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On global exponential stability of generalized stochastic neural networks with mixed time-delays ☆

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Abstract

This paper is concerned with the global exponential stability analysis problem for a general class of stochastic neural networks with mixed time-delays. The mixed time-delays under consideration comprise both the discrete time-varying delays and the distributed time-delays. The main purpose of this paper is to establish easily verifiable conditions under which the delayed stochastic neural network is exponentially stable in the mean square in the presence of both the discrete and distributed delays. By employing a new Lyapunov–Krasovskii functional and conducting stochastic analysis, a linear matrix inequality (LMI) approach is developed to derive the criteria of the exponential stability. Furthermore, the main results are specialized to deal with the analysis problem for the global asymptotic stability within the same LMI framework. The proposed criteria can be readily checked by using some standard numerical packages such as the Matlab LMI toolbox. A simple example is provided to demonstrate the effectiveness and applicability of the proposed testing criteria.

Keywords: Stochastic neural networks; Discrete and distributed delays; Lyapunov–Krasovskii functional; Global exponential stability; Global asymptotic stability; Linear matrix inequality

1. Introduction

The well-known Hopfield neural networks were firstly introduced by Hopfield [14,15] in early 1980s. Since then, both the mathematical analysis and practical applications of Hopfield neural networks have gained considerable research attention. Hopfield neural networks have already been successfully applied in many different areas such as combinatorial optimization, knowledge acquisition and pattern recognition, see e.g. [20,21,28]. It should be pointed out that, these applications are largely dependent on the stability of the equilibrium point of neural networks. Stability, as one of the most important properties for neural networks, is crucially required when designing neural networks. It is often the case in practice that, the neural network is designed with only one equilibrium point, and this equilibrium point is expected to be globally stable. For example, the neural network that is applied to solve the optimization problem must have one unique equilibrium point and be globally stable.

In both the biological and artificial neural networks, the interactions between neurons are generally asynchronous, which give rise to the inevitable signal transmission delays. Also, in electronic implementation of analog neural networks,

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time-delay is usually time-varying due to the finite switching speed of amplifiers. It is known that time-delays may cause undesirable dynamic network behaviors such as oscillation and instability. Consequently, the stability analysis problems for delayed neural networks have received considerable research attention. So far, a large amount of results have appeared in the literature, see e.g. [1,2,7–10,19,26,27,29] and references therein, where the delay type can be constant, time-varying, or distributed, and the stability criteria can be delay-dependent or delay-independent. Note that continuously distributed delays have recently gained particular attention, since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths.

On the other hand, in real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that a neural network could be stabilized or destabilized by certain stochastic inputs [4]. Hence, the stability analysis problem for stochastic Hopfield neural networks has begun to attract research interests, and some initial results have been obtained, see e.g. [4,16,17,24]. It should be mentioned that, in most existing literature tackling stochastic neural networks, the time-delays have been assumed to be either discrete or distributed, and the stability criteria have been derived mainly based on the computation of matrix norms. To the best of the authors' knowledge, the exponential stability analysis problem for *stochastic* Hopfield neural networks with *both* the discrete and distributed time-delays has not been fully investigated, and remains important and challenging.

In this paper, we deal with the global exponential stability analysis problem for a class of stochastic Hopfield neural networks with simultaneous presence of both the discrete and distributed time-delays. By utilizing a novel Lyapunov–Krasovskii functional and using stochastic analysis tools, we show that the addressed stability analysis problem is solvable if two linear matrix inequalities are feasible. Hence, different from the commonly used matrix norm theories (such as the M-matrix method), a unified *linear matrix inequality* (LMI) approach is developed to establish sufficient conditions for the neural networks to be globally exponential stable in the mean square. Note that LMIs can be easily solved by using the Matlab LMI toolbox, and no tuning of parameters is required [5]. A numerical example is provided to show the usefulness of the proposed global stability condition.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the transpose and the notation $X\geqslant Y$ (respectively, $X>Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. We let $h>0$ and $C([-h,0];\mathbb{R}^n)$ denote the family of continuous functions φ from $[-h,0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\|=\sup_{-h\leqslant\theta\leqslant 0}|\varphi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, denote by $\|A\|$ its operator norm, i.e., $\|A\|=\sup\{|Ax|:|x|=1\}=\sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$ where $\lambda_{\max}(\cdot)$ (respectively, $\lambda_{\min}(\cdot)$) means the largest (respectively, smallest) eigenvalue of A. $l_2[0,\infty]$ is the space of square integrable vector. Moreover, let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geqslant 0},P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L^p_{\mathscr{F}_0}([-h,0];\mathbb{R}^n)$ the family of all $\mathscr{F}_0$-measurable $C([-h,0];\mathbb{R}^n)$-valued random variables $\xi=\{\xi(\theta):-h\leqslant\theta\leqslant 0\}$ such that $\sup_{-h\leqslant\theta\leqslant 0}\mathbb{E}|\xi(\theta)|^p<\infty$ where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. Problem formulation

Consider, on a probability space $(\Omega,\mathscr{F},\mathscr{P})$, the following stochastic neural network with discrete and distributed time-delays of the form:

$$\begin{aligned} \mathrm{d}u_i(t) = {} & \left[-d_iu_i(t)+\sum_{j=1}^{n}a_{ij}f_j(u_j(t))+\sum_{j=1}^{n}b_{ij}g_j(u_j(t-\tau_1(t)))+\int_{t-\tau_2}^{t}\sum_{j=1}^{n}c_{ij}h_j(u_j(s))\,\mathrm{d}s+J_i\right]\mathrm{d}t \\ & +\sigma_i(t,u_1(t),\ldots,u_n(t),u_1(t-\tau_1(t)),\ldots,u_n(t-\tau_1(t)))\,\mathrm{d}w(t),\quad i=1,\ldots,n, \end{aligned} \tag{2.1}$$

where n is the number of the neurons in the neural network, $u_i(t)$ denotes the state of the ith neural neuron at time t, $f_j(u_j(t))$, $g_j(u_j(t))$ and $h_j(u_j(t))$ are the activation functions of the jth neuron at time t. The constants a_{ij}, b_{ij} and c_{ij} denote, respectively, the connection weights, the discretely delayed connection weights, and the distributively delayed connection weights, of the jth neuron on the i neuron. J_i is the external bias on the ith neuron, d_i denotes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs. $\tau_1(t)$ is the time-varying discrete time-delay with bound τ_1^*, i.e.,

$$0\leqslant\tau_1(t)\leqslant\tau_1^*, \tag{2.2}$$