Robotics and Autonomous Systems 79 (2016) 156–171

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

# Feasible acceleration count: A novel dynamic stability metric and its use in incremental motion planning on uneven terrain

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## HIGHLIGHTS

- A novel dynamic stability metric called Feasible Acceleration Count (FAC) is proposed.
- FAC gives a measure of the space of feasible/stable accelerations at a given state.
- It acts as a unified metric for quantifying stability and maneuverability of robot.
- It quantifies the efficiency of state space exploration in sampling based planners.
- An incremental planner with a novel node selection criteria based on FAC is proposed.

## ARTICLE INFO

*Article history:*
Received 20 May 2014
Received in revised form
10 November 2015
Accepted 16 November 2015
Available online 17 December 2015

*Keywords:*
Uneven terrain
Feasible acceleration count
Incremental motion planning
RRT

## ABSTRACT

Application of wheeled mobile robots has gradually progressed from the confines of structured indoor environments to rough outdoor terrains. Material transport and exploration are some of the few areas where wheeled robots are required to navigate over uneven terrains. Stable and efficient navigation of wheeled robots over uneven terrains requires a framework which can correctly ascertain the stability and maneuverability for a given robot's state. Most existing works on uneven terrain navigation assume a one-to-one correspondence between postural stability and maneuverability. In this paper, we show that such characterization is incomplete as states having high postural stability may have restricted maneuverability depending on underlying terrain topology. We thus, present a novel metric called *Feasible Acceleration Count* (FAC), introduced in our earlier works as a unified measure of robot stability and maneuverability. The metric gives the measure of the space of feasible accelerations available to the robot at a given state. The feasibility is decided by a set of inequalities which depends not only on robot's state but also on surface normals at the wheel ground contact point. This unique feature of the FAC metric makes it a more appropriate choice for motion planning on uneven terrains than metrics like Tip-Over. We further show that since space of feasible accelerations is a direct characterization of the space of possible motions at a given state, the metric *FAC*, also quantifies the quality of state space exploration achieved at each step of incremental sampling based planners. We build on top of this aspect of *FAC* and present an incremental trajectory planner with a novel node selection criteria for navigation of generic four wheeled robots and articulated systems like mobile manipulators on uneven terrain.



## 1. Introduction

With the advent of outdoor robotics and as more and more robots operate outdoors they are entailed to navigate over terrains that are uneven. These require some paradigm changes in robot motion planning methodologies. In particular, one is required to go beyond usual geometric and kinematic motion planning towards algorithms that integrate notion of stability and maneuverability into trajectory planning algorithm. Most existing literature on uneven terrain navigation assume a direct correlation between postural stability and maneuverability. However, such characterization is incomplete as states with high postural stability may have restricted maneuverability depending on the topology of the underlying terrain. In this paper, an exact mathematical correlation between stability and maneuverability of a robot's state on uneven terrain is presented through a metric called, *Feasible Acceleration Count* (*FAC*). As the name suggests, the proposed metric gives a measure of the space of feasible accelerations available to the robot at a given state. The feasibility is decided by a set of

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http://dx.doi.org/10.1016/j.robot.2015.11.007
0921-8890/

inequalities which depends not only on the robot states, the forces and moments acting on it but also explicitly on the topology of the underlying terrain. It is straightforward to note that the space of accelerations at a given state also characterizes the space of possible motions at that state. Thus, the concept of *FAC* proposed in this paper is central to the adaptation of sampling based planners like Rapidly Exploring Random Trees (RRT) [1–3] for motion planning on uneven terrain. To understand this further, note that sampling based planners like RRT rely on integrating the evolution model of the system for discrete set of control inputs. This integration procedure results in an incremental construction of a tree like data structure for the exploration of the state space. Thus, the number of discrete control inputs available for integration directly determines the quality of exploration of the state space. In RRT like frameworks, number of discrete control inputs would depend only on the resolution of discretization. However, as we show later, motion planning on uneven terrain is associated with generic state and control dependent differential constraints. Thus, the space of control inputs available for expansion of tree would depend on the state of the robot and the terrain parameters. If accelerations are taken as control inputs, then *FAC* gives an exact measure of space of available control inputs and consequently the quality of state space exploration. We build on top of this aspect of *FAC* and present an incremental trajectory planner with a novel node selection criteria for navigation of generic four wheeled robots and articulated systems like mobile manipulators on uneven terrains.

### *1.1. Related work*

Stability of wheeled mobile robots on uneven terrains has been addressed in many existing literature, either in isolation or in the context of motion planning on uneven terrain. One of the most popular metrics for defining the stability of wheeled mobile robots on uneven terrain has been the *force angle measure* or *tip-over* margin, proposed in [4,5]. It has been used for motion planning on uneven terrains in works like [6–9], where the objective was to obtain paths, along which at each point the robot posture satisfies the *tip-over* stability constraints. An implicit assumption in these cited works is that high stability of the robot as given by *tip-over* margin also corresponds to high maneuverability. Hence, as such these works do not involve any analysis to ascertain whether how well the robot can maneuver along the computed paths. However, this one to one correspondence between *tip-over* stability and maneuverability is not justified. For example, consider Fig. 1 which shows two example scenarios where the robots have identical state, forces, moments and wheel ground contact location. The tip-over margin concept would not differentiate between the two cases shown in Fig. 1. But it is apparent that the topology of the underlying terrain shown in case A is more favorable for motion generation than that shown in case B. In particular the surface normals at wheel ground contact point play an important role in robot maneuverability, but is completely neglected by *tip over* margin concept.

An alternative approach is presented in works like [10–12], where robot's stability is modeled through a set of constraints which enforce the requirement of permanent contact and no-slip at all the wheel ground contact points. In contrast to *tip-over* margin which primarily captures the postural stability, these set of constraints model stability from the point of view of robot's ability to maneuver and hence is more appropriate for motion planning on uneven terrain. We further strengthen this observation later in the paper. The current proposed work also follows the approach of modeling stability through permanent contact and no-slip constraints. However, we propose some significant and critical advancements in the current state of the art which are necessary for obtaining stable and highly agile trajectories on uneven terrain.

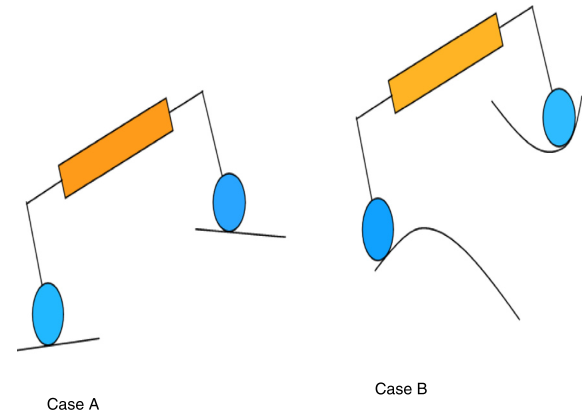


**Fig. 1.** Figure showing front view of two robots which have similar states, forces and moments but are operating on different terrain conditions. Tip-over stability concept would not differentiate between the two cases. However it is quite apparent that the terrain in case A is more favorable for motion than terrain B.

### *1.2. Contributions*

The contributions of the proposed work are synthesized from our earlier results [13–15]. However, in contrast to these cited works, the current paper presents a more in-depth analysis of various factors associated with motion planning on uneven terrain. The important aspects of the proposed work can be summarized as follows: Firstly, it provides a more rigorous mathematical representation of permanent contact and no-slip constraints by deriving them from the full 3D dynamics of the robot. This is in contrast to planar model of [10] and point mass model of [11]. The presented 3D constraint model is also an improvement over the procedure presented in [12] where the dynamics are projected into pitch, yaw and roll plane separately. Such procedure requires three different stability computations for each state of the robot. In contrast, the presented framework considers the full 3D dynamic model at once. Moreover, the number of variables in stability computation is the same as that in each planar analysis of [12]. Secondly, we incorporate a framework for ascertaining the 6D evolution of the robot's state for a given control input on uneven terrain. Such framework not only forms the crux of obtaining the full 3D dynamics of the robot and consequently the stability constraints, but is also imperative for constructing sampling based motion planners on uneven terrain. Works like [10–12] do not talk about any such evolution model. Thirdly, we map the solution space of the permanent contact and no-slip constraints to a unified metric called *Feasible Acceleration Count* (FAC) which provides a combined measure of stability, maneuverability and quality of state space exploration achieved at each step of incremental sampling based planners on uneven terrain. This explicit correlation that we describe between FAC and the efficiency of incremental sampling based planners on uneven terrain motion planning problems is a unique feature of the current proposed work and was not highlighted in [13,14] and [15]. Fourthly, we develop an incremental trajectory planner with RRT like data structure for motion planning on uneven terrain. However, in contrast to the conventional RRT framework, the proposed incremental planner has a new and novel *FAC* based node selection criteria which has been specifically carved for motion planning on uneven terrain. The proposed planner is applied to generic four wheeled non-holonomic robots as well as to articulated systems like mobile manipulator.

### *1.3. Layout of the paper*

The rest of the paper is organized as follows. Section 2 presents the framework for computing the 6 *dof* evolution of a generic