamical behaviors heavily. And time-delay systems are frequently encountered in various areas realistically, and time-delay is often a source of instability and oscillations in the system. Therefore, dynamics in a neural network often have time-delays due to many reasons, such as the finite switching speed of amplifiers in electronic neural networks or the finite signal propagation time in biological networks. As a result, either delay-independent or delay-dependent sufficient conditions have been proposed to verify the asymptotical or exponential stability of delayed neural networks (see, e.g., [1–5,7–10,12–14,16,17]).

On the other hand, since the neuron states are not often fully available in the network outputs in many applications, the neuron state estimation problem is also important for many applications to utilize the estimated neuron state. In recent years, the state estimation problem for neural networks has recently drawn particular research interests, see [6,11,15] for some recent results. Through available output measurements, the problems addressed in [6,11,15] are to estimate the neuron states in which the dynamics of the estimation error is globally asymptotically or exponentially stable. For the recurrent neural networks (RNNs) with mixed discrete and distributed delays, [11] first studies the state estimation problem and obtains some state estimation conditions. However, the proposed criteria cannot be applicable for systems with time-varying delays. In [15], though the obtained results can be applicable for systems with time-varying delays, they cannot cope with cases when the derivative of time-varying delay equals or is greater than 1. In [6], the state estimation problem for neural networks with time-varying delay is investigated and the derivative of a time-varying delay can take any value. Yet, the proposed methods in [6] cannot deal with the

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neural networks with distributed delays. So far, to the best of the authors' knowledge, when the derivative of a time-varying delay is just restrictive and the activation functions are assumed to be of more general descriptions, the state estimation problem for the RNNs with time-varying and distributed delays (DRNNs) has not been fully investigated yet, and remains to be challenging. It is, therefore, our intention in this paper to tackle this problem and provide an LMI-based condition for the desired state estimators.

By introducing the equivalent descriptor form and some free-weighting matrices, the aim of the paper is to estimate the neuron states via available output measurements such that the estimation error converges to zero exponentially. A numerically efficient LMI approach is developed to solve the addressed problem, and the explicit expression of the set of desired estimators is characterized. Two examples are used to illustrate the effectiveness of the proposed methods.

Notation: Throughout this paper, for the symmetric matrices $X, Y, X>Y$(respectively, $X\geqslant Y$) means that $X-Y>0$ ($X-Y\geqslant 0$) is a positive-definite (respectively, positive-semidefinite) matrix; $\lambda_{\max}(A), \lambda_{\min}(A)$ denote the maximum eigenvalue and minimum eigenvalue of the matrix A, respectively. $A^\tau, A^{-\tau}$ represent for the transposes of matrices A and A^{-1}, respectively. The symmetric term in a symmetric matrix is denoted by $*$, i.e.,

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^\tau & Z \end{bmatrix}.$$

2. Problem formulations

Considering the following neural networks with discrete delays:

$$\begin{aligned}\dot{z}(t) = & -Cz(t) + Ag_1(z(t)) + Bg_2(z(t-\tau(t))) \\ & + D\int_{t-\varrho(t)}^{t} g_3(z(s))\,\mathrm{d}s + I(t),\end{aligned} \tag{1}$$

where $z(\cdot)=(z_1(\cdot), z_2(\cdot), \ldots, z_n(\cdot))^\tau \in R^n$ is the neuron state vector; $g_i(z(\cdot)) = (g_{i1}(z_1(\cdot)), \ldots, g_{in}(z_n(\cdot)))^\tau \in R^n, i=1,2,3$ represents for neuron activation functions; $I(t) = (I_1(t), \ldots, I_n(t))^\tau \in R^n$ is a time-varying input vector; $C = diag\{c_1, c_2, \ldots, c_n\}$ is a diagonal matrix with $c_i>0$; and A, B, D are the connection weight matrix, the delayed weight matrix and the distributively delayed connection weight matrix, respectively. Here, $\tau(t), \varrho(t)$ denote the time-varying delay and the distributed time-varying delay satisfying

$$\begin{aligned} & 0\leqslant\tau(t)\leqslant\tau_m, \quad \dot{\tau}(t)\leqslant\mu, \\ & 0\leqslant\varrho(t)\leqslant\varrho_m, \end{aligned} \tag{2}$$

and τ_m, μ, ϱ_m are constants.

Remark 1. For the state estimation tasks addressed in [11,15], the derivatives of the delays considered are 0 or less than 1. However, the delays in the paper are time-varying and one of their derivatives is just restrictive but not necessarily less than 1, which means that our results are more meaningful than the ones in [11,15].

The following assumption is made on the neuron activation functions.

Assumption 1. For $i \in \{1,2,\ldots,n\}$, the neuron activation functions in (1) are bounded and satisfy

$$\begin{aligned} & \sigma_i^- \leqslant \frac{g_{1i}(x)-g_{1i}(y)}{x-y} \leqslant \sigma_i^+, \\ & \delta_i^- \leqslant \frac{g_{2i}(x)-g_{2i}(y)}{x-y} \leqslant \delta_i^+, \\ & \rho_i^- \leqslant \frac{g_{3i}(x)-g_{3i}(y)}{x-y} \leqslant \rho_i^+, \\ & \forall x,y\in R,\ x\neq y,\ \ i=1,2,\ldots,n, \end{aligned} \tag{3}$$

where $\sigma_i^+, \sigma_i^-, \delta_i^+, \delta_i^-, \rho_i^+, \rho_i^-$ are constants.

Remark 2. The constants $\sigma_i^+, \sigma_i^-, \delta_i^+, \delta_i^-, \rho_i^+, \rho_i^-$ in Assumption 1 are allowed to be positive, negative or zero. Hence, the activation functions can be nonmonotonic, more general than the usual sigmoid functions or those forms in [6,15].

It is noted that, for either biological or artificial neural networks, it is usually met with the case that the state of the neural network is not completely accessible and all the information one can have is just the output of the neural network. Therefore, estimating the neuron state from the given output is necessary to realize some specific design objectives in many practical applications. It is meaningful to construct an estimator to approximate the state of the neural network (1) in an exponential way.

Suppose the output form the system (1) to be of the form

$$y(t) = Ez(t) + F(t, z(t)), \tag{4}$$

where $y(t) = (y_1(t), \ldots, y_n(t))^\tau \in R^m$ is the measurement output of system (1), $E \in R^{m\times n}$ is a constant matrix and $F(t,z(t)) = (F_1(t,z_1(t)), \ldots, F_n(t,z_n(t)))^\tau \in R^m$ is the nonlinear disturbance and satisfies the following Lipschitz condition:

$$\|F(t,x)-F(t,y)\| \leqslant \|J(x-y)\|, \tag{5}$$

where $J\in R^{n\times n}$ is a known constant matrix.

In order to estimate the neuron state of (1), we introduce the following full-order state estimation

$$\begin{aligned}\dot{\bar{z}}(t) = & -C\bar{z}(t) + Ag_1(\bar{z}(t)) + Bg_2(\bar{z}(t-\tau(t))) \\ & + D\int_{t-\varrho(t)}^{t} g_3(\bar{z}(s))\,\mathrm{d}s + I(t) + K[y(t) \\ & - E\bar{z}(t) - F(t,\bar{z}(t))],\end{aligned} \tag{6}$$

where $\bar{z}(t)$ is the state estimate and $K \in R^{n\times m}$ is the estimator gain matrix to be designed. Let $\varepsilon(t) = (\varepsilon_1(t), \ldots, \varepsilon_n(t))^\tau := \bar{z}(t) - z(t)$ be the state estimation error. Then, with