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Dexterous workspace optimization of an asymmetric six-degree of freedom Stewart–Gough platform type manipulator

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HIGHLIGHTS

- Actuator types 8 produce superior dexterous workspace characteristics.
- Actuator 8 is unique type that produce dexterous workspace beyond LCI > 0.5.
- Radiuses of base platforms are commonly optimized bigger than the moving platform.
- Best platform orientation angles for $D_1^5D_3$ &TSPM vary between $-2°$ and $6°$.
- Moving platform radius can be used as a weighting factor to homogenize Jacobian.

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ABSTRACT

In this paper, an asymmetric *G*eneralized *S*tewart–Gough *P*latform (*GSP*) type parallel manipulator is designed by considering the type synthesis approach. The asymmetric six-*D*egree *O*f *F*reedom (*DOF*) manipulator optimized in this paper is selected among the GSPs classified under the name of 6D. The dexterous workspace optimization of *A*symmetric parallel *M*anipulator with t*E*n *D*ifferent *L*inear *A*ctuator *L*engths (*AMEDLAL*) subject to kinematics and geometric constraints is performed by using the *P*article *S*warm *O*ptimization *(PSO)*. The condition number and *M*inimum *S*ingular *V*alue (*MSV*) of homogenized Jacobian matrix are employed to obtain the dexterous workspace of AMEDLAL. Finally, the six-DOF AMEDLAL is also compared with the optimized *T*raditional *S*tewart–Gough *P*latform *M*anipulator (*TSPM*) considering the volume of the dexterous workspace in order to demonstrate its kinematic performance. Comparisons show that the manipulator proposed in this study illustrates better kinematic performance than TSPM.

1. Introduction

Parallel kinematic mechanisms have been broadly studied by the robotics community recently due to their high stiffness and high load/weight ratio compared to the serial robot manipulators. Researchers have tried to find the most promising parallel manipulator structures. Therefore several authors have directed their studies to meet this challenge especially last decades [1–11]. Designing all possible types of novel manipulator structures is called as type synthesis which is one of the most important issues for parallel kinematic mechanisms [12–18]. Robotics community considers this problem (finding the new types of parallel mechanism) as the type synthesis while Gao et al. [19] have defined the same problem as a geometrical constraint problem. They used six distance and/or angular constraints between six couples of points, lines and/or planes geometric primitives located on the base and moving platforms. Their study illustrated that there exists 3850 types of GSPs. Several authors studied the design and kinematic properties of the mechanisms proposed by Gao et al. [20–25].

A problem arises for selecting feasible structures among the 3850 GSP configurations. In this study two additional criteria are employed to obtain possible practical GSPs. The first criterion disregards planar joints which restricts motion on the plane only since these joints are not preferred for practical applications in general [26]. The second criterion implies symmetrical conditions defined by Tsai [27]. The first criterion reduces the possible combinations of GSPs to 195 while the second condition possibly leads to feasible structures. There are only three unique types of symmetrical GSPs since they include a single type of distance constraint only. Most of these 195 GSPs have asymmetrical structures since they have both angular and distance constraints. Different from symmetrical mechanisms, several types of asymmetrical mechanisms can be designed by using different numbers of legs that can include more than one actuator. Thus, researchers can easily construct new

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mechanisms for specific tasks without limiting themselves with regular numbers of legs & actuators. In addition, some applications may need different velocity and rigidity characteristics on different directions. For instance loading–unloading of goods in the conveyor belts require higher speed in the transversal direction than the other directions [28]. There are several studies addressing asymmetrical mechanisms in the literature. Gallardo-Alvarado et al. [29] presented a three-legged parallel manipulator composed of asymmetrical limbs. Refaat et al. [30] introduced four families of three-DOF translational–rotational asymmetrical parallel mechanisms. Karouia and Hervé [31] proposed three-DOF asymmetrical non-over constrained spherical parallel mechanisms. Abedinnasab and Vossoughi [32] presented a 4-legged 6-DOF redundantly actuated parallel mechanism.

In this paper, a dimensional optimization is performed for an asymmetric six-DOF parallel manipulator that is designed by taking the type synthesis approach into account. Some useful criteria that might help selecting more practical structures are also proposed to researchers. The possible practical structures of GSPs which are determined by using the proposed criteria are illustrated with tables under the class name of 3D3A, 4D2A, 5D1A and 6D. The $D_1^5D_3$ type mechanism studied in this paper is a member of 6D class manipulator having only one limb that causes asymmetry. The structure of $D_1^5D_3$ is composed of five-distance constrains between two points and one distance constraint between a line and a point located both on the base and moving platforms, respectively. Geometrical description, inverse kinematic and Jacobian matrix of the manipulator is presented. Dexterity of the asymmetric manipulator may not directly be obtained due to the dimensional irregularities of elements of Jacobian matrix. Therefore a weighting factor method is used for normalizing the elements of the Jacobian matrix. The condition number and MSV of the homogenized Jacobian matrix are employed to perform dexterous workspace optimization of ten different linear actuator lengths subject to kinematics and geometric limitations. The radiuses, leg attachment points of the base & moving platforms and orientation angles for the moving platform are used as the design variables. Particle Swarm Optimization is an evolutionary computation technique developed by Kennedy and Eberhart in 1995 [33] and is used as the optimization algorithm. The 6-DOF AMEDLAL are also compared with the optimized TSPM considering the dexterous workspace volumes in order to demonstrate their kinematic performances.

2. Type synthesis of 6-DOF GSPs as a geometric constraint problem

The type synthesis aims to find all possible types of robot structures and looks for finding the new, low-cost and simple parallel mechanisms [34,35]. Achieving new structures satisfying these criteria may not be guaranteed in the first designed manipulator. After several researches one may find such a manipulator that has extraordinary characteristics like very simple mechanism, low cost, higher reachable and dexterous workspace. For this purpose, Gao et al. [19] used six Distance (D) and/or Angular (A) constraints between six couples of points, lines and/or planes geometric primitives located on the base and the moving platforms. Their study illustrated that there exists 3850 types of GSPs grouped in four main classes namely 3D3A, 4D2A, 5D1A and 6D. Every class name is formed using the first letter and the number of the constraints i.e. GSPs in class of 3D3A has 3 distance and 3 angular constraints between its base and moving platforms.

In this study, two additional criteria are proposed to select more possible practical structures among these 3850 GSPs. The first criterion disregards planar joints which are rarely preferred in practical applications due to the restriction of the motion in the plane only [26]. The second criterion refers the symmetrical conditions given by Tsai [27]. The symmetrical condition refers the followings: (i) number of limbs must be equal to the number of DOF of manipulator, (ii) types and numbers of joints in each limb must be organized in the same manner, (iii) each limb must have the same type of actuator. The number of distance and angular constrains are reduced to four by applying the first criterion (Table 1). The first criterion reduces possible combinations of GSPs to 195, while the second condition possibly leads researches to feasible structures. As illustrated in Tables 2a–2d, possible numbers of the GSPs for each class is obtained as 20 for 3D3A, 35 for 4D2A, 56 for 5D1A and 84 for 6D classes.

It should be noted that a GSP is formed by combining the constraints (illustrated in Table 1) according to the formula given by Gao [19]. As an example, the $D_1^2D_4^2D_3A_1$ manipulator included in the 5D1A class has following constrains namely two numbers of D_1 and D_4, and one number of D3 and A1. It can easily be observed that the classes including both angular and distance constraints (3D3A, 4D2A and 5D1A) do not contain the symmetrical GSPs due to structures of the manipulators needing different types of joints. Some of the GSPs in the 6D class are also not symmetrical since the GSPs having different types of distance constraints cannot satisfy the symmetrical conditions. There are only four types of symmetrical GSPs (D_1^6, D_2^6, D_3^6 and D_4^6) in 6D class since they are composed of a single type of distance constraint only. The mathematical equations for D_2^6 and D_3^6 GSPs are completely identical since the distance constraints of both structures lie between a line and a point. The only difference is that the line and point primitives are located on different platforms. Therefore the number of the symmetrical GSPs in 6D class reduces to three namely D_1^6, D_3^6 and D_4^6.

3. Kinematic and Jacobian matrix modeling of the asymmetric manipulator

In this section, geometric description, inverse kinematic and Jacobian matrix of the $D_1^5D_3$ manipulator selected from the asymmetrical group of 6D is described in detail.

(a) *Geometrical description*

The asymmetric six-DOF $D_1^5D_3$ type manipulator (Fig. 1) has both base and moving platforms connected through six extensible links driven by active prismatic actuators. The six legs are designed for providing the required distance constraints. The mechanism has five distance constraints between five point pairs and a distance constraint between a line and a point on the base and moving platforms, respectively. Distance constraint between five points pairs are obtained by using *SPS* joint type while the last constraint is obtained by using a *CPS* joint type. The *O* and *U* coordinate systems are attached to the centers of the base and moving platforms, respectively. $\vec{P}$ illustrates the vector between *O* and *U* coordinate systems. The $\vec{a}_i$ and $\vec{b}_i$ vectors in Fig. 1 are directed from the centers of the *O* and *U* coordinate systems to the points A_i on the base and the points B_i on the moving platforms, separately. The base platform of the mechanism has five points named as A_i $(i = 2, 3, \ldots, 6)$ and a line L_1 passing through the points L_{p1} and L_{p2} while the moving platform has only six arbitrarily distributed points B_i where $i = 1, 2, \ldots, 6$. The vector $\vec{c}_1$ is positioned between the coordinate system *O* and f_1 point at the middle of the cylindrical joint on the base platform. The d_i (denoted as the five distance constraints) is the distance between A_i and B_i points on the base and moving platforms, respectively where $i = 2, 3, \ldots, 6$. The d_1 (determined as the first distance constraint) is the distance between the line L_1 and the point B_1 on the base and moving platforms, respectively.

(b) *Inverse kinematics and Jacobian matrix*

The inverse kinematics of this mechanism is composed of two constraint equations. The first one is the distance constraint between two points which are located on the base and moving platforms