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New results for global robust asymptotic stability of BAM neural networks with time-varying delays ☆

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ABSTRACT

In this paper, the global robust asymptotic stability for a class of delayed bidirectional associative memory (BAM) neural networks with interval uncertainty is studied. Some less conservative conditions are presented for the BAM neural networks with multiple time-varing delays based on the Lyapunov functional approach. We also give one example to demonstrate the applicability and effectiveness of our results, and compare the results with the previous robust stability results derived in the literature.

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1. Introduction

Kosko proposed a new class of neural networks called bidirectional associative memory (BAM) neural networks in [1,2]. These models generalized the single-layer auto-associative Hebbian correlator to a two-layer pattern-matched heteroassociative circuit. This class of networks has good application perspective in pattern recognition, optimization solving problems and automatic control engineering. As we all known,these applications require that the equilibrium point of the designed neural network model is globally asymptotically stable. Therefore, the investigation of stability of BAM neural networks is also of great interest. The stability of this class of neural networks has been extensively studied in the literature in the recent years, and many different sufficient conditions ensuring the stability of BAM neural networks have been given. Due to the finite switching speed of neuron amplifiers and, the finite speed of signal propagation, time delays are unavoidable in VLSI implementation of neural systems. It is known that the delay parameters in a neural system may affect the dynamical properties of the equilibrium point. Therefore, it is of great importance to study the equilibrium and stability properties of neural networks with time delays. In recent years, many researchers have investigated the BAM neural networks with constant delays [11,15,16,19]. However, the constant delay is only an idealization of variable delay and the systems with variable delays are more significant than those with constant delays. And many researchers have investigated the stability of BAM neural networks with time-varying delays [3–9,17,22–24].

On the other hand, in hardware implementation of neural networks, the network parameters of neural system may subject to some changes due to the tolerances of electronic components employed in the design. In such cases, the stability of a neural network may often be destroyed. However it is desired that the stability properties of neural network should not be affected by the small deviations in the values of the parameters. In other words, the neural network must be globally robust stable. Therefore, Global robust stability of BAM neural network models with time delays has been studied by many researchers and some important robust stability results have been reported in [10–15,18].

Motivated by the preceding discussion, in this paper, we are going to investigate the global robust asymptotic stability for BAM neural networks with multiple time-varying delays. Based on the Lyapunov functional approach, some new sufficient conditions for the global robust asymptotic stability of the equilibrium of this class of neural networks are presented. One example is also given to compare the results with the previous robust stability results derived in the literature [11,13]. And by making comparisons we can obtain that our conditions can be considered as the alternative results to the previously published results.

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The rest of this paper is organized as follows. In the Section 2, the problem to be investigated is stated and some definitions and lemmas are listed. In Section 3, improved global robust asymptotic stability conditions for BAM neural networks with time-varying delays are obtained. In Section 4, One example is given to show the effectiveness of the proposed results. Finally, some conclusions are presented in Section 5.

Notations: Throughout this paper, I denotes the identity matrix with appropriate dimensions. R^n denotes the n-dimensional Euclidean space. For real symmetric matrices X, the notation $X<0$ means that the matrix X is negative definite. For $u=(u_1,u_2,\ldots,u_n)^T \in R^n$, $\|u\|_2 \triangleq (\sum_{i=1} u_i^2)^{1/2}$ is the vector 2-norm. For matrix $A=(a_{ij})_{n\times n} \in R^{n\times n}$, $\lambda_M(A)(\lambda_m(A))$ represents the maximum (minimum) eigenvalue of matrix A, $\|A\|_2 \triangleq \sqrt{\lambda_{max}(A^T A)}$ is the spectral norm of matrix A.

2. Model description and preliminaries

Dynamical behavior of a hybrid BAM neural network with time-varying delays can be described by the following set of differential equations:

$$\begin{cases} \dot{u}_i(t) = -a_i u_i(t) + \sum_{j=1}^{m} w_{ij}\tilde{f}_j(v_j(t)) \\ \quad + \sum_{j=1}^{m} w_{ij}^{\tau}\tilde{f}_j(v_j(t-\tau_j(t))) + I_i, \ i=1,2,\ldots,n \\ \dot{v}_j(t) = -b_j v_j(t) + \sum_{i=1}^{n} v_{ji}\tilde{g}_i(u_i(t)) \\ \quad + \sum_{i=1}^{n} v_{ji}^{\tau}\tilde{g}_i(u_i(t-\sigma_i(t))) + J_j, \ j=1,2,\ldots,m \end{cases} \tag{1}$$

or be rewritten in the following vector-matrix form:

$$\begin{cases} \dot{u}(t) = -Au(t) + W\tilde{f}(v(t)) + W^{\tau}\tilde{f}(v(t-\tau(t))) + I, \\ \dot{v}(t) = -Bv(t) + V\tilde{g}(u(t)) + V^{\tau}\tilde{g}(u(t-\sigma(t))) + J, \end{cases} \tag{2}$$

where n and m denote the number of neurons in two layers, respectively; a_i and b_j denote the neuron charging time constants and passive decay rates, respectively; w_{ij} and v_{ji} are the synaptic connection strengths; w_{ij}^{τ} and v_{ji}^{τ} are the delayed synaptic connection strengths; $\tilde{f}_j(\cdot)$ and $\tilde{g}_i(\cdot)$ represent the activation functions of the *jth* neuron from the neural field F_Y and the *ith* neuron from the neural field F_X, respectively; I_i and J_j denote the external inputs; $u(t)=(u_1(t),u_2(t),\ldots,u_n(t))^T \in R^n$ and $v(t)=(v_1(t),v_2(t),\ldots, v_m(t))^T \in R^m$ are the state vectors associated with neurons from the neural field F_X and the neurons from the neural field F_Y, respectively; $\tau_j(t)$ and $\sigma_i(t)$ are time-varying differentiable functions which satisfy

$$0<\tau_j(t)\le\tau_j, \quad \dot{\tau}_j(t)=\mu_j<1, \ j=1,2,\ldots,m,$$

$$0<\sigma_i(t)\le\sigma_i, \quad \dot{\sigma}_i(t)=\nu_i<1, \ i=1,2,\ldots,n. \tag{3}$$

The initial condition of system (2) is assumed to be

$$x(s)=\varphi(s)\in R^n, \quad y(s)=\phi(s)\in R^m, \tag{4}$$

for

$$s \in \left[-\max\left\{\max_{j=1,2,\ldots,m}\tau_j, \max_{i=1,2,\ldots,n}\sigma_i\right\}, 0\right].$$

In this paper, to establish the robust stability of the model given in (2), it is necessary to make the following assumptions.

(A_1) The activation functions $\tilde{f}_j(\cdot)$ and $\tilde{g}_i(\cdot)$ are bounded. That is, for all $x,y\in R$, there exist positive constants $F_j>0$ and $G_i>0$, such that

$$|\tilde{f}_j(y)| \le F_j, \quad j=1,2,\ldots,m,$$

$$|\tilde{g}_i(x)| \le G_i, \quad i=1,2,\ldots,n.$$

(A_2) The activation functions $\tilde{f}_j(\cdot)$ and $\tilde{g}_i(\cdot)$ are Lipschitz continue, that is, there exist constants $k_j>0$ and $l_i<0$, such that

$$|\tilde{f}_j(x)-\tilde{f}_j(y)| \le k_j|x-y|, \quad j=1,2,\ldots,m, \ \forall x,y\in R,$$

$$|\tilde{g}_i(\tilde{x})-\tilde{g}_i(\tilde{y})| \le l_i|\tilde{x}-\tilde{y}|, \quad i=1,2,\ldots,n, \ \forall \tilde{x},\tilde{y}\in R.$$

As we discussed in Section 1, there are some uncertainties in neural networks due to the existence of modeling errors, external disturbance and other uncertain factors and these uncertainties will lead to some deviations in the values of the parameters in (2). Since in practice these deviations are bounded in general, the quantities $a_i, b_j, w_{ij}, w_{ij}^{\tau}, v_{ji}$ and v_{ji}^{τ} may be intervalized as follows: for $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$,

$$\begin{cases} A_I := \{A=diag(a_i): 0<\underline{A}\le A\le\overline{A}, & i.e., \ 0<\underline{a}_i\le a_i\le\overline{a}_i\}, \\ B_I := \{B=diag(b_j): 0<\underline{B}\le B\le\overline{B}, & i.e., \ 0<\underline{b}_j\le b_j\le\overline{b}_j\}, \\ W_I := \{W=(w_{ij})_{n\times m}: \underline{W}\le W\le\overline{W}, & i.e., \ \underline{w}_{ij}\le w_{ij}\le\overline{w}_{ij}\}, \\ W_I^{\tau} := \{W^{\tau}=(w_{ij}^{\tau})_{n\times m}: \underline{W^{\tau}}\le W^{\tau}\le\overline{W^{\tau}}, & i.e., \ \underline{w^{\tau}}_{ij}\le w_{ij}^{\tau}\le\overline{w^{\tau}}_{ij}\}, \\ V_I := \{V=(v_{ji})_{m\times n}: \underline{V}\le V\le\overline{V}, & i.e., \ \underline{v}_{ji}\le v_{ji}\le\overline{v}_{ji}\}, \\ V_I^{\tau} := \{V=(v_{ji}^{\tau})_{m\times n}: \underline{V^{\tau}}\le V^{\tau}\le\overline{V^{\tau}}, & i.e., \ \underline{v^{\tau}}_{ji}\le v_{ji}^{\tau}\le\overline{v^{\tau}}_{ji}\}. \end{cases} \tag{5}$$

In the following, we restate some definitions and some results from the vector-matrix theory, that are important in the context of this paper.

Definition 1. The BAM neural network (2) with the parameter ranges defined by (5) is called globally robust asymptotic stable if there is a unique equilibrium $p^*=(u^*,v^*)=(u_1^*,u_2^*,\ldots,u_n^*,v_1^*,v_2^*,\ldots,v_m^*)$ is globally asymptotic stable for all $A\in A_I$, $B\in B_I$, $W\in W_I$, $V\in V_I$, $W^{\tau}\in W_I$, $V^{\tau}\in V_I$.

Definition 2 (*Horn and Johnson [20]*). A real $n\times n$ matrix $S=(s_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $s_{ii}>0$ and $s_{ij}<0$, $i,j=1,2,\ldots,n$, $i\ne j$, and the real part of every eigenvalue of S is positive.

Lemma 1 (*Horn and Johnson [20]*). *Let $S=(s_{ij})_{n\times n}$ be a matrix with positive diagonal entries and non-positive off-diagonal entries. Then, S is a nonsingular M-matrix if and only if there exists a positive vector $\Psi=(\psi_1,\psi_2,\ldots,\psi_n)>0$ such that $S\Psi>0$.*

Lemma 2 (*Horn and Johnson [20]*). *Let $S=(s_{ij})_{n\times n}$ be a matrix with positive diagonal entries and non-positive off-diagonal entries. if S is a nonsingular M-matrix, then at least one row of S is strictly diagonally dominant. That is*

$$s_{ii} > \sum_{j=1, j\ne i}^{n} |s_{ij}|,$$

for at least one index i.

Lemma 3 (*Cao and Wang [21]*). *For $\forall A\in[\underline{A},\overline{A}]$, we have*

$$\|A\|_2 \le \|A^*\|_2 + \|A_*\|_2,$$

where $A^*=\frac{1}{2}(\underline{A}+\overline{A}), A_*=\frac{1}{2}(\overline{A}-\underline{A})$.

Lemma 4. *For any vectors $x,y\in R^n$ and positive definite matrix $Q\in R^{n\times n}$, the following equality holds*:

$$2x^Ty \le x^TQx + y^TQ^{-1}y.$$

3. Global robust stability analysis

In this section, we present a new result which states the conditions that guarantee the global robust asymptotic stability of the equilibrium point of BAM neural system (2). It should be noted that the assumption (A_1) and (A_2) guarantee the existence

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