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Asymptotical synchronization for a class of coupled time-delay partial differential systems via boundary control ☆

Kai-Ning Wu, Tian Tian, Liming Wang *, Wei-Wei Wang

Department of Mathematics, Harbin Institute of Technology at Weihai, Weihai, 264209, PR China

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ABSTRACT

The asymptotical synchronization for coupled delay partial differential systems (PDSs) with boundary control is considered in this paper. First, the synchronization error dynamics are introduced and we turn the asymptotical synchronization problem into the asymptotical stabilization problem. The boundary controller is also presented. Then, by employing Lyapunov functional method, under the given boundary controller, sufficient conditions are provided which guarantee the asymptotical synchronization for the coupled delay PDSs. These sufficient conditions are given by linear matrix inequalities (LMIs), which are easy to be solved by the developed standard algorithms. Both the cases of node delay and coupling delay are considered. Finally, numerical examples are given to illustrate the effectiveness of the proposed theoretical results.

1. Introduction

As a kind of collective behavior of networks, synchronization exists in a widespread field, ranging from natural systems to manmade networks and it has received substantial attention in recent years, see [1–11] and the references therein for a survey. As is well known, delay is unavoidable in practice and it may degrade the properties of a given system, including the synchronization. A great deal of concerns on the synchronization with time delay have also been paid [12–20].

In the real world, there are many phenomena, such as the ones in chemical engineering, neurophysiology and biodynamics, in which state variables depend not only on the time but also on the spatial position. These phenomena are generally modeled in spatial–temporal domain by partial differential systems (PDSs). Recently, increasing concerns have been raised on the study of PDSs [21–25]. The synchronization of coupled PDSs also has recently received some initial research interests [26–31]. In [26], the authors considered the synchronization of delayed neural networks with reaction–diffusion terms under impulsive control, and sufficient conditions were obtained which guaranteed the synchronization and depended on the diffusion coefficients. In [27], the p-norm exponential synchronization of stochastic neural networks with leakage delay and reaction–diffusion terms was studied and by using the Lyapunov stability theory and stochastic analysis approaches, and a periodically intermittent controller is proposed to guarantee the exponential synchronization. In [29,30], we considered the asymptotical synchronization and H_∞ synchronization for coupled PDSs with or without delays.

It is well known that, for the PDS, the boundary control strategy is an effective control method which needs fewer actuators and may be easier to be applied. Boundary control for the PDS has also attracted many interests and lots of results were published [21–25,32–35]. In [32], the authors presented the backstepping method to study the boundary control for the PDS and many results have been published after that classical paper. Then, in [33], the authors generalized the backstepping method to the delay case. In [34], the combination of backstepping-based state-feedback control and flatness-based trajectory planning and feedforward control is considered for the design of an exponentially stabilizing tracking controller for a linear diffusion–convection–reaction system with spatially and temporally varying parameters and nonlinear boundary input. In [35], exponential synchronization via boundary control for a class of networked linear spatio-temporal dynamical networks is presented by using Lyapunov's direct method, the vector-valued Wirtinger's inequality and the technique of integration by parts. However, time-delay is not considered in that paper and this motivates the current paper.

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* Corresponding author. Tel./fax: +86 631 5687572.
E-mail addresses: kainingwu@hitwh.edu.cn (K.-N. Wu), HITmathtian@163.com (T. Tian), hitwlm@163.com (L. Wang), awei-awei@126.com (W.-W. Wang).

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In this paper, we consider the boundary control synchronization for coupled node-delay PDSs with Neumann boundary conditions. First, we turn the asymptotical synchronization problem into the asymptotical stabilization problem by introducing coupled synchronization error dynamics. We design the boundary controller and provide the sufficient conditions to guarantee the asymptotical synchronization under the designed boundary controllers. Then, we briefly state a criterion for the synchronization of coupled PDSs with coupling delay. Finally, numerical examples are given to illustrate the effectiveness of the proposed design methods.

The remainder of this paper is organized as follows. Section 2 gives preliminaries and problem formulation. The sufficient conditions on asymptotical synchronization of the coupled time-delay PDSs are provided in Section 3 on both cases of node and coupling delay. Numerical examples are given in Section 4 and a brief conclusion ends this paper in Section 5.

2. Model description and preliminaries

In this paper, we mainly consider the following N-coupled PDSs with node-delay

$$\begin{cases} y_{i,t}(x,t) = \Theta y_{i,xx}(x,t) + Ay_i(x,t) + By_i(x,t-\tau) + \sum_{j=1}^{N} g_{ij} y_j(x,t) \\ y_{i,x}(x,t)|_{x=0} = Cu_i(t), \ \ y_{i,x}(x,t)|_{x=L} = 0 \\ y_i(x,t) = \varphi_i(x,t), \quad t \in [-\tau, 0], \ \ i = 1, 2, \dots, N, \end{cases} \tag{2.1}$$

where $y_i(x,t) \triangleq [y_{i1}(x,t), \dots, y_{in}(x,t)]^T \in \mathbb{R}^n$ is the state variable of the i-th subsystem, and the subscripts x and t stand for the partial derivatives with respect to x and t, respectively. The variables $x \in [0, L] \subset \mathbb{R}$ and $t \in [0, \infty)$ are the spatial variable and time variable. $\Theta, A, B \in \mathbb{R}^{n\times n}$ are real matrices and Θ is symmetric positive definite matrix. $C \in \mathbb{R}^{n\times m}$ is the control input matrix and $u_i(t) \in \mathbb{R}^m$ is the boundary control input of the i-th subsystem. $G \triangleq (g_{ij})_{N\times N}$ is the coupling matrix of the network, in which g_{ij} is defined as follows: If there exists a connection between node i and node j $(i \neq j)$, then $g_{ij} \neq 0$; otherwise, $g_{ij} = 0$ $(i \neq j)$ and the diagonal elements are defined by $g_{ii} = -\sum_{j\neq i} g_{ij}, i = 1, 2, \dots, N$.

Remark 2.1. The coupling matrix G is not required to be symmetric or irreducible in this paper.

Let $s(x,t) \triangleq [s_1(x,t), \dots, s_n(x,t)]^T \in \mathbb{R}^n$ be the function to which all $y_i(x,t)$'s are expected to synchronize and $s(x,t)$ satisfies the following PDS:

$$\begin{cases} s_t(x,t) = \Theta s_{xx}(x,t) + As(x,t) + Bs(x,t-\tau) \\ s_x(x,t)|_{x=0} = s_x(x,t)|_{x=L} = 0 \\ s(x,t) = \varphi(x,t), \ \ t \in [-\tau, 0]. \end{cases} \tag{2.2}$$

Define the synchronization error as follows:

$$e_i(x,t) = y_i(x,t) - s(x,t). \tag{2.3}$$

From (2.1) and (2.2), we have the following error system of the i-th node

$$\begin{cases} e_{i,t}(x,t) = \Theta e_{i,xx}(x,t) + Ae_i(x,t) + Be_i(x,t-\tau) + \sum_{j=1}^{N} g_{ij} e_j(x,t) \\ e_{i,x}(x,t)|_{x=0} = Cu_i(t), \ \ e_{i,x}(x,t)|_{x=L} = 0 \\ e_i(x,t) = \varphi_i(x,t) - \varphi(x,t), \ \ t \in [-\tau, 0]. \end{cases} \tag{2.4}$$

We design the following state feedback controller for the i-th node of the PDSs (2.1)

$$u_i(t) = \int_0^L K(y_i - s)\, dx = \int_0^L Ke_i(x,t)\, dx, \tag{2.5}$$

where $K \in \mathbb{R}^{m\times n}$ is the control gain matrix to be determined.

Denote

$$e(x,t) = [e_1^T(x,t) \cdots e_N^T(x,t)]^T, \tag{2.6}$$

and in the sequel, the variables (x,t) are compressed for the simplicity. $A \otimes B$ means the Kronecker product of two matrices A and B. Identity matrix of $n \times n$ dimension will be denoted by I_n. We can get the following synchronization error system:

$$\begin{cases} e_t = (I_N \otimes \Theta) e_{xx} + (I_N \otimes A + G \otimes I_n) e + (I_N \otimes B) e(x, t-\tau) \\ e_x|_{x=0} = \int_0^L (I_N \otimes CK) e\, dx \\ e_x|_{x=L} = 0 \\ e(x,t) = \psi(x,t), \ \ t \in [-\tau, 0], \end{cases} \tag{2.7}$$

where $\psi(x,t) = [\psi_1^T(x,t), \dots, \psi_N^T(x,t)]^T = [\varphi_1^T(x,t) - \varphi^T(x,t), \dots, \varphi_N^T(x,t) - \varphi^T(x,t)]^T$.

For the convenience of analyses and applications in the sequel, we present some useful notation, definitions and lemmas to end this section.

We adopt the following notation. For a symmetric matrix M, $M > 0 (M < 0, M \leq 0)$ means that it is positive definite (negative definite, semi-negative definite, respectively). $\mathcal{W}^{1,2}([0,L]; \mathbb{R}^n)$ is a Sobolev space of absolutely continuous n-dimensional vector functions $\omega(x) : [0, L] \to \mathbb{R}^n$ with square integrable derivatives $\frac{d^l \omega(x)}{dx^l}$ of the order $l \geq 1$.

Definition 2.2. The coupled time-delay PDSs (2.1) are of asymptotical synchronization if the synchronization error $e(x,t)$ satisfies $\lim_{t\to\infty} e(x,t) = 0$ for all $x \in [0, L]$.

Lemma 2.3 (*Wu et al. [25]*). *Let $z \in \mathcal{W}^{1,2}([0,L]; R^n)$ be a vector function with $z(0) = 0$ or $z(L) = 0$. Then, for a matrix $S > 0$, one has the following integral inequality:*

$$\int_0^L z^T(s) S z(s)\, ds \leq 4L^2 \pi^{-2} \int_0^L \left(\frac{dz}{ds}\right)^T S \left(\frac{dz}{ds}\right) ds.$$

3. Asymptotical synchronization for coupled time-delay PDSs

In this section, at first, we present an LMI-based criterion on asymptotical synchronization for the coupled node-delay PDSs (2.1). Using the Lyapunov functional method and Wirtinger's inequality, we obtain the sufficient condition to guarantee the synchronization via the boundary control. Then we briefly state a criterion of synchronization for the coupled PDSs with coupling delay.

Now we give the main theorem of this paper.

Theorem 3.1. *For the coupled node-delay PDSs (2.1), if there exists an $m \times n$ matrix K satisfying the following LMI:*

$$\Psi \triangleq \begin{bmatrix} -0.5L^{-2}\pi^2 (I_N \otimes \Theta) & I_N \otimes \Theta CK \\ (I_N \otimes \Theta CK)^T & \Psi_{22} \end{bmatrix} < 0, \tag{3.1}$$

where

$\Psi_{22} \triangleq 2I_{nN} + 2(I_N \otimes B)(I_N \otimes B^T) + 2(I_N \otimes A + G \otimes I_n) - 2(I_N \otimes \Theta CK),$