Robotics and Autonomous Systems 62 (2014) 131–141

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

Inverse-free control of a robotic manipulator in a task space

Mirosław Galicki *

Faculty of Mechanical Engineering, University of Zielona Góra, Podgórna 50, 65-246 Zielona Góra, Poland
Institute of Medical Statistics, Computer Science and Documentation, Friedrich Schiller University Jena, Jahnstrasse 3, D-07740 Jena, Germany

HIGHLIGHTS

- A class of kinematic inverse-free nonlinear manipulator controllers is proposed.
- The control laws are shown to be asymptotically stable.
- Adaptive inverse-free controllers generating bounded controls are also offered.
- The numerical simulations confirm theoretical results.

ARTICLE INFO

Article history:
Received 9 January 2013
Received in revised form
24 October 2013
Accepted 22 November 2013
Available online 6 December 2013

Keywords:
Inverse-free control
Robotic manipulator
Lyapunov stability theory

ABSTRACT

This paper addresses the control problem in a task space of the redundant and/or non-redundant manipulators with both known and parametric unknown kinematics and dynamics. A computationally simple class of the inverse-free control algorithms is proposed for the end-effector trajectory tracking. These controllers use some suitably constructed non-singular matrix whose inverse estimates the product of the manipulator Jacobian by its transposition. Moreover, by introducing a suitably defined sliding vector and nonlinear errors of the parameters estimation, the new controllers generate bounded and continuous signals. Based on the Lyapunov stability theory, inverse-free control schemes proposed are shown to be asymptotically stable provided that some reasonable assumptions are fulfilled during the manipulator movement. The performance of the proposed control strategies is illustrated through computer simulations for a planar redundant manipulator of three revolute kinematic pairs which accomplishes trajectory tracking by the end-effector in a two-dimensional task space.

1. Introduction

Recently, an interest has increased in applying redundant manipulators in useful practical tasks which are specified in terms of a time parameterized geometric path (a trajectory) to be tracked by the end-effector. Redundant degrees of freedom make it possible to achieve some useful objectives such as collision avoidance in the work space with obstacles, joint limit avoidance and/or avoiding the singular configurations. An effective approach to the motion control problem for redundant robotic manipulators is the so-called kinematic control. It is based on an inverse kinematic transformation which determines a reference joint (manipulator) trajectory corresponding to the end-effector trajectory given in the task space. One may distinguish several approaches in this context. Among them, we mainly concentrate on the two major ones. The first approach is the extended or augmented task space formulation of the inverse kinematics problem presented in works [1–5]. It is based on extending the dimension of the task space by incorporating as many additional constraints as the degree of the redundancy. These additional constraints are obtained based on, e.g., various types of optimization criteria. Consequently, the resulting system becomes non-redundant. Unfortunately, this approach usually introduces additional algorithmic singularities related to the rank of the so-called extended Jacobian matrix, and hence, can cause the joint velocity to become unbounded even though the manipulator is not in a singular configuration. Moreover, the dimensionality of the inverse kinematics problem increases. The second approach, discussed in the works [6–9], is based on the generalized pseudo-inverse of the manipulator Jacobian matrix. However, application of the pseudo-inverse techniques is both computationally time consuming and assumes a full rank of the Jacobian matrix along the trajectory. In order to tackle the singular configurations potentially met when tracking the end-effector trajectory, the use of a damped least-squares (DLS) inverse of the Jacobian matrix has been proposed in works [10–13] in lieu of the pseudo-inverse. Nevertheless, this technique suffers from the tracking errors due to a long-term numerical integration drift.

* Correspondence to: Institute of Medical Statistics, Computer Science and Documentation, Friedrich Schiller University Jena, Jahnstrasse 3, D-07740 Jena, Germany.

E-mail addresses: M.Galicki@ibem.uz.zgora.pl, miroslav.galicki@mti.uni-jena.de.

0921-8890/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.robot.2013.11.005

Recently [14–18], several tracking controllers with unknown robot kinematics and/or dynamics have been proposed. Nevertheless, control schemes in [15,17] require the inverse of the adaptive Jacobian and are, in fact, applicable to non-redundant manipulators. Moreover, adaptive control scheme in [17] may generate finite but arbitrarily large values of the torques/forces. In the works [16,15], commonly bounded parameter estimations were generated by utilizing the projection algorithms [19,20]. Nevertheless, this approach introduces discontinuity in the regression matrix and consequently results in discontinuous control. Furthermore, trajectory tracking algorithm presented in [14] requires full knowledge of the manipulator Jacobian matrix and its pseudo-inverse. In order to eliminate the computation of the regression matrix, an adaptive set point controller for manipulator with unknown both kinematic and dynamic equations has been proposed in very recent work [18].

This study addresses the problem of the task space control of a redundant and/or non-redundant manipulator in real time so that the end-effector tracks a prescribed trajectory. Our approach is based on the Lyapunov stability theory and, in contrast to others, does not require any pseudo-inverse nor the inverse of the augmented Jacobian. Instead, some non-singular matrix whose inverse estimates the product of the Jacobian matrix by its transposition, is constructed herein to eliminate the extended Jacobian inverse or pseudo-inverse. This methodology is then generalized and applied to redundant and/or non-redundant manipulators with unknown both robot kinematics and dynamics. By suitable definition of a sliding vector and constructing some non-singular matrix whose inverse approximates the product of the estimated Jacobian matrix by its transposition, we derive a new asymptotically stable inverse-free control scheme with unknown both manipulator kinematics and dynamics. Moreover, by introducing the nonlinear errors of the parameters estimation, the control scheme proposed herein, in contrast to adaptive algorithms known from the literature, prevents generating arbitrarily large values of the torques/forces. Furthermore, the resulting controls are continuous. The remainder of the paper is organized as follows. Section 2 formulates the problem of the end-effector trajectory tracking and summarizes useful properties of the robot kinematics. Our main results are presented in Section 3 where we mention some properties of the robot dynamic equations and propose a class of inverse-free controllers. Moreover, we provide conditions on the controller gains to ensure asymptotic stability. Section 4 presents a computer example of the end-effector trajectory tracking problem for a redundant manipulator with the three revolute kinematic pairs, operating in a two-dimensional task space. A comparison of the controller proposed with DLS approach is also carried out in this section. Finally, some concluding remarks are made in Section 5.

2. Problem formulation

The direct kinematic and differential mappings between joint coordinates q of the manipulator and task space coordinates p of the end-effector can be written as

$$p = f(q, X), \qquad \dot{p} = J(q, X)\dot{q} \tag{1}$$

where $\mathbb{R}^n \ni q = (q_1, \dots, q_n)^T$ is the vector of generalized coordinates of the manipulator (its configuration); n stands for the number of kinematic pairs of the V-th class; $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ denotes an m-dimensional nonlinear (with respect to vector q) mapping constructed from the kinematic equations of the manipulator; $f(q, X) = (f_1(q, X), \dots, f_m(q, X))^T$; $J = J(q, X) = [J_{ij}]_{1\le i\le m,\ 1\le j\le n} = \frac{\partial f}{\partial q}$ is the $(m \times n)$ Jacobian matrix; X stands for an ordered set of kinematic parameters $X = (X_1, \dots, X_k)^T$, such as link lengths, joint offsets; k denotes the number of kinematic parameters. On account of the redundant and/or non-redundant manipulator considered herein, the relation $n \ge m$ holds.

The control aim is to design a task space controller which generates velocity vector $\dot{q}$ in such a way that the task error $e = (e_1, \dots, e_m)^T = f(q, X) - \phi(t)$, where $\phi(t) = (\phi_1(t), \dots, \phi_m(t))^T$ is a given trajectory to be tracked belonging to a class of continuously differentiable mappings; $t \in [0, \infty)$, tends to asymptotically approach zero, i.e.

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (f(q(t, X)) - \phi(t)) = 0. \tag{2}$$

Without loss of generality, all the kinematic pairs of the robot are assumed to be revolute. Moreover, we limit ourselves to analysis of robotic manipulators whose kinematic equations may be described as sine and/or cosine polynomials. These assumptions imply that there exist some constants c_0, c_0' such that

$$\|J\| \le c_0, \qquad \left\|\frac{\partial J}{\partial q}\right\| \le c_0' \tag{3}$$

where $\|\cdot\|$ stands for the Euclidean (Frobenius) matrix norm. Moreover, desired trajectory $\phi(\cdot)$ is assumed to fulfil the following inequalities:

$$\|\dot{\phi}\| \le c_1 \qquad \|\ddot{\phi}\| \le c_1', \tag{4}$$

where c_1, c_1' denote some positive constants. As is known, a redundant manipulator is able both to track reference trajectory ϕ by the end-effector (a primary task) and to accomplish some useful objectives e.g. collision avoidance or joint limit avoidance (a secondary task) which represent state constraints imposed on the manipulator motion. In order to involve these state (inequality) constraints in the manipulator controller, suitable exterior penalty functions are introduced. The idea of exterior penalty function approach is to construct a modified criterion by adding a function called an exterior penalty function to prior cost function and then to optimize such unconstrained criterion function. Physically, finite positive values of exterior penalty function represent a penalty imposed by violating state constraints. Moreover, by fulfilment of state inequality constraints, exterior penalty function equals zero. As is known, exterior penalty function methods take into account only active state constraints thus reducing a computational burden and generate bounded signals even in small neighbourhoods of the constraint boundaries. To be more precise, we introduce some exterior penalty function $W(q)$ to satisfy state constraints. Our additional (secondary) task is to minimize $W(q)$ along trajectory $q(\cdot)$ provided by the manipulator controller designed in the next section. For the purpose of the manipulator control, we assume that

$$\left\|\frac{\partial W}{\partial q}\right\| \le c_2, \qquad \left\|\frac{\partial^2 W}{\partial q^2}\right\| \le c_2', \tag{5}$$

where c_2, c_2' denote some positive constants. Throughout the paper, $\lambda_{\min}(t)$ denotes the square root of the minimal eigenvalue of the matrix JJ^T at time instant t.

3. Inverse-free control of robotic manipulator

We will begin our considerations from the analysis of a kinematic controller designed to track end-effector trajectory ϕ. Then, by suitable definition of a sliding vector, we generalize this control law in the next subsection in such a way as to take into account both kinematic and dynamic uncertainties of the robotic manipulator.