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Global exponential stability of uncertain neural networks with discontinuous Lurie-type activation and mixed delays ☆

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ABSTRACT

This paper deals with the problem of the global exponential stability of a class of uncertain neural networks with discontinuous Lurie-type activation and mixed delays. By establishing a new sufficient condition, we first prove the existence of the equilibrium point by using the Leray–Schauder alternative theorem. Then, by employing a new Lyapunov functional, we obtain the global exponential stability of the equilibrium point of the uncertain neural network. In the end, some comparisons and numerical examples are given to show the improvement of the conclusions in this paper.

1. Introduction

In recent years, neural networks have been extensively investigated since their wide applications, such as pattern recognition, associative memories, automatic control, optimization, image processing and other areas (see [1–8]). It is well known that the applications of neural networks heavily depend on their stability. Therefore, in order to successfully employ neural networks in applications, we should study the stability of the designed neural network in advance. However, due to the finite switching speed of the neuron amplifiers and transmissions of signals in a network, time delays are actually unavoidable in the electronic implementation. Meanwhile, in both biological and artificial neural networks, the delay may create the loss of stability, since it may originate the onset of nonvanishing oscillations (see [9–25]). On the other hand, the estimation errors are unavoidable when we measure the vital data of deterministic neural networks such as the neuron fire rate and the synaptic interconnection weights. Moreover, in deterministic neural networks, the stability of neural networks can often be destroyed by its compulsory uncertainty issuing from the existence of modeling errors, external disturbance and parameter fluctuations. Hence, it is important and necessary to study the robust stability of neural networks with time delays in presence of uncertainties (see [13–15,26,27]).

In the past years, the stability of uncertain neural network has received considerable attentions, and varieties of interesting results have been presented in the literatures (see [10,13–15,26–28]). For example, in [10], authors investigated the problem of the existence, uniqueness and global asymptotic stability of the equilibrium point for the class of neural networks with multiple time delays and parameter uncertainties. By means of the homeomorphism theory and Lyapunov functional method, Arik in [13] studied the global asymptotic stability problem of dynamical neural networks with multiple time delays under parameter uncertainties. Guo et al. in [26] presented a systematic method for analyzing the robust stability of a class of interval neural networks with uncertain parameters and time delays. In [28], we studied the global robust exponential stability of the neural networks with possibly unbounded activation functions.

However, most of the results concerning the robust stability of neural networks are based on a common assumption that the activations are continuous or even Lipschitz continuous (see [9,10,13–15,29,30]). In practice, as mentioned by [31], discontinuous or non-Lipschitz neuron activations have been introduced into neural network systems due to their theoretical and practical significance in recent years. Nowadays, more and more scholars observe the importance of neural network with

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discontinuous activations (see [18,32–35,35–37]). For example, Qin et al. in [18] studied the global exponential stability and global convergence in finite time of neural networks with discontinuous activations. In [38], a novel class of Cohen–Grossberg neural networks with delays and inverse H ö lder neuron activation functions are presented. In [37], authors integrated a class of delayed neural networks with discontinuous activations by means of the Leray–Schauder theorem and Viability theorem. In [23], authors investigated the global dynamics of equilibrium point for delayed competitive neural networks with different time scales and discontinuous activations.

Inspired by previous studies, in this paper, we will study the existence, global exponential stability of uncertain neural networks with discontinuous Lurie-type activation and mixed delays. The remainder of this paper is arranged as follows. In Section 2, we state some preliminaries including some necessary definitions and lemmas. Our main results are contained in Sections 3 and 4, where the sufficient conditions are given to guarantee the existence and global exponential stability of equilibrium for neural networks in this paper. Section 5 presents some illustrative numerical examples to verify our results.

Notation: Given the column vector $x=(x_1,x_2,\dots,x_n)^T$, where the superscript T is the transpose operator, and $\|x\|:=(\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$. Let $A=(a_{ij})\in\mathbb{R}^{n\times n}$ and define $\|A\|=\sqrt{\lambda_M(A^TA)}$, where $\lambda_M(A)$ stands for the operation of taking the maximum eigenvalue of A. $I\in\mathbb{R}^{n\times n}$ is the $n\times n$ identity matrix. For a real symmetric matrix A, $A<0$ means that A is negative definite.

2. Preliminaries

In this section, we will introduce some definitions and lemmas, which will be used in the remainder of this paper. The neural network we consider in this paper is described as follows:

$$\frac{dx_i(t)}{dt}=-d_ix_i(t)+\sum_{j=1}^n a_{ij}g_j(x_j(t))+\sum_{j=1}^n b_{ij}g_j(x_j(t-\tau_j))+\sum_{j=1}^n c_{ij}\int_{t-h_j}^t g_j(x_j(s))\,ds+u_i, \tag{1}$$

or equivalently,

$$\dot{x}(t)=-Dx(t)+Ag(x(t))+Bg(x(t-\tau))+C\int_{t-h}^t g(x(s))\,ds+U, \tag{2}$$

where $x=(x_1,x_2,\dots,x_n)^T\in\mathbb{R}^n$ denotes the state of the neurons; $D=\mathrm{diag}(d_1,d_2,\dots,d_n)^T\in\mathbb{R}^n$ is a positive diagonal matrix; $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$, $C=(c_{ij})_{n\times n}$, are the $n\times n$ real connection weight matrices representing the weighting coefficients of the neurons; $g=(g_1(x_1),g_2(x_2),\dots,g_n(x_n))^T\in\mathbb{R}^n$ represents the neuron activations.

In this paper, we will study the neural network (1) with general activation functions. That is, the activation functions are only assumed to be non-increasing. For convenience, we define

$$\mathcal{M}\triangleq\{\phi:\mathbb{R}\to\mathbb{R}\,|\,s\phi(s)\ge 0,\quad\text{and}\quad D^+\phi(s)\ge 0, s\in\mathbb{R}\}. \tag{3}$$

For any $\phi\in\mathcal{M}$, we define

$$K[\phi(t)]=[\phi(t^-),\phi(t^+)] \tag{4}$$

In this paper, we always assume the following assumptions hold,

($\mathbf{A}_1$) The activation function of neural network (1) satisfies the Lurie-type condition. That is, $g_i\in\mathcal{M}$, and there exists $k_i>0$ such that

$$x_ig_i(x_i)\le k_ix_i^2$$

for any $x_i\in\mathbb{R}$.

Here, the constants $k_i, i=1,2,\dots,n$, are generally called to be Lurie constants.

($\mathbf{A}_2$) The parameters $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$, $C=(c_{ij})_{n\times n}$, $D=\mathrm{diag}(d_i)$ are assumed to be intervalised as follows:

$$D_I:=\{D=\mathrm{diag}(d_i):0\le\underline{D}\le D\le\overline{D}, \text{i.e ., } \underline{d_i}\le d_i\le\overline{d_i}, \forall i\}$$
$$A_I:=\{A=(a_{ij}):\underline{A}\le A\le\overline{A}, \text{i.e., } \underline{a_{ij}}\le a_{ij}\le\overline{a_{ij}}, \forall i,j\}$$
$$B_I:=\{B=(b_{ij}):\underline{B}\le B\le\overline{B}, \text{i.e., } \underline{b_{ij}}\le b_{ij}\le\overline{b_{ij}}, \forall i,j\} \tag{5}$$

Next lemma is Leray–Schauder alternative theorem, which plays an important role in proving the existence of the equilibrium point of the neural network (2).

Lemma 2.1 (*Granas and Dugundji [39]*). *If X is a Banach space, $C\subseteq X$ is nonempty convex with $0\in C$ and $G:C\to P_{k(c)}(C)$ is an USC multifunction which maps bounded sets into relatively compact sets, then one of the following statements is true:*

(1) *the set $\Gamma=x\in C: x\in\lambda G(x), \lambda\in(0,1)$ is unbounded;*
(2) *the $G(\cdot)$ has a fixed point in C, i.e., there exists $x\in C$ such that $x\in G(x)$.*

Definition 2.1. x^* is said to be an equilibrium point of neural network (2) if there exists $\gamma^*\in K[g(x^*)]$ such that

$$-Dx^*+(A+B)\gamma^*+CH\gamma^*+U=0, \tag{6}$$

where $H=\mathrm{diag}(h_i)$, $i=1,2,\dots,n$. And γ^* is said to be an output equilibrium point of system (2) corresponding to x^*.

Definition 2.2 (*Forti et al. [35]*). For any continuous function $\phi:[-\tau,0]\to\mathbb{R}^n$ and any measurable selection $\psi:[-\tau,0]\to\mathbb{R}^n$, such that $\psi(s)\in K[g(\phi(s))]$ for a.a. $s\in[-\tau,0]$, by an initial value problem (IVP) associated to (2) with initial condition (ϕ,ψ), we mean the following problem: find a couple of functions $[x,\gamma]:[-\tau,T)\to\mathbb{R}^n\times\mathbb{R}^n$, such that x is a solution of (2) on $[-\tau,T)$ for some $T>0$, γ is an output associated to x, and

$$\begin{cases}\dot{x}(t)=-Dx(t)+A\gamma(t)+B\gamma(t-\tau)+C\int_{t-h}^t\gamma(s)\,ds+U, & \text{for } a.a.\ t\in[0,T)\\ x(r)=\phi(r), & \forall r\in[-\tau,0]\\ \gamma(r)=\psi(r), & \text{for } a.a.\ r\in[-\tau,0]\end{cases} \tag{7}$$

Lemma 2.2 (*Faydasicok and Arik [40]*). *Let $x=(x_1,x_2,\dots,x_n)^T\in\mathbb{R}^n$, if*

$$A\in A_I:=\{A=(a_{ij}):0\le\underline{A}\le A\le\overline{A}, \text{i.e ., } 0\le\underline{a_{ij}}\le a_{ij}\le\overline{a_{ij}}, \forall i,j\}$$

then, for any positive diagonal matrix P, the following inequality holds:

$$x^T(PA+A^TP)x\le x^T(PA^*+A^{*T}P+\|PA_*+A_*^TP\|_2I)x,$$

where $A^*=\frac{1}{2}(\underline{A}+\overline{A}), A_*=\frac{1}{2}(\overline{A}-\underline{A})$.

Lemma 2.3 (*Qin et al. [28]*). *Let $x=(x_1,x_2,\dots,x_n)^T\in\mathbb{R}^n$, if*

$$B\in B_I:=\{B=(b_{ij}):0\le\underline{B}\le B\le\overline{B}, \text{i.e ., } 0\le\underline{b_{ij}}\le b_{ij}\le\overline{b_{ij}}, \forall i,j\},$$

then, the following inequality holds:

$$\|B\|_2\le \mathrm{b},$$

where